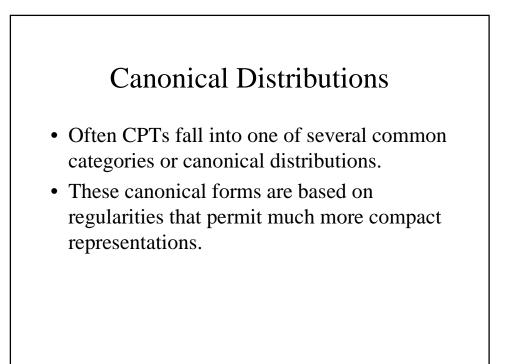
### **Representation Size**

- At first glance it would appear that the space to represent a Bayes Net is necessarily quadratic (possible number of arcs) in the number of random variables.
- But recall we also must represent the CPT of each node, which in general will have size exponential in the number of parents of the node.



### Deterministic Nodes

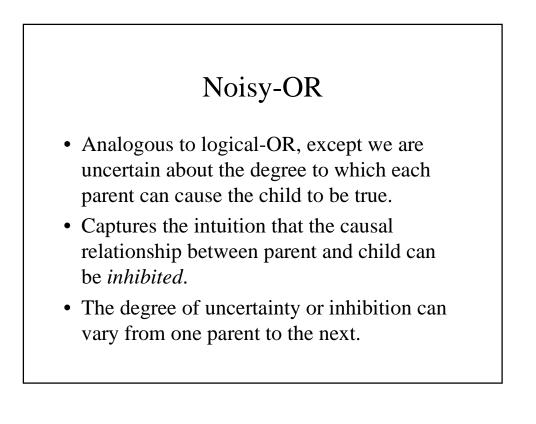
- A deterministic node has its value specified exactly by the values of its parents, with no uncertainty.
- Different types of deterministic nodes, including logical and numerical.



- Value of a child node is determined by a logical expression over the parent nodes.
- Might be negation, disjunction, conjunction, or a more complex expression.
- Example: The relationship between Boolean variables *Canadian*, *US*, and *Mexican* as parents and the child node *North American* is the child is a disjunction of the parents.

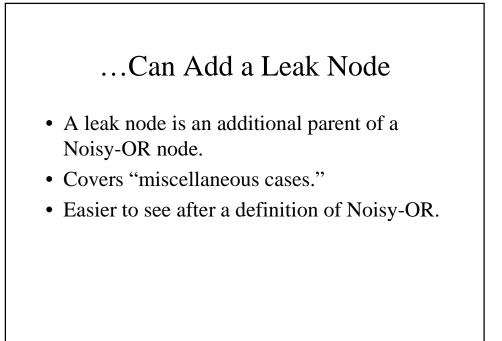
### Deterministic Nodes: Numerical

- The value of the child node is a numerical function of the parent values.
- If the parent nodes are prices of the same model car at different dealers, and the child is the price paid by a careful shopper, the child is the minimum of the parents.
- Might also have differences, sums, or more complex numerical functions.



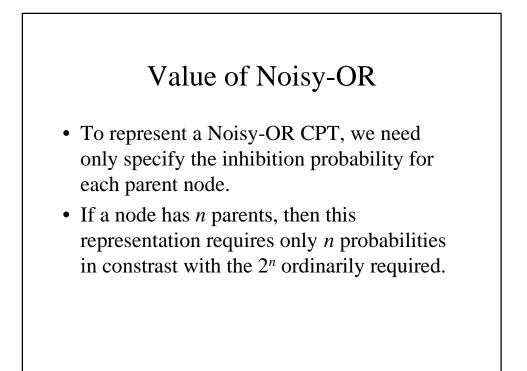
# Noisy-OR Assumptions

- Parents and child are Boolean variables.
- Inhibition of one parent is independent of the inhibitions of any other parents; e.g., whatever prevents *Malaria* from causing *Fever* is independent of whatever inhibits *Flu* from causing *Fever*.
- All possible causes are listed. In practice this constraint is not an issue because...



