

CS 525 - Spring 2011 -Midterm Examination
Tuesday, March 8, 2011, 2:30-3:45PM

Each question is worth the same number of points. You may bring on standard-size sheet of paper, handwritten on both sides into the exam. No other electronic devices, notes, or books allowed. **You need to give reasoning and justify all of your answers**, citing the appropriate theorems where necessary.

1. For the following choice of A and b solve the system of equations $Ax = b$. If there are multiple solutions, describe the full solution set. If there are linear dependence relations between the rows of the coefficient matrix, state them.

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & -1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

2. Consider the following linear program

$$\begin{array}{ll} \text{minimize} & 9x_1 + x_2 \\ \text{subject to} & x_1 + x_2 \geq 4 \\ & 3x_1 - x_2 \geq -2 \\ & x_1, x_2 \geq 0 \end{array}$$

- (a) Write down the dual of this problem.
 - (b) Find solutions for the primal and dual.
 - (c) Suppose the right-hand side of the first constraint is changed from 4 to 6. Without performing any additional simplex iterations or referring to the tableau, give a lower bound on the optimal primal objective value of the modified problem. Explain.
3. Consider the following linear program, where c_1, c_2, c_3 are constants:

$$\begin{array}{ll} \text{maximize} & c_1x_1 + c_2x_2 + c_3x_3 \\ \text{subject to} & -1 \leq x_1 \leq 1 \\ & -1 \leq x_2 \leq 1 \\ & -1 \leq x_3 \leq 1 \end{array}$$

- (a) Write down the dual of this problem.
- (b) Write down the KKT conditions for this problem.
- (c) Find optimal solutions of the primal and dual problems that jointly satisfy the KKT conditions.
- (d) Write the optimal cost of the primal problem solely in terms of the constants c_1, c_2, c_3 .