

Index

1-1: 3

A

a posteriori error estimates: 169
 A -invariant: 150, 152
 a.e.: 69
 absolutely homogeneous: 5, 9, 53
 absorbing: 104
 addition: 1, 5
 additivity of a map: 2, 6
 affine: 102, 163
 affine combination: 102
 affine hull of M : 102
 agrees with g on L : 21, 25
 AGTASMAT: 108, 135
 Alaoglu's theorem: 63, 112, 119
 algebraic complement: 10, 14
 almost everywhere: 69
 Alternation Theorem: 110
 analysis: 8, 11
 angle: 115
 angle preserving: 153
 annihilate: 151
 annihilator: 16, 20
 Anselone's Theorem: 141
 approximate (left) inverse: 60
 approximate evaluation of linear functional: 19, 23
 approximate identity: 84, 136
 approximate inverse: 60, 135, 140, 173
 approximation power: 87
 Approximation Theory: 72, 134
 argmin: 124
 Arzela-Ascoli: 49
 associative: 3
 Axiom of Choice: 11, 14

B

ba: 35
 ba from convex set in ips: 118
 ba wrto a discrete norm: 110
 Babuška-Brezzi Condition: 129
 Baire Category theorem: 87
 balanced: 105
 ball of radius r and center Y : 35
 Banach space: 83
 Banach-Steinhaus: 90
 basic wisdom: 15
 basis: 13
 basis for X : 13
 basis map: 13
 Bessel's inequality: 130

best approximation: 35
 best approximation from L to μ : 73
 best rule in the sense of Sard: 72
 bi-orthonormal: 17
 bidual: 11, 78
 bilinear fl: 129
 bilinear form: 115
 bound for a lm, lower: 55, 60
 bound for a lm, upper: 55
 boundary: 32
 bounded: 54
 bounded below: 55, 56, 129
 bounded bilinear form: 128
 bounded linear map: 54, 90
 bounded map: 54, 90
 Bounded Pointwise Convergence: 84
 bounded quadratic form: 125
 bounded sequence: 38
 bounded set: 35
 bounded set in nls: 90
 bounded variation: 70
 bounded, one function by another: 77
 boundedly compact: 47, 50
 boundedly invertible: 56
 Brakhage: 141
 broken line: 65, 139, 145
 broken line interpolant: 37
 Brouwer fixed point theorem: 173
 Bs: 83
 BVP: 95

C

c(onjugate) dual: 153
 canonical: 11
 canonical embedding: 78
 Caratheodory's Theorem: 103
 cardinality: 2
 cartesian product: 2, 97
 category, first or second: 87
 Cauchy: 40
 Cauchy sequence: 40
 Cauchy's formula: 158
 Cauchy-Bunyakovski-Schwarz: 117
 CBS: 117
 centralized wrto Y : 46
 chain rule: 165
 characteristic function: 4
 characteristic function of the set I : 131
 characterization of ba from a convex set: 106
 characterization of ba from lss in ips: 118
 characterization of continuity: 33

Chebyshev Alternation Theorem: 110
 Chebyshev points: 93
 closed: 32
 Closed Graph Theorem: 97
 closed interval: 3
 closed interval $[x \dots y]$ in a ls: 101
 closed lss: 57
 closed map: 97
 closed range: 56, 95, 137
 closure: 32
 closure point: 32
 cluster point: 32
 codimension: 10, 14
 coercive: 128
 coercive bilinear form: 128
 coercive quadratic form: 125
 collectively compact: 141
 collocation: 134, 170
 column: 2, 8
 column map: 8
 compact: 45
 compact map: 135
 compact perturbation of the identity: 137
 complement: 32
 complement of Y in X : 2
 complete: 41, 85, 130
 completely continuous: 135
 completion: 125, 127
 completion of a ms: 41
 complex: 5
 composite trapezoidal rule: 23
 composition: 3
 computing the norm of a lm: 59
 computing the representer: 124
 conic section: 26
 conjugate index: 67
 consistent: 91
 constructive instance of HB: 107
 continuity of a map, definition of: 30
 continuity, characterization of: 33
 continuity, uniform: 36
 continuous: 30
 continuous dual: 63
 contraction, summary on: 43
 convergence of a sequence in ms: 37
 convergence of a sequence in ts: 38
 convergence, of a filter basis: 39
 convergence, quadratic: 169
 convergent: 91, 149
 convergent to 0: 149
 converges (in norm) to: 83
 converges pointwise: 83
 converges strongly: 83
 converges to: 37, 38

convex combination: 102
 convex fl: 111
 convex hull: 101
 convex set: 101
 contraction: 40
 coordinate fl: 13, 17
 coordinate space: 5
 coordinates: 13
 correct: 25
 countable: 87
 cubic spline interpolation: 123

