

Ensembles of Classifiers

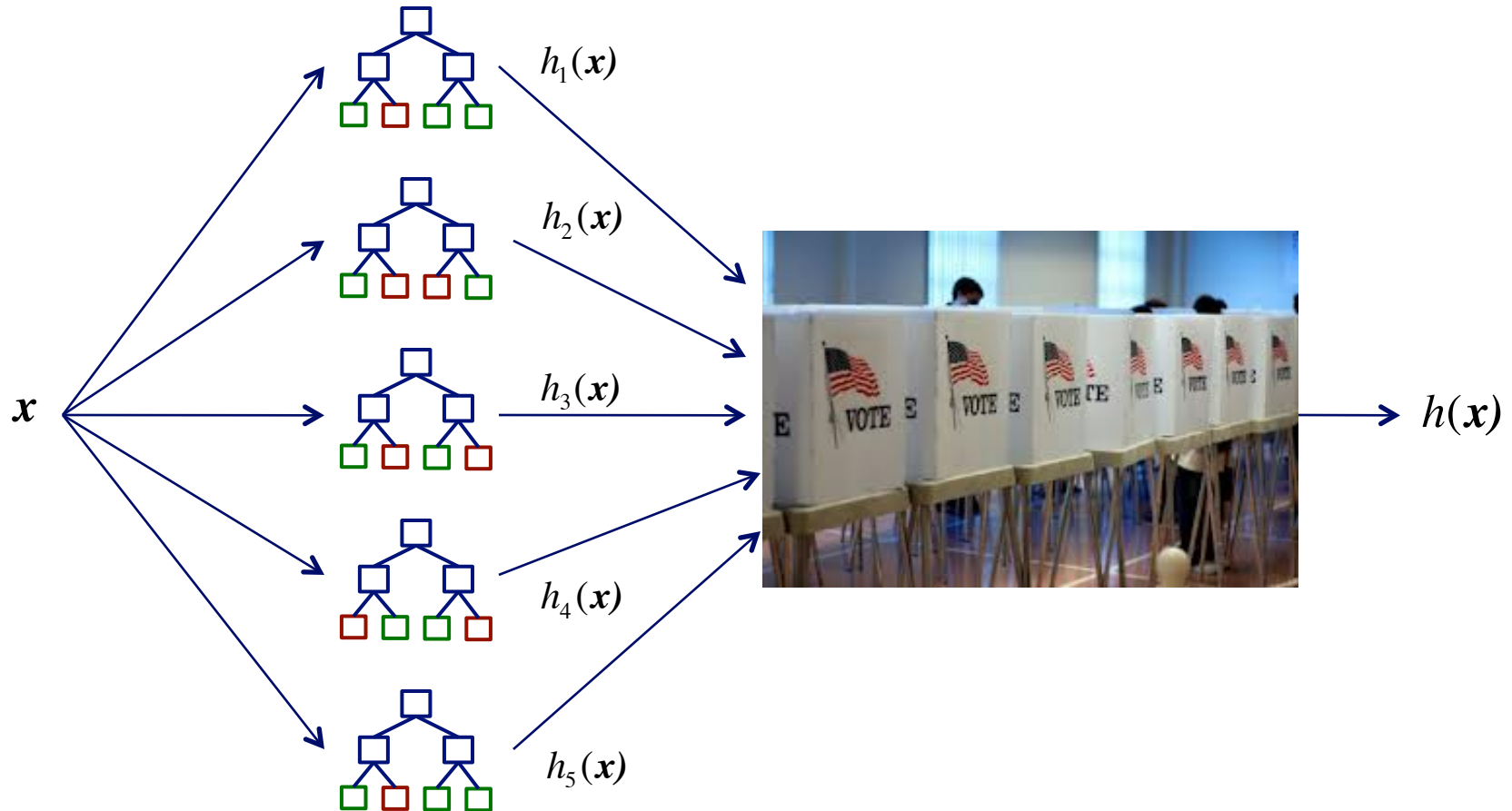
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Goals for the lecture

you should understand the following concepts

- ensemble
- bootstrap sample
- bagging
- boosting
- random forests
- error correcting output codes

