CS 525 - Fall 2011 - Homework 9
For extra credit

assigned 11/30/11, due 12/8/11

1. Do Exercise 7-1-3

2. Do Exercise 7-2-2

3. (a) Write down the KKT conditions for the problem

$$\begin{aligned}
\text{minimize} \quad & x_1^2 + x_2^2 + x_3^2 \\
\text{subject to} \quad & x_1 + x_2 + x_3 \geq 1 \\
& x_1 - x_2 - x_3 \geq 1 \\
& x_1 + x_2 - x_3 \geq 1 \\
& x \geq 0
\end{aligned}$$

   (b) Find an optimal primal dual pair which solves the KKT conditions.

4. Consider the equality constrained least-squares problem

$$\begin{aligned}
\text{minimize} \quad & \|Ax - b\|_2^2 \\
\text{subject to} \quad & Gx = h
\end{aligned}$$

   where $A \in \mathbb{R}^{m \times n}$ with $\text{rank}(A) = n$ and $G \in \mathbb{R}^{p \times n}$ with $\text{rank}(G) = p$. Write down the the KKT conditions, and derive expressions for the optimal primal solution $\bar{x}$ and dual solution $\bar{u}$.

5. Consider the quadratic program

$$\begin{aligned}
\text{minimize}_x \quad & c_1 x_1 + c_2 x_2 + c_3 x_3 \\
\text{subject to} \quad & x_1^2 + x_2^2 + x_3^2 \leq 1
\end{aligned}$$

   Here x is the variable and c_1, c_2, and c_3 are constants.

(a) Write down the Lagrangian for this problem. Be careful about the sign of the Lagrange multiplier!

(b) By minimizing with respect to x, write down the dual problem.

(c) Solve the dual problem.

(d) Use the dual optimal solution to solve the original quadratic program.