D

data map: 16
 defective: 149
 degree of a rule: 26
 degree of approximation: 85, 87
 dense (in T): 32
 derivative, Fréchet: 163
 diagona(liza)ble: 149
 diameter: 35
 difference, of two sets in a ls ($M - N$): 5, 9
 difference, set-theoretic ($M \setminus N$): 5, 9
 differentiation of a map: 163
 dimension: 14
 Dimension Formula: 15
 dimension of \mathbb{F}^n : 14
 Dini-Lipschitz: 93
 direct sum: 14
 direct sum decomposition: 14, 18
 directional derivative: 164
 Dirichlet: 92, 127
 discrete topology: 30
 discretization: 169, 173
 disjoint projectors: 158
 distance: 35
 distance from noninvertible lm's: 61
 divergence: 91
 divided difference: 27, 79, 110
 domain: 3
 dual: 10, 16, 17, 20, 21, 25, 78
 dual basis: 66
 dual is always complete: 63
 dual problem: 79
 Duality formula for distance from convex set: 107
 Duality in Approximation Theory: 79, 128
 Dubovitskii-Milyutin Separation Theorem: 106

E

eigenpair: 149
 eigenstructure of a compact lm: 160
 eigenvalue: 149
 eigenvector: 149
 elimination: 64, 116

elliptic pde: 127
 embedding: 11
 empty sequence: 2
 empty set: 2
 epigraph: 111
 epimorphism: 3
 equality of maps: 3
 equicontinuous: 49
 equivalent norms: 57, 96
 equivalent topologies: 31
 error estimate: 65
 Euclidean: 30, 34
 Euler equation: 133
 Euler's method: 86
 everywhere dense: 87
 existence of ba: 65
 expanded Chebyshev points: 93
 extended: 5, 9
 extension by continuity: 85, 113
 extreme point: 112
 extreme set: 113

F

Factor Lemma: 6, 10
 factor map: 10, 58, 95
 factor norm: 58
 factor space: 54, 58
 factoring: 26
 factorization: 19
 factorization, minimal: 19
 Farkas' Lemma: 106
 filter basis: 39
 finest: 152
 Finite Intersection Property: 113
 Finite Intersection property w.r.to Y : 46
 finitely generated: 14
 FIP: 113
 first kind: 137
 first-order ODE: 43
 first-order system: 100
 fixed point: 39, 60
 fixed point iteration: 39, 60, 83, 94, 149
 flat: 102
 flat spanned by M : 102
 form: 115
 form is function(al): 115
 Fourier series: 92
 Fredholm Alternative: 15, 138
 Fréchet derivative: 163
 Fréchet space: 54
 full Fredholm Alternative: 138
 function: 3, 6
 functional: 10, 115
 functional, convex: 111

fundamental: 130
 fundamental set: 85
 Fundamental Theorem of Calculus: 9, 74

G

Galerkin: 133
 Galerkin's method: 134, 144
 Gateaux: 164
 Gateaux-differentiable at u : 164
 Gauss rule: 91, 108
 generalized eigenspace of A belonging to z : 159
 Golomb-Weinberger interval: 73
 gradient: 127, 164
 Gram-Schmidt process: 131
 Gram-Schmidt-orthogonalization: 28, 120
 Gramian: 17, 120
 graph: 97
 Green's function: 142, 146
 GW-interval: 123

H

Haar space: 110
 Haar's system: 131
 Hahn-Banach Theorem: 76
 Hahn-Banach theorem, general: 78
 half spaces: 107
 Hamel basis: 15, 21, 89
 Hausdorff: 47
 Hausdorff Maximality Theorem: 15, 21, 51
 HB: 76
 HB Theorem for $C(T)$: 107
 Heaviside: 70
 Heine-Borel Theorem: 48
 Hermitian: 153
 hermitian: 115, 154
 higher order: 144
 Hilbert space: 117
 homogeneity of a map: 6
 Hs: 117
 hypercircle: 70, 122
 hyperplane: 64
 Hölder's inequality: 67, 69

I

idempotent (map): 18
 identify $(\mathbb{F}^n)'$ with \mathbb{F}^n : 11
 identity map: 3
 identity, useful: 62, 65
 ill posed: 136
 image: 3
 Implicit Function Theorem: 172
 inadequacy of rules: 24
 increasing sequence of lprojectors: 89
 indiscrete (or, trivial) topology: 30

initial guess: 39
 injective: 3
 inner product: 115
 inner product space: 115
 integral equation: 39
 integral equations of the second kind: 137
 integration by parts: 126
 interior: 32
 interior point of: 32
 interior, relative: 103
 interpolates: 19
 interpolation: 19, 23, 70, 119, 133
 interpolation functionals: 25, 27
 invariant subspace: 157
 inverse: 3
 Inverse Function Theorem: 173
 inverse of a basis: 17
 invertible: 3
 ips: 115
 isolated eigenvalue: 159
 isolated point: 32
 isolating parts of the spectrum: 157
 isometric: 67, 85
 isometry: 63, 66, 78
 iterated projection method: 146