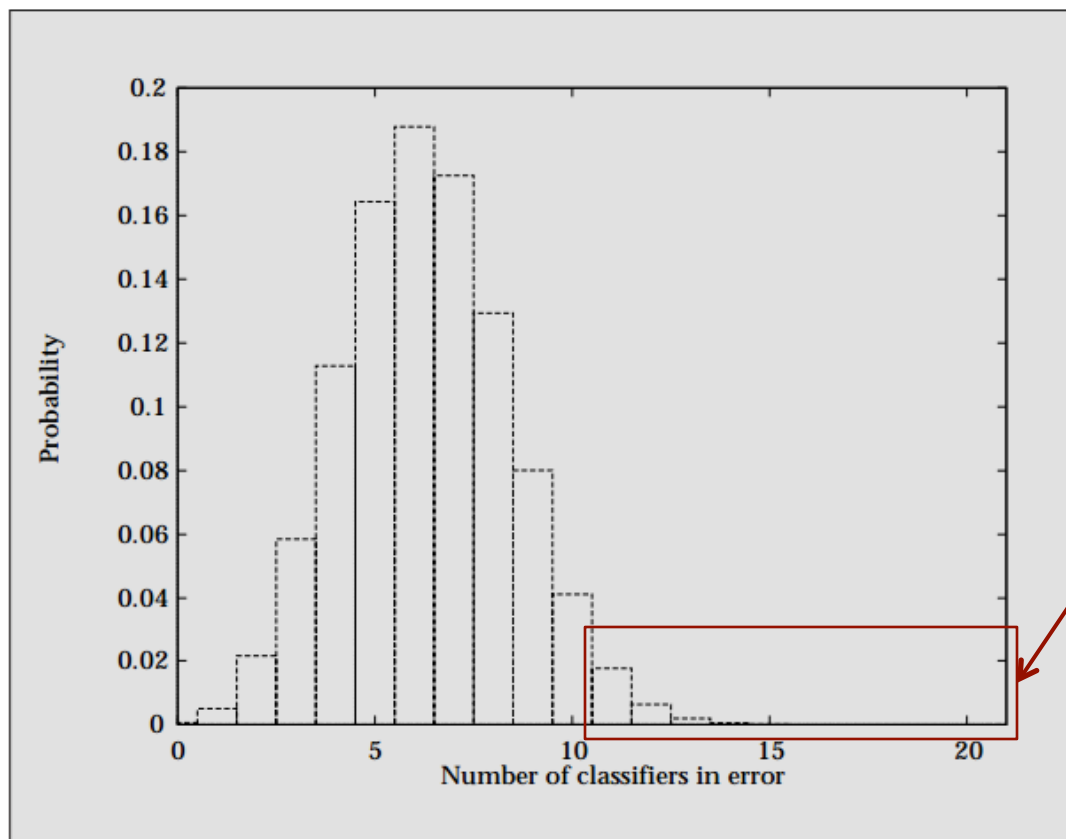
What is an ensemble?



a set of learned models whose individual decisions are combined in some way to make predictions for new instances

When can an ensemble be more accurate?

- when the errors made by the individual predictors are (somewhat) uncorrelated, and the predictors' error rates are better than guessing (< 0.5 for 2-class problem)
- consider an idealized case...



error rate of ensemble
is represented by
probability mass in this box
= 0.026

Figure 1. The Probability That Exactly ℓ (of 21) Hypotheses Will Make an Error, Assuming Each Hypothesis Has an Error Rate of 0.3 and Makes Its Errors Independently of the Other Hypotheses.

How can we get diverse classifiers?

- In practice, we can't get classifiers whose errors are completely uncorrelated, but we can encourage diversity in their errors by
 - choosing a variety of learning algorithms
 - choosing a variety of settings (e.g. # hidden units in neural nets) for the learning algorithm
 - choosing different subsamples of the training set (*bagging*)
 - using different probability distributions over the training instances (*boosting*)
 - choosing different features and subsamples (*random forests*)

Bagging (Bootstrap Aggregation)

[Breiman, *Machine Learning* 1996]

learning:

given: learner L , training set $D = \{ \langle \mathbf{x}_1, y_1 \rangle \dots \langle \mathbf{x}_m, y_m \rangle \}$

for $i \leftarrow 1$ to T do

$D^{(i)} \leftarrow m$ instances randomly drawn with replacement from D

$h_i \leftarrow$ model learned using L on $D^{(i)}$

classification:

given: test instance x

predict $y \leftarrow$ plurality_vote($h_1(x) \dots h_T(x)$)

regression:

given: test instance x_t

predict $y \leftarrow$ mean($h_1(x) \dots h_T(x)$)

Bagging

- each sampled training set is a *bootstrap replicate*
 - contains m instances (the same as the original training set)
 - on average it includes 63.2% of the original training set
 - some instances appear multiple times
- can be used with any base learner
- works best with *unstable* learning methods: those for which small changes in D result in relatively large changes in learned models

