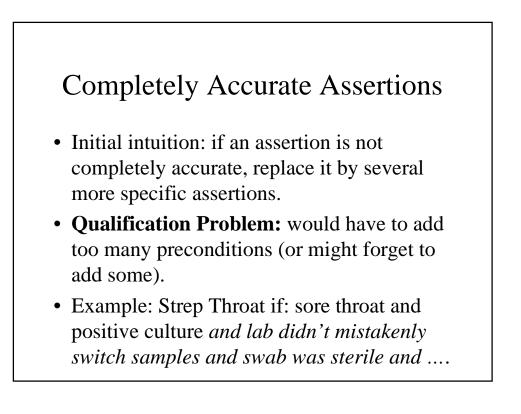
Lecture 2: Uncertainty

Deductive Logic-based Systems

- Automated Theorem Provers
- Rule-based Expert Systems
- Planners and Schedulers
- Robots
- Constraint Satisfaction Systems
- Natural Language Processing

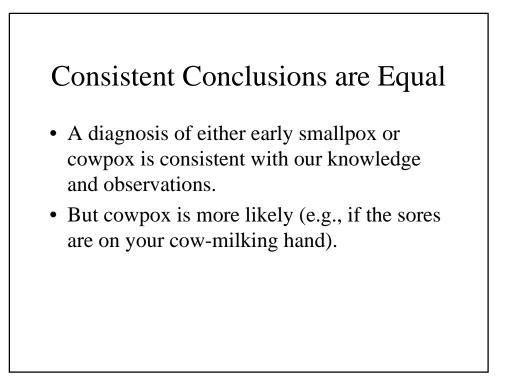
Assumptions Inherent in Deductive Logic-based Systems

- All the assertions we wish to make and use are **universally true**.
- Observations of the world (percepts) are **complete** and **error-free**.
- All conclusions consistent with our knowledge are **equally viable**.
- All the desirable inference rules are **truth**-**preserving**.



Complete and Error-Free Perception

- Errors are common: biggest problem in use of Pathfinder for diagnosis of lymph system disorder is human error in feature detection.
- Some tests are impossible, too costly, or dangerous. "We could determine if your hip pain is really due to a lower back problem if we cut these nerve connections."



Truth-Preserving Inference

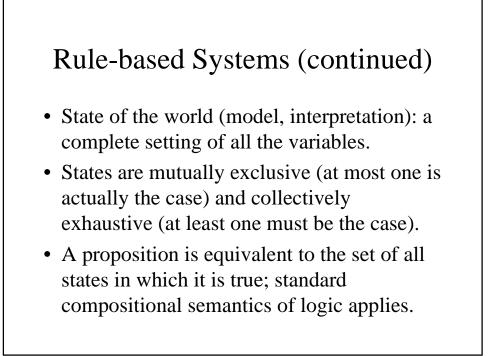
- Might want to use abductive or inductive inference.
- Even if our inference rules are truthpreserving, if there's a slight probability of error in our assertions or observations, during chaining (e.g., resolution) these probabilities can compound quickly, *and we are not estimating them*.

Solution: Reason Explicitly About Probabilities

- Full joint distributions.
- Certainty factors attached to rules.
- Dempster-Shafer Theory.
- Qualitative probability and non-monotonic reasoning.
- Possibility theory (within fuzzy logic, which itself does not deal with probability).
- Bayesian Networks.

Start with the Terminology of Most Rule-based Systems

- Atomic proposition: assignment of a value to a variable (parameter).
- Domain of possible values: variables may be Boolean, Discrete (finite domain), or Continuous (floating point in practice).
- Compound assertions can be built with standard logical connectives.

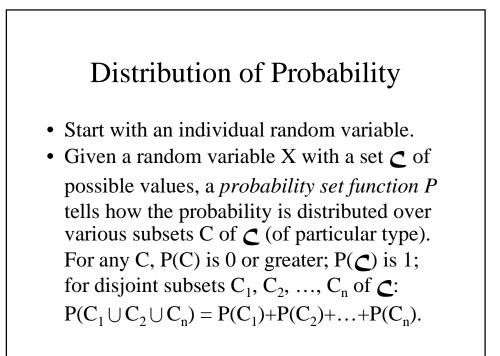


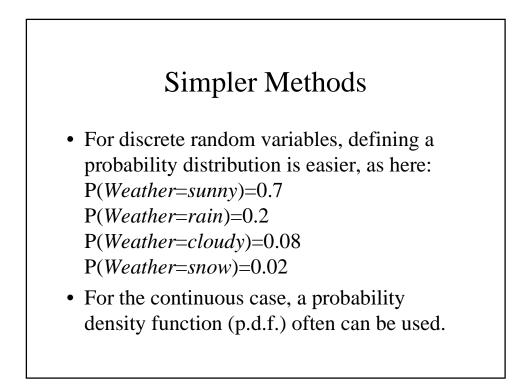
To Add Probability

- Replace variables with random variables.
- State of the world (setting of all the random variables) will be called an **atomic event**.
- Apply probabilities or *degrees of belief* to propositions: P(*Weather=sunny*) = 0.7.