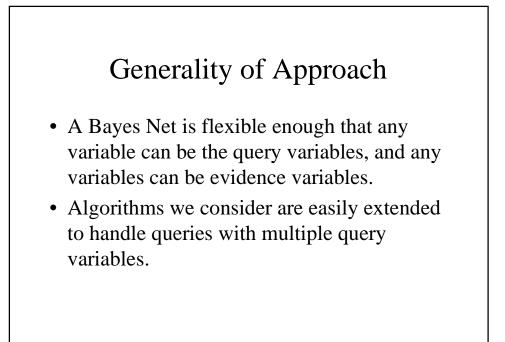
# Definition of Noisy-OR

- A child node is *false* only if its *true* parents are inhibited.
- The probability of such inhibition is the product of the inhibition probabilities for each parent.
- So the probability the child node is *true* is 1 minus the product of the inhibition probabilities for the *true* parents.



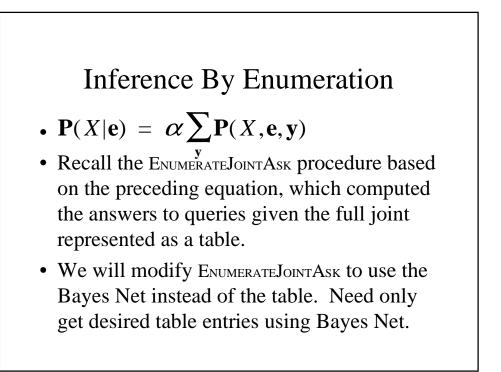
## The Basic Inference Task in Bayesian Networks

- Given some observed **event** (some assignment of values to a set of **evidence variables**), compute the posterior probability distribution over a set of **query variables**.
- Variables that are neither evidence variables nor query variables are **hidden variables**.
- Most common query:  $\mathbf{P}(X|\mathbf{e})$ .



### **Example Query**

- **P**(Burglary | JohnCalls=true, MaryCalls=true)
  - = <0.284,0.716>
- How can we compute such answers?
- One approach is to compute entire full joint distribution represented by the network, and use our earlier algorithm. But this would defeat the entire purpose of Bayes Nets.



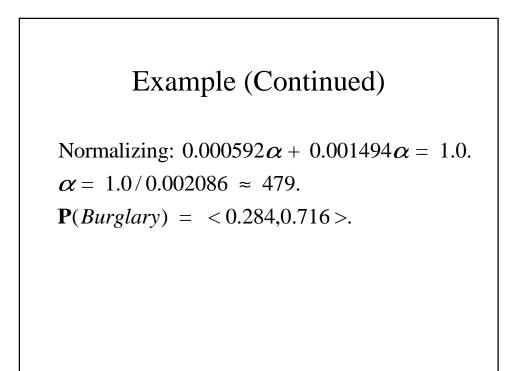
# Getting a Full Joint Table Entry from a Bayes Net Recall: P(x1,...,xn) = <sup>n</sup> P(xi/Parents(Xi)) A table entry for X₁ = x₁,...,Xn = xn is simply P(x₁,...,xn) which can be calculated based on the Bayes Net semantics above. Recall example: P(a,j,m,¬b,¬e) = P(j/a) P(m/a) P(a/¬b,¬e)P(¬b)P(¬e)

# Example of Full Procedure Query: P(Burglary | JohnCalls=true, MaryCalls=true) P(B/j,m) = α∑∑∑P(B,e,a,j,m) Solve without the normalization constant for both B = true and B = false, and then compute the normalization constant and the final probabilities.

Example (Continued)  

$$P(b|j,m) = \alpha \sum_{e} \sum_{a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$

$$= P(b) \sum_{e} P(e) \sum_{a} P(a|b,e)P(j|a)P(m|a).$$
 Go to  
CPTs in Bayes Net to pull out numbers (Fig. 16.8).  
Repeat with  $\neg b$  instead of *b* throughout. Resulting  
numbers are 0.000592 and 0.001494 respectively.



### Some Observations

- The number of terms in the sum is exponential in the number of *hidden variables*.
- Many sub-expressions are repeated on multiple branches. Each could be computed once and saved ... leads to the idea of *variable elimination*.