Empirical evaluation of bagging with C4.5

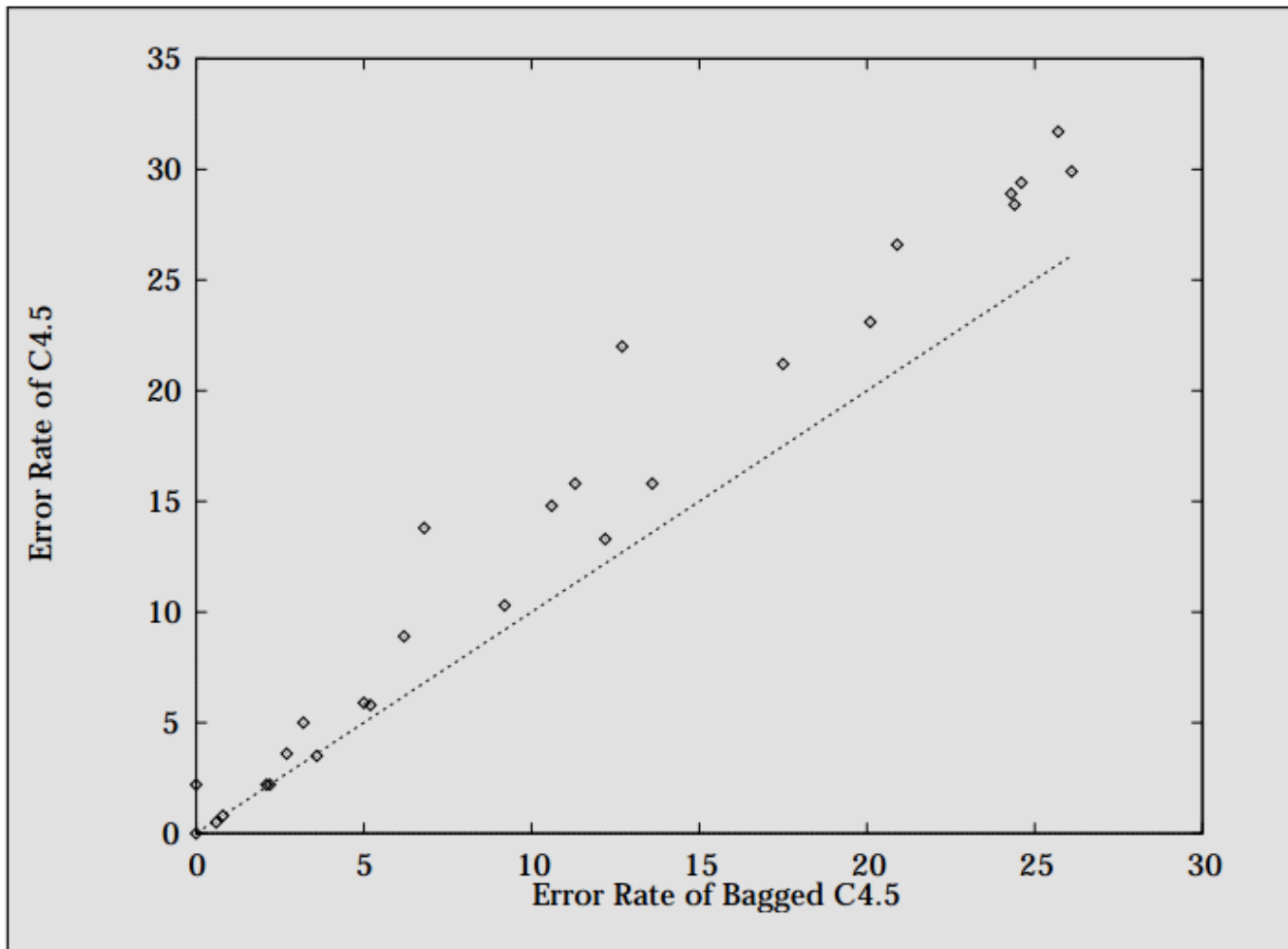


Figure from Dietterich, *AI Magazine*, 1997

Bagging reduced error of C4.5 on most data sets; wasn't harmful on any

Boosting

- Boosting came out of the PAC learning community
- A *weak PAC learning* algorithm is one that cannot PAC learn for arbitrary ε and δ , although its hypotheses are slightly better than random guessing
- Suppose we have a *weak PAC learning* algorithm L for a concept class C . Can we use L as a subroutine to create a strong PAC learner for C ?
 - **Yes, by boosting!** [Schapire, *Machine Learning* 1990]
 - The original boosting algorithm was of theoretical interest, but assumed an unbounded source of training instances
- A later boosting algorithm, AdaBoost, has had notable practical success

