Relational Algebra

Chapter 4, Part A

Relational Query Languages

- *Query languages*: Allow manipulation and retrieval of data from a database.
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- Query Languages != programming languages!
  - QLs not expected to be "Turing complete".
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:

1. *Relational Algebra*: More operational, very useful for representing execution plans.
2. *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-operational, *declarative*.)

☞ *Understanding Algebra & Calculus is key to*
☞ *understanding SQL, query processing!*

Preliminaries

- A query is applied to *relation instances*, and the result of a query is also a relation instance.
  - *Schemas* of input relations for a query are fixed (but query will run regardless of instance!)
  - The schema for the *result* of a given query is also fixed! Determined by definition of query language constructs.
- Positional vs. named-field notation:
  - Positional notation easier for formal definitions, named-field notation more readable.
  - Both used in SQL

Example Instances

- "Sailors" and "Reserves" relations for our examples.
- We'll use positional or named field notation, assume that names of fields in query results are `inherited' from names of fields in query input relations.

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational Algebra

- Basic operations:
  - *Selection* (σ) Selects a subset of rows from relation.
  - *Projection* (π) Deletes unwanted columns from relation.
  - *Cross-product* ($\times$) Allows us to combine two relations.
  - *Set-difference* ($-$) Tuples in reln. 1, but not in reln. 2.
  - *Union* ($\cup$) Tuples in reln. 1 and in reln. 2.
- Additional operations:
  - Intersection, *join*, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, operations can be *composed*! (Algebra is "closed".)

Projection

- Deletes attributes that are not in *projection list*.
- *Schema* of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to eliminate *duplicates*! (Why??)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname,rating}(S2)$

age
35.0
55.5

$\pi_{age}(S2)$

Selection

- Selects rows that satisfy *selection condition.*
- No duplicates in result! (Why?)
- *Schema* of result identical to schema of (only) input relation.
- *Result* relation can be the *input* for another relational algebra operation! (*Operator composition.*)

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$\sigma_{rating>8}(S2)$

sname	rating
yuppy	9
rusty	10

$\pi_{sname,rating}(\sigma_{rating>8}(S2))$

Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be ***union-compatible***:
  - Same number of fields.
  - \`Corresponding' fields have the same type.
- What is the *schema* of result?

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 \cap S2$

Cross-Product

- Each row of S1 is paired with each row of R1.
- *Result schema* has one field per field of S1 and R1, with field names \`inherited' if possible.
  - *Conflict*: Both S1 and R1 have a field called *sid*.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

☛ *Renaming operator*: $\rho\,(C(1 \rightarrow sid1, 5 \rightarrow sid2),\ S1 \times R1)$

Joins

- *Condition Join*: $R \bowtie_c S = \sigma_c(R \times S)$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$S1 \bowtie_{S1.sid < R1.sid} R1$

- *Result schema* same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently
- Sometimes called a *theta-join*.

Joins

- *Equi-Join*: A special case of condition join where the condition *c* contains only ***equalities***.

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

$S1 \bowtie_{sid} R1$

- *Result schema* similar to cross-product, but only one copy of fields for which equality is specified.
- *Natural Join*: Equijoin on *all* common fields.

Division

- Not supported as a primitive operator, but useful for expressing queries like:
  *Find sailors who have reserved **all** boats.*
- Let A have 2 fields, x and y; B have only field y:
  - $A/B = \{\langle x \rangle \mid \exists \langle x, y \rangle \in A \ \forall \langle y \rangle \in B\}$
  - i.e., **A/B contains all x tuples (sailors) such that for *every* y tuple (boat) in B, there is an xy tuple in A.**
  - *Or*: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B, the x value is in A/B.
- In general, x and y can be any lists of fields; y is the list of fields in B, and $x \cup y$ is the list of fields of A.

Examples of Division A/B

A

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

B1

pno
p2

B2

pno
p2
p4

B3

pno
p1
p2
p4

A/B1

sno
s1
s2
s3
s4

A/B2

sno
s1
s4

A/B3

sno
s1

Expressing A/B Using Basic Operators

- Division is not essential op; just a useful shorthand.
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- *Idea*: For A/B, compute all x values that are not \`disqualified' by some y value in B.
  - x value is *disqualified* if by attaching y value from B, we obtain an xy tuple that is not in A.

Disqualified x values: $\pi_x((\pi_x(A) \times B) - A)$

A/B: $\pi_x(A) -$ all disqualified tuples

Find names of sailors who've reserved boat #103

- Solution 1: $\pi_{sname}((\sigma_{bid=103} \text{Reserves}) \bowtie Sailors)$
- Solution 2: $\rho\ (Temp1, \sigma_{bid=103} \text{Reserves})$

  $\rho\ (Temp2, Temp1 \bowtie Sailors)$

  $\pi_{sname}(Temp2)$
- Solution 3: $\pi_{sname}(\sigma_{bid=103}(\text{Reserves} \bowtie Sailors))$

Find names of sailors who've reserved a red boat

- Information about boat color only available in Boats; so need an extra join:

  $\pi_{sname}((\sigma_{color='red'} Boats) \bowtie \text{Reserves} \bowtie Sailors)$
- A more efficient solution:

  $\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'} Boats) \bowtie \text{Res}) \bowtie Sailors)$

☛ *A query optimizer can find this given the first solution!*

Find sailors who've reserved a red or a green boat

- Can identify all red or green boats, then find sailors who've reserved one of these boats:

  $\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'} Boats))$

  $\pi_{sname}(Tempboats \bowtie \text{Reserves} \bowtie Sailors)$
- Can also define Tempboats using union! (How?)
- What happens if $\vee$ is replaced by $\wedge$ in this query?

Find sailors who've reserved a red *and* *a green boat*

- Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):

$$\rho\ (Tempred, \pi_{sid}((\sigma_{color='red'} Boats) \bowtie Reserves))$$

$$\rho\ (Tempgreen, \pi_{sid}((\sigma_{color='green'} Boats) \bowtie Reserves))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

Find the names of sailors who've reserved all boats

- Uses division; schemas of the input relations to / must be carefully chosen:

$$\rho\ (Tempsids, (\pi_{sid,bid} Reserves) / (\pi_{bid} Boats))$$

$$\pi_{sname}(Tempsids \bowtie Sailors)$$

- To find sailors who've reserved all 'Interlake' boats:

$$\ldots\ldots \ / \ \pi_{bid}(\sigma_{bname='Interlake'} Boats)$$

Summary

- The relational model has rigorously defined query languages that are simple and powerful.
- Relational algebra is more operational; useful as internal representation for query evaluation plans.
- Several ways of expressing a given query; a query optimizer should choose the most efficient version.