CS 525 - Fall 2011 - Homework 8*

assigned 11/16/11 - due 11/30/11

1. Which of the following sets are convex?
   (a) $\{x \in \mathbb{R} \ : \ x^2 = 1\}$.
   (b) $\{x \in \mathbb{R} \ : \ x^2 \leq 1\}$.
   (c) $\{x \in \mathbb{R} \ : \ x^2 \geq 1\}$.

2. Let A be an $m \times n$ matrix. Verify that the set
$$\{x \in \mathbb{R}^n \ : \ Ax = b\,, \ x \geq 0\}$$
is a convex set.

3. A point $x \in \mathbb{R}^n$ is a *convex combination* of points $x_1, \ldots, x_m \in \mathbb{R}^n$ if there are scalars $\lambda_1, \ldots, \lambda_m$ such that
$$x = \sum_{k=1}^{m} \lambda_k x_k \quad \sum_{k=1}^{m} \lambda_k = 1 \quad \lambda_k \geq 0$$
Show that a set C is convex if and only if every convex combination of a a finite number of points from C is contained in C. *Hint: Note that a convex combination of two points is a line segment. Use proof by induction to show that any convex combination of m points lies in C provided that any convex combination of $m-1$ points lies in C.*

4. Prove that $x = (1/2, 1)$ is a global solution of
$$\begin{array}{ll} \text{minimize} & 2x_1^2 + 4x_1x_2 + 5x_2^2 - 6x_1 - 12x_2 \\ \text{subject to} & -1 \leq x_1 \leq 1 \\ & -1 \leq x_2 \leq 1 \end{array}$$
Hint: Use the optimality conditions in Section 7.1

5. Do exercise 7-2-1.

*Hard copy to be submitted **in class** on the due date. No late homework accepted.