AdaBoost

[Freund & Schapire, Journal of Computer and System Sciences, 1997]

given: learner L , # stages T , training set $D = \{ \langle \mathbf{x}_1, y_1 \rangle \dots \langle \mathbf{x}_m, y_m \rangle \}$

```
for all  $i$  :  $w_1(i) \leftarrow 1/m$  // initialize instance weights
for  $t \leftarrow 1$  to  $T$  do
    for all  $i$  :  $p_t(i) \leftarrow w_t(i) / (\sum_j w_t(j))$  // normalize weights
     $h_t \leftarrow$  model learned using  $L$  on  $D$  and  $p_t$ 
     $\varepsilon_t \leftarrow \sum_i p_t(i) (1 - \delta(h_t(\mathbf{x}_i), y_i))$  // calculate weighted error
    if  $\varepsilon_t > 0.5$  then
         $T \leftarrow t - 1$ 
        break
     $\beta_t \leftarrow \varepsilon_t / (1 - \varepsilon_t)$ 
    for all  $i$  where  $h_t(\mathbf{x}_i) = y_i$  // downweight correct examples
         $w_{t+1}(i) \leftarrow w_t(i) \beta_t$ 
```

return:

$$h(\mathbf{x}) = \arg \max_y \sum_{t=1}^T \left(\log \frac{1}{\beta_t} \right) \delta(h_t(\mathbf{x}), y)$$

Implementing weighted instances with AdaBoost

- AdaBoost calls the base learner L with probability distribution p_t specified by weights on the instances
- there are two ways to handle this
 1. Adapt L to learn from weighted instances; straightforward for decision trees and naïve Bayes, among others
 2. Sample a large ($\gg m$) unweighted set of instances according to p_t ; run L in the ordinary manner

