Comments on the draft of Applied Linear Algebra
by Carl de Boor

With an author of the stature of Carl de Boor one accepts an opportunity to share the particular vision of the subject matter that he has acquired. Suggestions for improvements here and there seem inappropriate. The garment is seamless.
In 1997 Wiley(Science) published Linear ALgebra by Peter D. Lax. Peter confided to me that his was the least succesful mathematics book that Wiley has ever published. In fact it is a lovely book that is on my shelves and has opened my eyes to several aspects of the area in which I work. The explanation (of poor sales) is not hard to find.
At the Courant Institute only graduate level courses are offered, so a level of sophistication is assumed that is well beyond that of most undergraduates in the U.S.A.. My guess is that no one adopted the book as a text for a course because Linear Algebra is almost always a junior level course and, sinking lower) the students are expected to have
encountered Elimination and Null Spaces as part of sophomore Calculus.
I expect that it was only specialists in (and teachers of) Linear Algebra
who bought the book.

I expect a similar fate for de Boor's book. People will buy it who know and admire the author. Instructors will shun it because the book is, above
all, INTIMIDATING. As I read it I feel that I am not worthy to share in the mysteries of Linear Algebra. Anyone with the smarts and the stamina
to master this book will emerge with a superb arsenal of weapons to attack open research problems involving Linear Algebra. The authors approach is
iconoclastic. He attaches (quite rightly) great weight to notation and is not afraid to break away from tradition in order to obtain more economy and elegance than do most presentations.

Let me give an illustration of what I mean by "intimidating". THere is an
illuminating example on p.177 of the Euclidean algorithm via elimination.
The key words, striking fear to the heart, are "It turns out that". Gulp.
The autor then displays only the crucial part of the system of equations.
How they turned up is not immediately clear. I would have written:
Now write out pq = f - r as a set of n linear equations that govern the coefficients of unknown q. Since r is also unknown the first d-1 equations
are no use but, minor miracle, r drops out of the picture for equations d:n and, write these out explicitly, what remains is upper triangular and
invertible (because ....) and suffices to determine uniquely the coefficients
of q. The author could say that I am too soft and hold too many hands.

I could say much about Prerequisites and Background. Imagine, if you can, a junior level engineer who has never met the notion of upper semicontinuity.
THis is part of the background Theorem(17.8) also on p.177. I cannot see why such a student could not succesfully master the important ideas of bound and free unknowns and of ran and null despite such shocking ignorance.
Even more startling is the mention of Noetherian rings and Hilbert's basis theorem.
I bet that over half the instructors of Linear Algebra do not know these terms and I cannot see why they are relevant to the applications of Linear Algebra.
AS a matter of fact, as a graduate student I took an advanced course on Ring Theory and took my oral exam on Group THeory adn Group Representations so I am not reacting through ignorance.
Even though I have taught Linear Algebra for many years I did not know what an Assignment is before I read this book and the author does not seem to allow for the fact that students are advised to take Linear Algebra BEFORE Real Analysis because it is an easier subject.

When I was a student texts, by pure mathematicians, aimed to teach Linear Algebra withput mentioning a matrix, except perhaps in the last week of the course. I still think this is the nicest way to understand (but perhaps not to teach) the subject. THe author may point out that he brings in matrices from the beginning and perhaps that is why his title is Applied Linear Algebra. Frankly I found the applciations section small and disappointing. Markov chains are an important and fashoinable topic of reserch at the moment. Yet we get barely half a page which I would summarise as Perron-Frobenius lite. No indication of why random walks on graphs are posing serious challenges.

I would like to say a word about RIGOR, a quality dear to the author's heart.
For myself, when I have eventually understood some topic then I find great pleasure in laying out the arguments in the most economical and elegant manner.
However to get to that understanding formality just gets in the way. To the outsider rigor seems to mean being very pedantic about trivial details.
In most cases it is only one or two details that are tricky and, to a student,

it is not clear which are the key points.   Why do I say all this?
I think a student would say to herself: my God, I have to internalize so many
maps with rather subtle differences in order to get started on this subject,
I doubt I will ever do it. The start up price is just too high. Are column
maps and row maps fancy names for elements of a vector space and for linear
functionals or are there subtle differences?

I have been critical of the book from the point of view of a student but,
for myself I love some of the notational devices: there should be a universal
notation for the monomials that hides the variable. Pete Stewart favors the
numeral 1 turned backwards for the identity and \reverse1 ^j for the jth power.
The authors ()^j takes up more room but is self explanatory. Also I like ? for
the unknowns. I wish there were more on the matrices invoved in both univariate
and multivariate splines.

Almost every page elicits some response from me but I think I have given enough
to help you.