# Relational Learning

www.cs.wisc.edu/~page/cs760/

# Goals for the lecture

you should understand the following concepts

- rule-set learning
- relational learning
- the FOIL algorithm

# Rule sets as a hypothesis space

- we can use propositional rule sets as a hypothesis space for a learning algorithm
- each rule is a conjunction of tests + a class that is implied (predicted) when the conjunction is satisfied

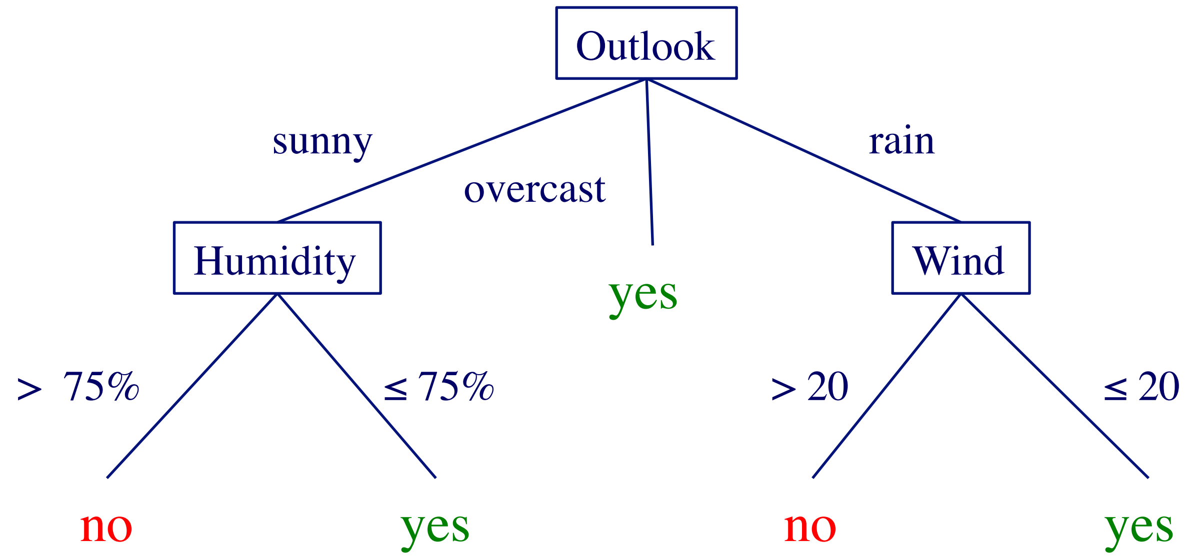
$$\text{Class=yes} \leftarrow \text{Outlook=sunny} \land \text{Humidity} \leq 75\%$$