J

Jackson Theorem: 89
 Jacobian: 164
 Jordan canonical form: 152

K

Kantorovich: 169
 kernel: 8, 20, 39
 Krasnoselski: 140
 Kronecker delta: 17

L

Lagrange form: 23, 27, 110
 Lagrange polynomial: 27
 Laplacian: 126
 Lax-Milgram lemma: 129
 least-squares approximation: 27
 least-squares method: 134
 Lebesgue function: 93
 Lebesgue inequality: 72
 Lebesgue integrable: 68
 Lebesgue integration: 69
 Lebesgue measure: 68
 Lebesgue's Inequality: 134
 left inverse: 3
 Legendre polynomials: 131
 lfl: 10
 lfl, approximate evaluation of: 23

lim inf: 45
 lim inf Y_n : 124, 126
 lim inf of a sequence of lss's: 124
 lim sup: 45
 limit inferior: 45
 limit point: 44
 limit superior: 45
 linear: 6, 10, 54, 119
 linear combinations: 12
 linear hull: 13
 linear independence, test for: 22
 Linear Interpolation Problem: 25
 linear iteration: 43
 linear map: 6
 linear space: 5, 6
 linear subspace: 5
 linearly independent: 13
 linearly isometric: 67
 Lipschitz: 37
 Lipschitz constant: 37
 lm: 6
 lower bound: 60
 lower semicontinuous: 47, 90
 ls: 5, 6
 lss: 5
 LU factorization: 27

M

map: 3
 map norm: 55
 mapping: 6
 matrix: 2, 7, 66
 matrix representation: 149
 maximally 1-1: 13
 meagre: 87
 Meanvalue Estimate: 165
 measure: 68
 metric: 33
 metric space: 33
 metrizable: 50
 minimal (annihilating) polynomial: 152
 minimal factorization: 19
 minimizing: 118
 Minkowski fl: 104
 Minkowski's inequality for integrals: 70
 modulus of continuity: 36
 modulus of continuity, uniform: 36
 moment method: 134
 monic: 150
 monomials: 4, 8, 87
 monomorphism: 3
 monotone norm: 80
 ms: 33
 multiplier: 155

N

n-sequence: 2, 5
n-simplex: 103
n-tail: 38
 Neighborhood Assumption 1: 30
 Neighborhood Assumption 2: 32
 neighborhood system: 29
 net: 39
 Neumann series: 61, 156
 Neumann, von: 117
 Newton form: 23, 27
 Newton map: 167
 Newton's method: 167
 Newton-Cotes rule: 91
 nilpotent: 159
 nls: 53
 nodes: 72, 91
 nonlinear functional analysis: 173
 norm: 53
 norm metric: 53
 norm of a linear projector: 80
 norm, strictly convex: 112
 norm-preserving: 66
 normal: 120, 154
 normal equations: 27, 120
 normalized: 66
 Normalized Bounded Variation: 70
 normed linear space: 53
 not meagre: 87
 not thin: 87
 nowhere dense: 87
 nullspace: 8
 numerics: 11, 22, 26, 120, 129, 159, 170
 Nystrom's method: 140, 142

O

o.n.: 130
 ODE, *m*th order: 145
 ODE, nonlinear: 170
 of first (Baire) category: 87
 of radius: 30
 of second category: 87
 one-one: 3, 8
 onto: 3
 open: 32, 56, 97
 open ball around: 30
 open ball of radius r and center x : 35
 open interval: 3
 open map: 93
 Open Mapping Theorem: 96
 Open mapping/closed graph: 93
 operator: 6
 optimal: 121, 137
 optimal interpolation: 121

optimal recovery: 72
 optimality: 125
 order: 19
 orthogonal: 20, 116, 120
 orthogonal complement: 116, 119
 orthogonal projector: 118
 orthonormal: 28, 130

P

pair: 98
 parallel: 66
 Parallelogram Law: 117
 Parseval's Identity: 131
 PDE: 147
 perturbation: 134
 Picard iteration: 100
 Picard's iteration function: 43
 place holder: 4
 point spectrum: 149
 pointwise: 97
 pointwise bounded: 90
 pointwise convergence: 31, 39, 50, 83
 pointwise vector operations: 5, 97
 Poisson's equation: 126
 polarization: 116
 polynomial interpolation: 23, 26, 111
 positive definite: 33, 53, 115
 positive homogeneous: 77
 power form: 110
 power function: 4
 powerbounded: 149
 precision of a rule: 26
 preimage: 3
 product metric: 50
 product topology: 51
 projected equation: 134, 138
 projection method: 134, 141
 projector: 18
 proper convex combination: 112
 proper simplex: 103
 Pythagoras: 116