Empirical evaluation of boosting with C4.5

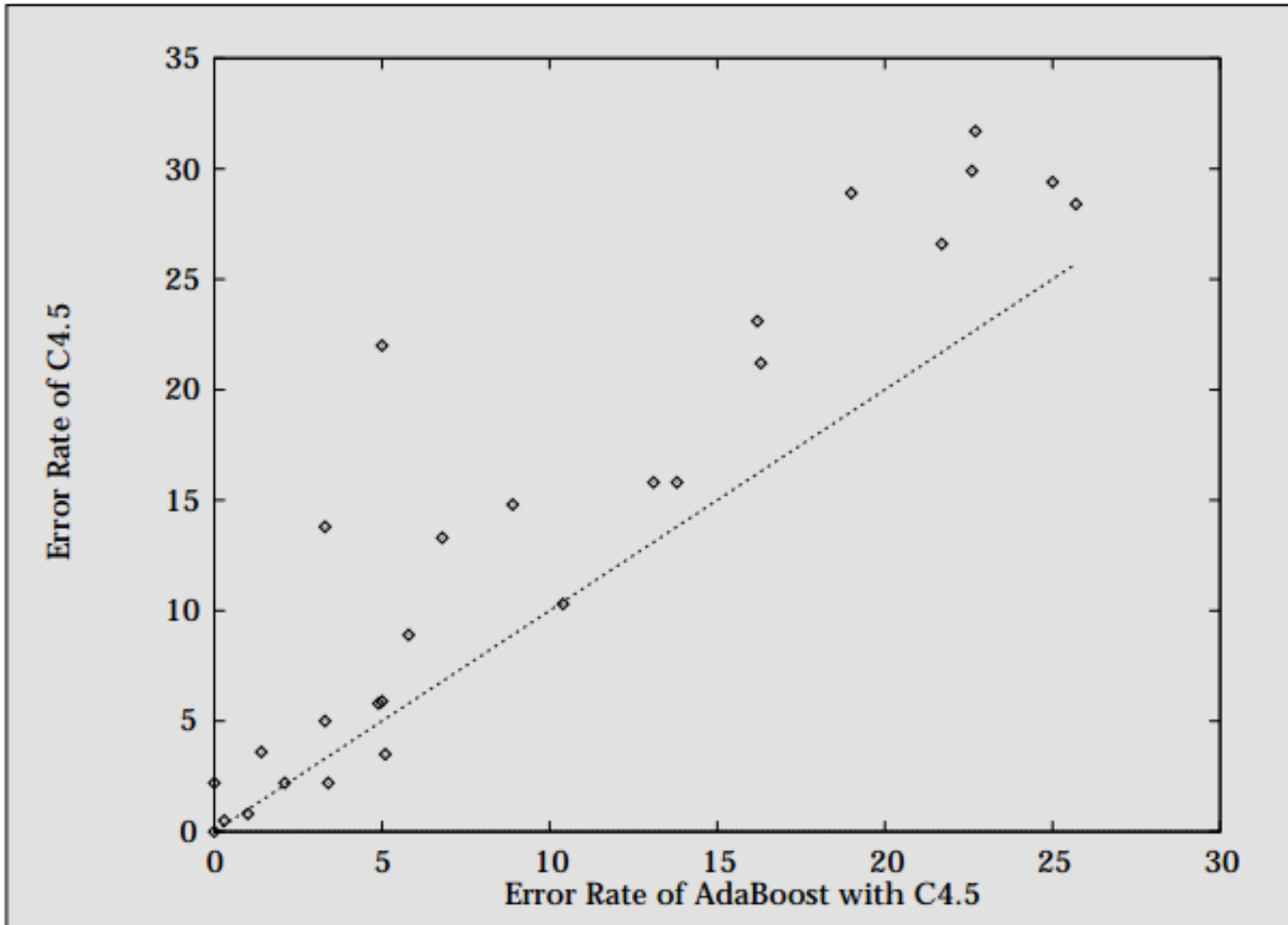


Figure from Dietterich, *AI Magazine*, 1997

Bagging and boosting with C4.5

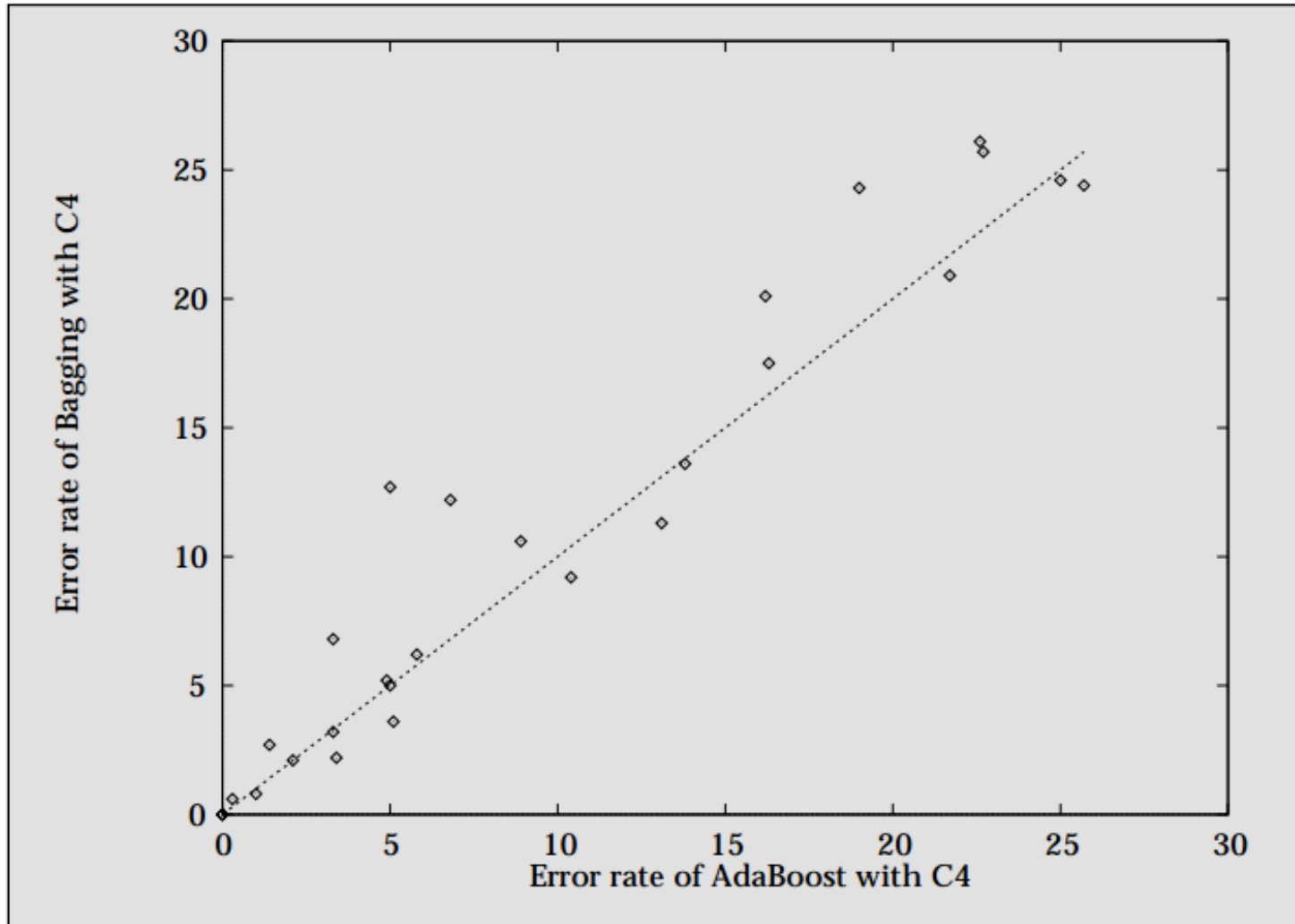


Figure from Dietterich, *AI Magazine*, 1997

Empirical study of bagging vs. boosting

[Opitz & Maclin, *JAIR* 1999]

- 23 data sets
- C4.5 and neural nets as base learners
- bagging almost always better than single decision tree or neural net
- boosting can be much better than bagging
- however, boosting can sometimes reduce accuracy (too much emphasis on outliers?)