$$\text{Class=yes} \leftarrow \text{Outlook=overcast}$$

$$\text{Class=yes} \leftarrow \text{Outlook=rain} \land \text{Win} \leq 20$$

# Decision trees and rules

Any decision tree can be converted into an equivalent set of rules



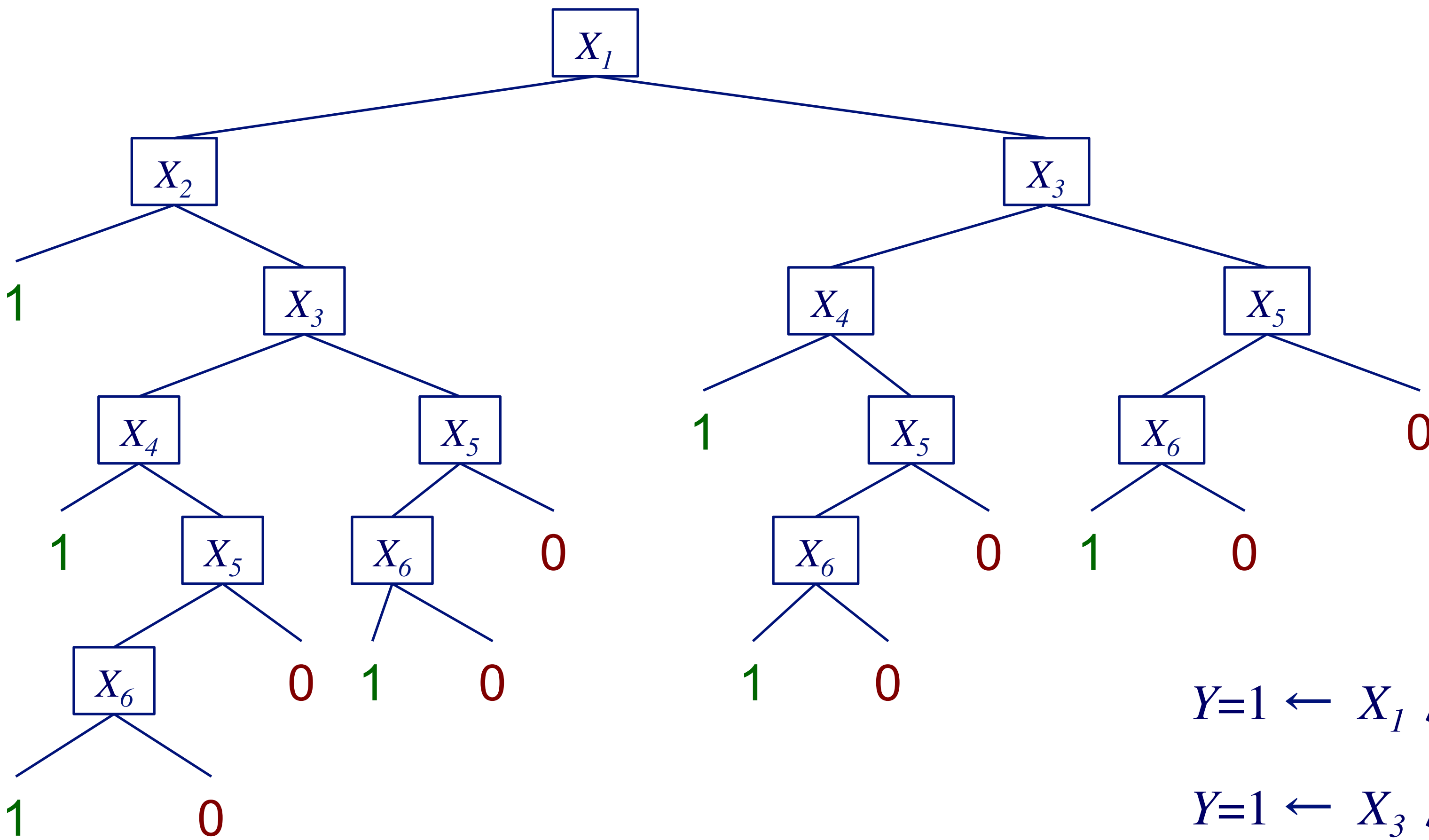

Class=yes ← Outlook=sunny ∧ Humidity≤75%

Class=yes ← Outlook=overcast

Class=yes ← Outlook=rain ∧ Wind≤20

# Decision trees and rules

a small set of rules can represent a large decision tree because of the *replication problem*




$$Y{=}1 \leftarrow X_1 \wedge X_2$$

$$Y{=}1 \leftarrow X_3 \wedge X_4$$

$$Y{=}1 \leftarrow X_5 \wedge X_6$$

# Rule learning

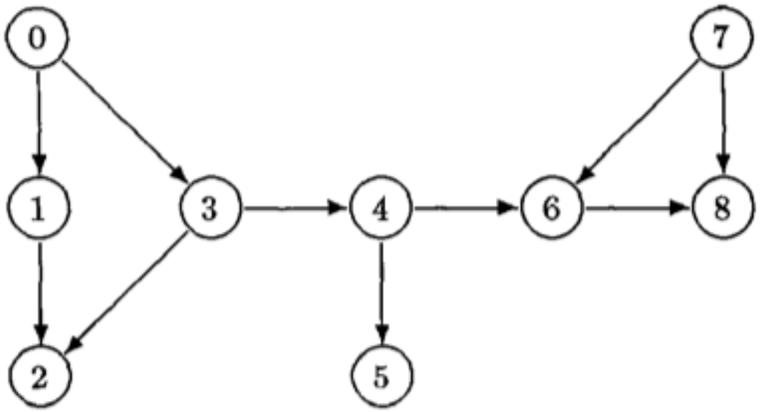
- rule sets can be learned by extracting them from decision trees (C4.5 has a module for this)
- there are also algorithms for learning rules directly, such as SLIPPER [Cohen & Singer, *AAAI* 1999]

- the rules we've considered so far are expressed in propositional logic – they're not well suited to representing multiple entities and relationships among them

- let's consider *relational learning* methods, which represent their hypotheses using a subset of first-order logic

# Relational learning example

- suppose we want to learn the general concept of can-reach in a graph, given a set of training instances describing a particular graph




$\oplus$: ⟨0,1⟩ ⟨0,2⟩ ⟨0,3⟩ ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ ⟨1,2⟩ ⟨3,2⟩ ⟨3,4⟩ ⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ ⟨4,5⟩ ⟨4,6⟩ ⟨4,8⟩ ⟨6,8⟩ ⟨7,6⟩ ⟨7,8⟩

