

Lecture 1: Introduction

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1 The Essence of Computation

What is computation? This is the most fundamental question in computer science. In order to understand what computation is, we need to clarify one common misunderstanding. To illustrate, consider the following example.

EXAMPLE 1.1. *Solve the quadratic equation*

$$x^2 + bx + c = 0 \quad (1)$$

Of course, we can perform some simple algebraic manipulations leading to the familiar quadratic formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \quad (2)$$

But does this bring us any closer to finding a numerical solution? The answer is no; while we can easily perform simple operations like addition and multiplication, it is unclear how we can calculate a square root. So this algebraic manipulation has gotten us nowhere.

1.1 An Approximation Scheme

So how do we calculate a square root? Before we can answer, we must consider two of the fundamental limitations of numerical computation:

- Limited storage - since there is only a limited amount of space to store any number, we cannot always get an *exact* numerical solution. Rather, we can only get a solution that is accurate up to the limit of our precision (for example, 16 digits).
- Limited time - Each operation (e.g. addition, multiplication) takes time to complete. We can only perform a limited number of operations during a particular calculation, or it will take too long to finish.

For example, consider the problem of evaluating a quadratic function $f(x) = ax^2 + bx + c$ at a point. The evaluation will take 5 steps: 3 multiplications ($x * x$, $a * x^2$, $b * x$) and two additions.

To compute a square root, we would like a method that can calculate a numerical solution that is accurate up to the limit of our representation and performs few operations. One such method uses an initial guess at the square root value x_{old} to calculate a more accurate value x_{new} . The calculation can be repeated using the new value as input, producing increasingly accurate solutions.

In later lectures we will see how this method works in general, but for now, consider the following formula for calculating the square root of 5:

$$x_{\text{new}} = \frac{x_{\text{old}}^2 + 5}{2x_{\text{old}}} \quad (3)$$

If we start with an initial value $x_{\text{old}} = 2.2$ and repeat the computation for 3 steps, Matlab produces the following results:

Phase 0:	$x = 2.2$	$x^2 = 4.84$	
Phase 1:	$x_{\text{old}} = 2.2$	$x_{\text{new}} = 2.23636363636364$	$x_{\text{new}}^2 = 5.00132231404959$
Phase 2:	$x_{\text{old}} = 2.23636363636364$	$x_{\text{new}} = 2.23606799704361$	$x_{\text{new}}^2 = 5.00000008740261$
Phase 3:	$x_{\text{old}} = 2.23606799704361$	$x_{\text{new}} = 2.23606797749979$	$x_{\text{new}}^2 = 5.00000000000000$

Using this procedure, only $3 \times 4 = 12$ simple arithmetic operations are required to arrive at a numerical solution that is accurate to the limit of Matlab's precision.