Q

quadratic convergence: 169
 quadrature: 91, 108, 140
 quadrature rule: 72
 quantification of continuity: 36
 question mark: 4
 quotient map: 9, 58
 quotient space: 9

R

r-net: 48
 range: 3
 rank: 19

rank-one linear map: 61
 Rayleigh-Ritz method: 125
 refines: 29, 69
 reflexive: 78
 regularization: 136
 relative interior: 103
 Remes algorithm: 110
 representation: 66
 representation of X' : 7, 11
 representer: 66, 124
 reproducing kernel: 123
 reproducing kernel Hs: 123
 residual: 60, 134
 residual reduction method: 134
 resolvent: 156
 restriction: 3, 10
 Riemann integration: 69
 Riesz Representation Theorem: 121
 Riesz' Lemma: 57
 right inverse: 3
 row: 2, 16
 row map: 16
 rule: 71, 91
 rule construction: 24
 Rule Construction Problem: 25
 rule for μ : 24
 rule, interpolatory: 26

S

saddle point: 55
 Sard interval: 73
 Sard's best rule: 72, 123
 scalar multiplication: 5, 6
 scalars: 6
 scale invariance: 54
 Schauder basis: 89, 91
 Schauder fixed point theorem: 173
 Schauder independent: 89
 Schauder span: 89
 Schur form: 153, 154
 second kind: 137, 139, 143
 second-kind integral equation: 39
 semi-inner product space: 117
 semicontinuous, lower: 47
 semicontinuous, upper: 47
 seminorm: 9, 53
 seminorm, extended: 9
 separable: 77
 separate points: 113
 Separation Theorem: 105
 sequence: 37
 sequentially compact: 45
 set of measure zero: 68
 set, strictly convex: 112

sharp: 56, 59, 74
 signum: 67
 similar: 149, 153
 simplex, proper: 103
 Singular Value Decomposition: 162
 singular values: 162
 size: 35
 skew-linear: 115
 skew-symmetric: 115
 spaces of smooth functions: 98
 Span: 89
 span of: 12
 spanning: 12
 Spectral Mapping Theorem: 150
 spectral projector: 150, 158
 spectral radius: 156
 Spectral Theorem for compact normal maps: 160
 Spectral Theorem for hermitian matrices: 154
 Spectral Theorem for normal matrices: 154
 spectrum: 155
 spectrum of a compact lm: 137
 spline interpolation, cubic: 123
 splitting: 39, 145
 Stability of difference schemes for PDE: 173
 stable: 91, 95, 137
 starlike: 102
 strictly convex: 112
 strong convergence: 83
 strong topology: 84
 stronger topology: 30
 subadditive: 9, 53, 77
 sublinear: 77
 subsequence: 45
 sum of sets in a ls ($M + N$): 5, 9
 superadditive: 78
 superconvergence: 147
 superlinear: 78
 support functional: 107
 support of a function: 3
 supremum of a subset of \mathbb{R} : 33
 surjective: 3
 SVD: 162
 symmetric: 33, 90
 Synge's hypercircle: 122
 synthesis: 8, 11

T

take away: 2
 take on one's norm: 59
 target: 3
 test for linear independence: 18, 22
 Theorem of the Alternative: 106, 106
 thick: 87
 thin: 87
 topological space: 30

topology of pointwise convergence: 39, 82
 topology, discrete: 30
 total degree: 9
 totally bounded: 48, 135
 touch: 163
 transfinite induction: 77
 transformation: 6
 translation invariance: 53
 transpose: 3, 16, 20
 trapezoidal rule: 65, 75
 triangle inequality: 33, 53, 117
 truncated Taylor series: 23
 truncation: 70
 truncation projector: 68
 ts: 30
 Tykhonov's Theorem: 50
 T_0 -space: 47
 T_1 -space: 47
 T_2 -space: 47

U

Uniform Boundedness Principle: 90
 uniform continuity: 36
 uniform convergence: 31
 uniform metric: 42
 uniform modulus of continuity: 36
 uniformly absorbing: 105
 uniformly bounded: 90
 uniformly compact: 141, 142
 uniformly elliptic: 128
 uniqueness: 40

unit ball: 54
 unit sphere: 55, 56
 unit vector: 7
 unitary: 153
 upper bound: 59
 upper semicontinuous: 47
 useful identity: 62, 65

V

value: 3
 Vandermonde: 26
 Vandermonde matrix: 27
 variation: 70
 vector addition: 6
 vector-valued: 100
 vectors: 6
 Volterra: 98, 155

W

weak: 126
 weak convergence: 84, 90, 136
 weak formulation: 125
 weak topology: 84
 weak* convergence: 84, 90
 weak*-topology: 82
 Weierstrass' Approximation Theorem: 87
 weights: 72, 91

Z

Zeitgeist: 181