$\ominus$: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩ ⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩ ⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩ ⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩ ⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩ ⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩ ⟨8,7⟩ ⟨8,8⟩

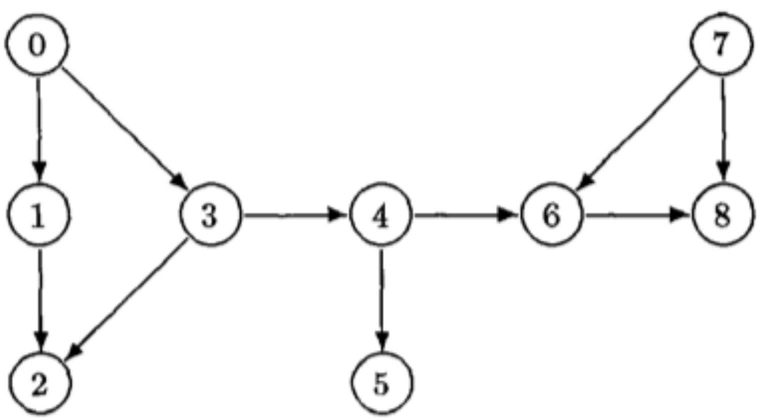
- how would you represent this task to a learner?

# Relational learning example

- a relational representation, such as first-order logic, can capture this concept succinctly and in a general way

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_2)$$

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_3) \land \text{can-reach}(X_3, X_2)$$




$\oplus$: ⟨0,1⟩ ⟨0,2⟩ ⟨0,3⟩ ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ ⟨1,2⟩ ⟨3,2⟩ ⟨3,4⟩ ⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ ⟨4,5⟩ ⟨4,6⟩ ⟨4,8⟩ ⟨6,8⟩ ⟨7,6⟩ ⟨7,8⟩

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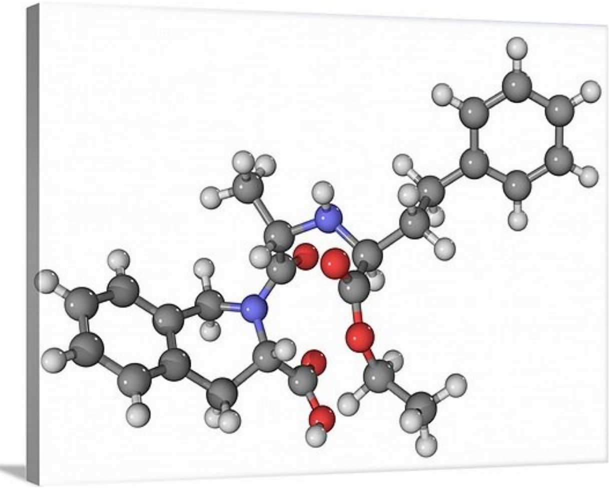
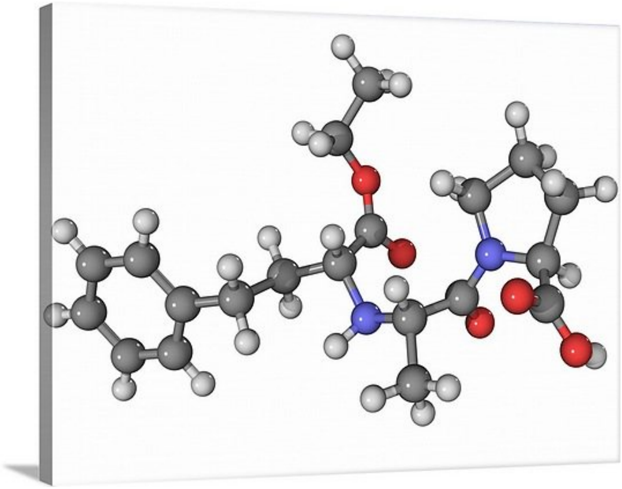
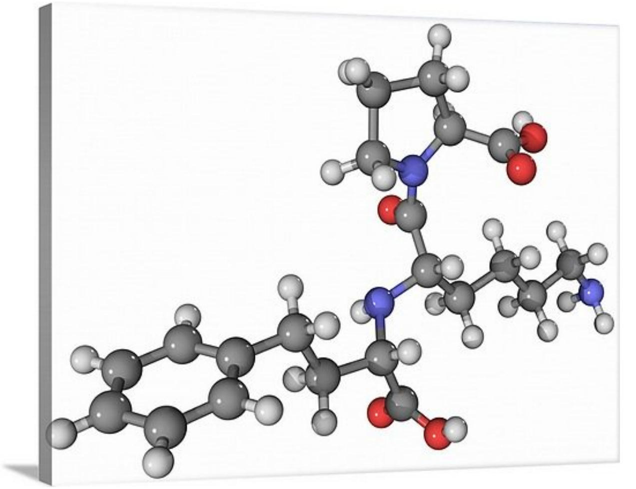
# Relational learning example

consider the task of learning a *pharmacophore*: the substructure of a molecule that interacts with a target of interest