Random forests

[Breiman, Machine Learning 2001]

given: candidate feature splits F , training set $D = \{ \langle \mathbf{x}_1, y_1 \rangle \dots \langle \mathbf{x}_m, y_m \rangle \}$

for $i \leftarrow 1$ to T do

$D^{(i)} \leftarrow m$ instances randomly drawn with replacement from D

$h_i \leftarrow$ randomized decision tree learned with $F, D^{(i)}$

randomized decision tree learning:

to select a split at a node

$R \leftarrow$ randomly select (without replacement) f feature splits from F
(where $f \ll |F|$)


choose the best feature split in R

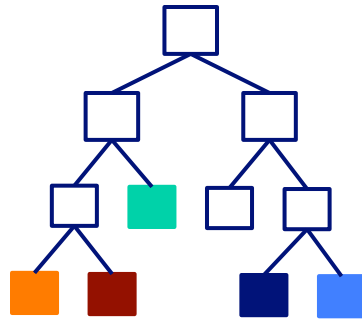
do not prune trees

classification/regression:

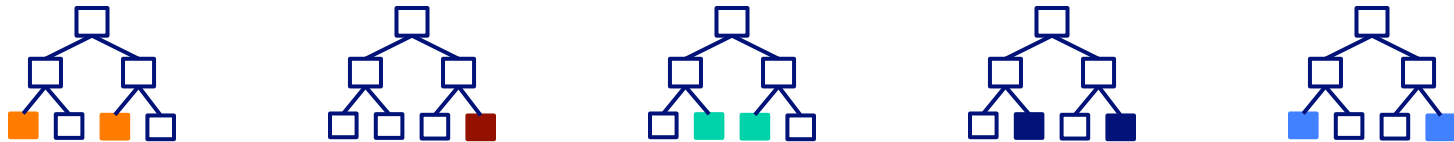
as in bagging

Learning models for multi-class problems

- consider a learning task with $k > 2$ classes 
- with some learning methods, we can learn one model to predict the k classes



- an alternative approach is to learn k models; each represents one class vs. the rest



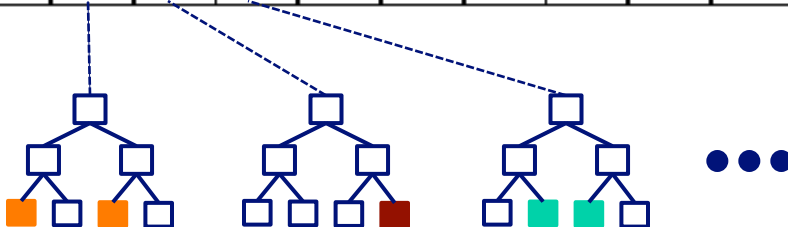
- but we could learn models to represent other encodings as well

Error correcting output codes

[Dietterich & Bakiri, *JAIR* 1995]

- ensemble method devised specifically for problems with many classes
 - represent each class by a multi-bit code word
 - learn a classifier to represent each bit function

Class	Code Word														
	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	1
1	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0
2	1	0	0	1	0	0	0	1	1	1	1	0	1	0	1
3	0	0	1	1	0	1	1	1	0	0	0	0	1	0	1
4	1	1	1	0	1	0	1	1	0	0	1	0	0	0	1
5	0	1	0	0	1	1	0	1	1	1	0	0	0	0	1
6	1	0	1	1	1	0	0	0	0	1	0	1	0	0	1
7	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1
8	1	1	0	1	0	1	1	0	0	1	0	0	0	1	1
9	0	1	1	1	0	0	0	0	1	0	1	0	0	1	1



Classification with ECOC

- to classify a test instance x using an ECOC ensemble with T classifiers
 1. form a vector $h(x) = \langle h_1(x) \dots h_T(x) \rangle$ where $h_i(x)$ is the prediction of the model for the i^{th} bit
 2. find the codeword c with the smallest Hamming distance to $h(x)$
 3. predict the class associated with c

- if the minimum Hamming distance between any pair of codewords is d , we can still get the right classification with $\lfloor \frac{d-1}{2} \rfloor$ single-bit errors

recall, $\lfloor x \rfloor$ is the largest integer not greater than x

Error correcting code design

a good ECOC should satisfy two properties

1. *row separation*: each codeword should be well separated in Hamming distance from every other codeword
2. *column separation*: each bit position should be uncorrelated with the other bit positions

