CS 525 - Fall 2011 - Homework 4*

assigned 10/5/11 - due 10/12/11

1. Do Exercise 3-4-2.

2. Do Exercise 3-4-3.

3. Solve the problem in Exercise 4-2-2 by adding dual labels to the tableau and applying Phase I and Phase II in the usual way. (Hint: If you need to add a row and column for Phase I, just use the usual `addrow` and `addcol` commands; the dual labels for the row and column will be left blank, which is OK.)

4. Do Exercise 4-4-3.

5. Consider the standard form LP

$$\begin{array}{ll} \text{minimize} & p^T x \\ \text{subject to} & Ax \geq b \\ & x \geq 0\,. \end{array} \tag{1}$$

   Let $u \in \mathbb{R}^m$, $u \geq 0$.

   (a) Prove that if x is feasible for the LP, then it also satisfies the inequality $u^T A x \geq u^T b$.

   (b) Prove that for any $u \geq 0$, the optimal value of the LP

$$\begin{array}{ll} \text{minimize}_x & p^T x \\ \text{subject to} & (A^T u)^T x \geq b^T u \\ & x \geq 0\,. \end{array} \tag{2}$$

   is less than or equal to the optimal value of (1).

*Hard copy to be submitted **in class** on the due date. No late homework accepted.

(c) Show that (2) is bounded below if $A^T u \leq p$.

(d) **EXTRA CREDIT:** Derive a necessary condition on u such that (2) is bounded below.

(e) **EXTRA CREDIT:** When the LP is bounded, derive an expression for the optimal value of (2). Your expression will depend on the vector u.

(f) **EXTRA CREDIT:** Formulate the problem of finding the best such bound, by maximizing the lower bound over $u \geq 0$ subject to the conditions when the LP (2) is bounded.

(g) **EXTRA CREDIT:** How does the optimal value of the resulting optimization in part (f) problem compare to the optimal value of LP (1)?