- instances for this task consist of interacting (+) and non-interacting molecules (-)







to represent each instance, we'd like to describe

- the (variable # of) atoms in the molecule
- the possible conformations of the molecule
- the bonds among atoms
- distances among atoms
- etc.

# Relational learning example

[Finn et al., *Machine Learning* 1998]

a multi-relational representation for molecules

| Molecule | Target_1 | ... | Target_n |
|---|---|---|---|
| mol1 | inactive | | i nactive |
| mol2 | active | | inactive |
| • | | | |
| • | | | |
| • | | | |

Molecular Bioactivity

| Molecule | Bond_ID | Atom_1_ID | Atom_2_ID | Bond_Type |
|---|---|---|---|---|
| mol1 | bond1 | a1 | a2 | aromatic |
| • | | | | |
| • | | | | |
| • | | | | |

Bonds

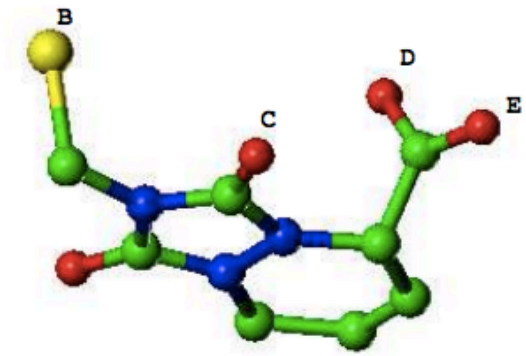
| Molecule | Conformer | Atom_ID | Atom_Type | X_Coordinate | Y_Coordinate | Z_Coordinate |
|---|---|---|---|---|---|---|
| mol1 | conf1 | a1 | carbon | 2.58 | −1.23 | 0.69 |
| • | | | | | | |
| • | | | | | | |
| • | | | | | | |

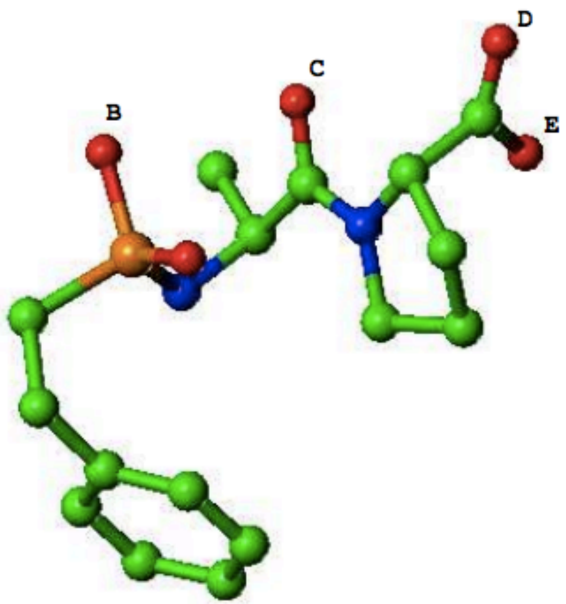
3D Atom Locations

# Relational learning example

[Finn et al., *Machine Learning* 1998]

a learned relational rule characterizing ACE inhibitors




```
Molecule A is an ACE inhibitor if
for some conformer Conf of A:
   molecule A contains a zinc binding site B;
   molecule A contains a hydrogen acceptor C;
   the distance between B and C in Conf is 7.9 +/- .75;
   molecule A contains a hydrogen acceptor D;
   the distance between B and D in Conf is 8.5 +/- .75;
   the distance between C and D in Conf is 2.1 +/- .75;
   molecule A contains a hydrogen acceptor E;
   the distance between B and E in Conf is 4.9 +/- .75;
   the distance between C and E in Conf is 3.1 +/- .75;
   the distance between D and E in Conf is 3.8 +/- .75.
```

# Relational representation

ACE_inhibitor(A) ← has_zinc_binding_site(A, B) ∧
has_hydrogren_acceptor(A, C) ∧
distance(B, C, 7.9, 0.75) ∧
has_hydrogen_acceptor(A, D) ∧
distance(B, D, 8.5, 0.75) ∧
distance(C, D, 8.5, 0.75) ∧
has_hydrogen_acceptor(A, E) ∧
distance(B, E, 4.9, 0.75) ∧
distance(C, E, 3.1, 0.75) ∧
distance(D, E, 3.8, 0.75)