Class	Code Word														
	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	1
1	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0
2	1	0	0	1	0	0	0	1	1	1	1	0	1	0	1
3	0	0	1	1	0	1	1	1	0	0	0	0	1	0	1
4	1	1	1	0	1	0	1	1	0	0	1	0	0	0	1
5	0	1	0	0	1	1	0	1	1	1	0	0	0	0	1
6	1	0	1	1	1	0	0	0	0	1	0	1	0	0	1
7	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1
8	1	1	0	1	0	1	1	0	0	1	0	0	0	1	1
9	0	1	1	1	0	0	0	0	1	0	1	0	0	1	1

6 bits apart

7 bits apart

$$d = 7 \text{ so this code can correct } \left\lfloor \frac{7-1}{2} \right\rfloor = 3 \text{ errors}$$

ECOC evaluation with C4.5

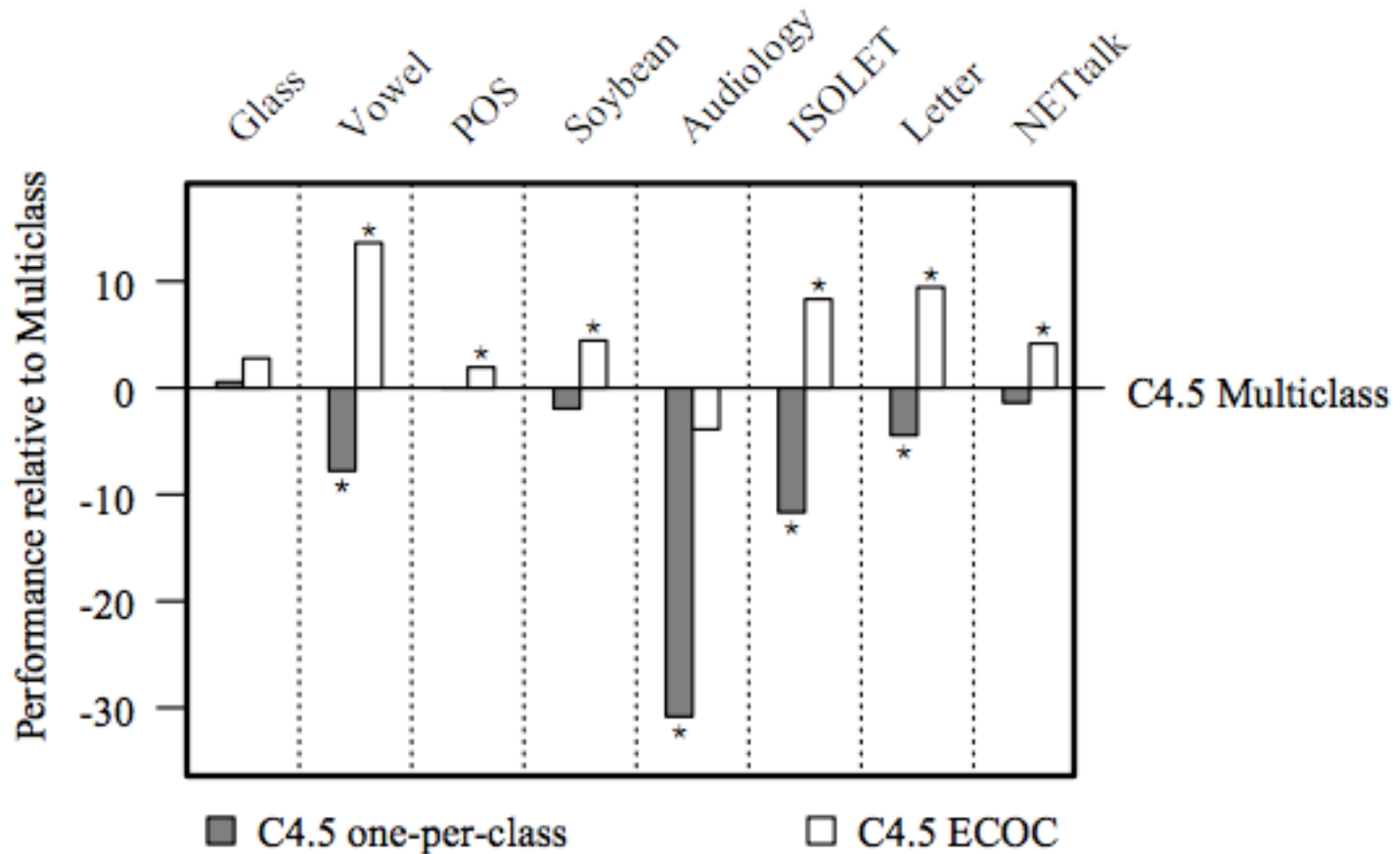


Figure from Bakiri & Dietterich, *JAIR*, 1995

ECOC evaluation with neural nets