Prior Probability

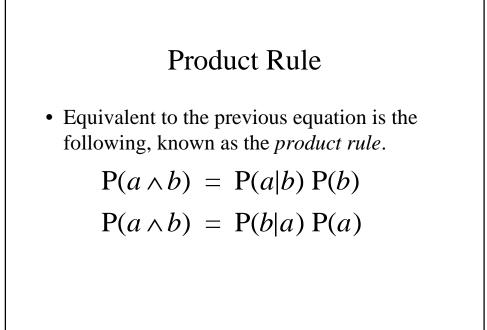
- The **unconditional** or **prior probability** of a proposition is the degree of belief accorded to it *in the absence of any other information*.
- Example: P(*Cavity* = *true*) or P(*cavity*)
- Example: P(*Weather = sunny*)
- Example: P(*cavity* \(Weather = sunny))





Conditional (Posterior) Probability

- P(*a*|*b*): the probability of *a* given that all we know is *b*.
- P(*cavity*|*toothache*) = 0.8: if a patient has a toothache, and no other information is available, the probability that the patient has a cavity is 0.8.
- To be precise: $P = \frac{P(a \wedge b)}{P(b)}$



The Axioms of Probability

- For any proposition $a, 0 \le P(a) \le 1$
- P(true) = 1 and P(false) = 0
- The probability of a disjunction is given by

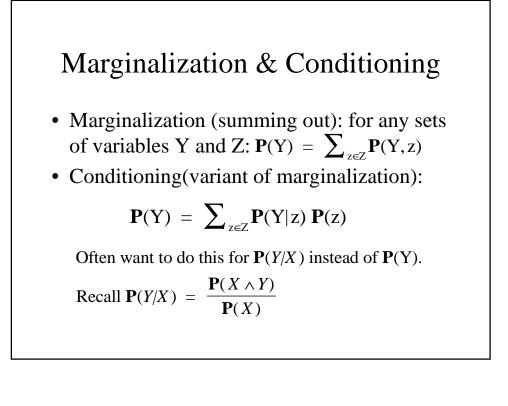
$$P(a \lor b) = P(a) + P(b) - P(a \land b)$$

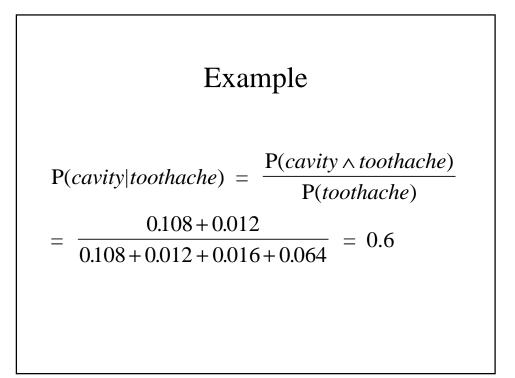
Boldface Notation

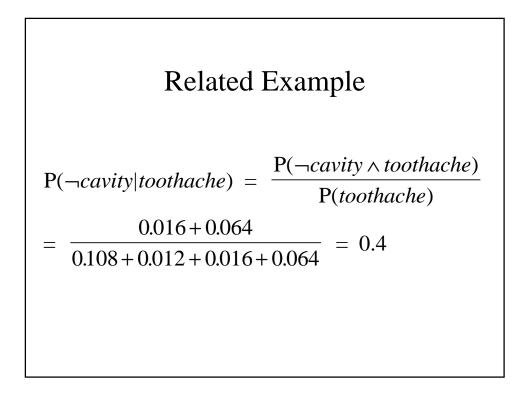
- Sometimes we want to write an equation that holds for a vector (ordered set) of random variables. We will denote such a set by **boldface** font. So *Y* denotes a random variable, but **Y** denotes a set of random variables.
- *Y* = *y* denotes a setting for *Y*, but **Y**=**y** denotes a setting for all variables in **Y**.

P Notation

- P(X = x) denotes the probability that the random variable X takes the value x; P(X) denotes a probability distribution over X.
- For example, if we want to say that observing a value for *Y* does not change the probability distribution over *X*, we can write P(X/Y) = P(X) rather than repeating for every combination of settings for *X* and *Y*.







Normalization

- In the two preceding examples the denominator (P(*toothache*)) was the same, and we looked at all possible values for the variable *Cavity* given *toothache*.
- The denominator can be viewed as a normalization constant *α*.
- We don't have to compute the denominator -- just normalize 0.12 and 0.08 to sum to 1.