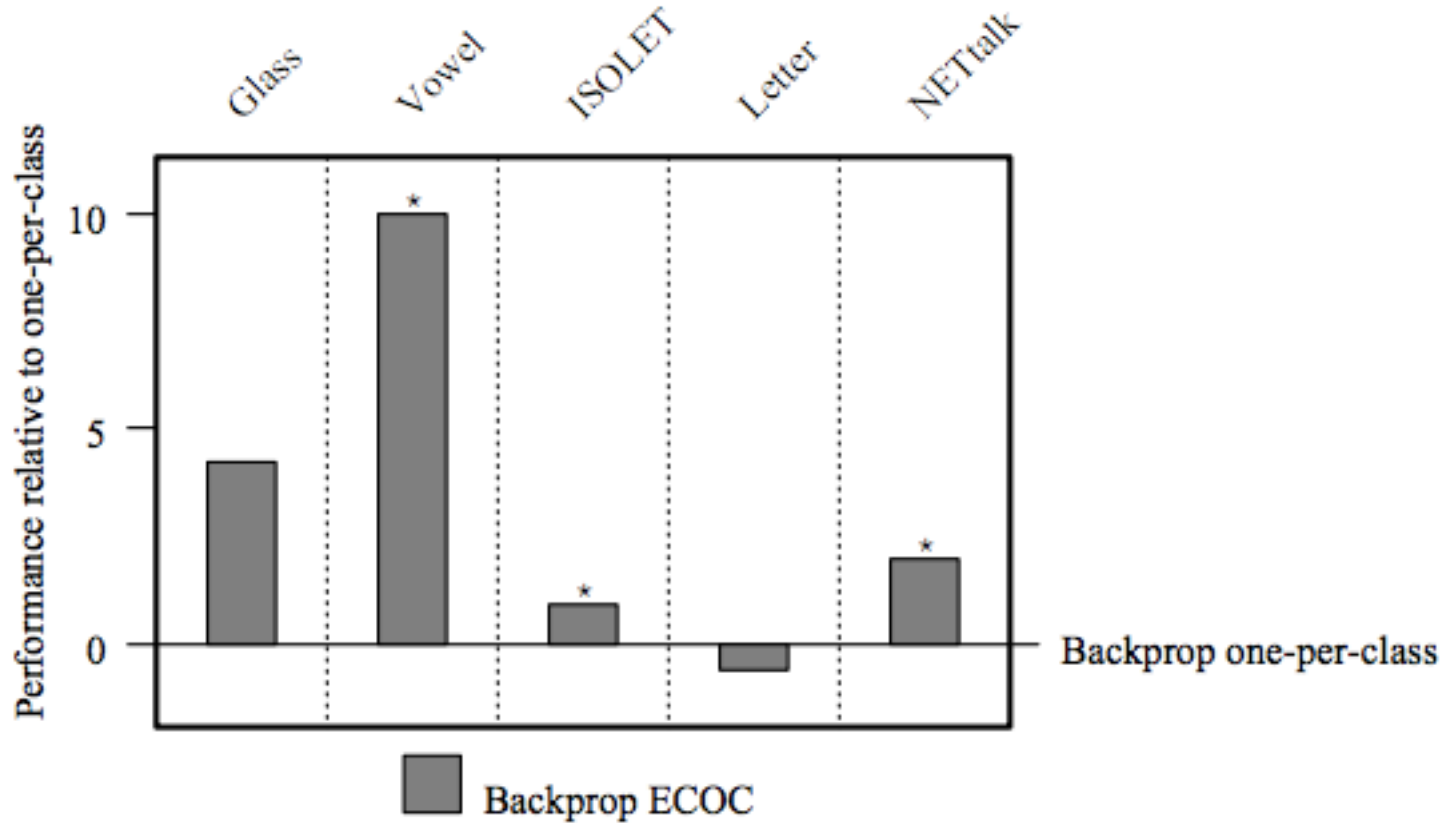


Figure from Bakiri & Dietterich, *JAIR*, 1995

Comments on ensembles

- They very often provide a boost in accuracy over base learner
- It's a good idea to evaluate an ensemble approach for almost any practical learning problem
- They increase runtime over base learner, but compute cycles are usually much cheaper than training instances
- Some ensemble approaches (e.g. bagging, random forests) are easily parallelized
- Prediction contests (e.g. Kaggle, Netflix Prize) usually won by ensemble solutions
- Ensemble models are usually low on the comprehensibility scale, although see work by

[Craven & Shavlik, *NIPS* 1996]

[Domingos, *Intelligent Data Analysis* 1998]

[Van Assche & Blockeel, *ECML* 2007]