To learn an equivalent rule with a feature-vector learner, what features would we need to represent?

has_zinc_binding_site
has_hydrogen_acceptor
zinc_binding_site_and hydrogen_acceptor_distance
hydrogen_acceptor_hydrogen_acceptor_distance
…

can easily encode distance between a pair of atoms; but this pharmacophore has 4 important atoms with 6 relevant distances among them

# Relational learning example

[Craven et al., *ECML* 1998]

- consider the task of classifying web pages according their roles
- here is a learned rule for recognizing home pages for CS courses

$$
\begin{aligned}
&\text{course}(A) \leftarrow \\
&\qquad \text{has-word}(A, \text{instructor}), \\
&\qquad \neg\ \text{has-word}(A, \text{good}), \\
&\qquad \text{link-from}(A, B), \\
&\qquad \text{has-word}(B, \text{assign}), \\
&\qquad \neg\ \text{link-from}(B, C)
\end{aligned}
$$

- test-set accuracy: 31 / 34

# Relational learning example

[Page et al., *AAAI* 2012]

- Data from electronic health records (EHRs) is being used to learn models for risk assessment, adverse event detection, etc.
- A patient's record is described by multiple tables in a relational DB

demographics

| PatientID | Gender | Birthdate |
|---|---|---|
| P1 | M | 3/22/63 |

diagnoses

| PatientID | Date | Physician | Symptoms | Diagnosis |
|---|---|---|---|---|
| P1 | 1/1/01 | Smith | palpitations | hypoglycemic |
| P1 | 2/1/03 | Jones | fever, aches | influenza |

labs

| PatientID | Date | Lab Test | Result |
|---|---|---|---|
| P1 | 1/1/01 | blood glucose | 42 |
| P1 | 1/9/01 | blood glucose | 45 |

genetics

| PatientID | SNP1 | SNP2 | … | SNP500K |
|---|---|---|---|---|
| P1 | AA | AB | | BB |
| P2 | AB | BB | | AA |

drugs

| PatientID | Date Prescribed | Date Filled | Physician | Medication | Dose | Duration |
|---|---|---|---|---|---|---|
| P1 | 5/17/98 | 5/18/98 | Jones | prilosec | 10mg | 3 months |

# The FOIL algorithm for relational learning

[Quinlan, *Machine Learning* 1990]

**given**:

- tuples (instances) of a target relation
- *extensionally* represented background relations

**do**:

- learn a set of rules that (mostly) cover the positive tuples of the target relation, but not the negative tuples

# Input to FOIL

- instances of target relation

$\oplus$: ⟨0,1⟩ ⟨0,2⟩ ⟨0,3⟩ ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ ⟨1,2⟩ ⟨3,2⟩ ⟨3,4⟩
⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ ⟨4,5⟩ ⟨4,6⟩ ⟨4,8⟩ ⟨6,8⟩ ⟨7,6⟩ ⟨7,8⟩

$\ominus$: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩
⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩
⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩
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⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩
⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩
⟨8,7⟩ ⟨8,8⟩

- extensionally defined background relations

$$\textit{linked-to} = \{\langle 0,1\rangle, \langle 0,3\rangle, \langle 1,2\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,5\rangle, \langle 4,6\rangle, \langle 6,8\rangle, \langle 7,6\rangle, \langle 7,8\rangle\}$$

# The FOIL algorithm for relational learning

FOIL uses a covering approach to learn a set of rules

```
LEARNRULESET(set of tuples T of target relation, background relations B)
{
    S = { }
    repeat
        R ← LEARNRULE(T, B)
        S ← S ∪ R
        T ← T – positive tuples covered by R
    until there are no (few) positive tuples left in T
    return S
}
```

# The FOIL algorithm for relational learning

LEARNRULE(set of tuples $T$ of target relation, background relations $B$)
{
    $R = \{ \}$
    repeat
        $L \leftarrow$ best literal, based on $T$ and $B$, to add to right-hand side of $R$
        $R \leftarrow R \cup L$
        $T \leftarrow$ new set of tuples that satisfy $L$
    until there are no (few) negative tuples left in $T$
    return $R$
}

# Literals in FOIL

- Given the current rule $R(X_1, X_2, \ldots X_k) \leftarrow L_1 \wedge L_2 \wedge \ldots \wedge L_n$
  FOIL considers adding several types of literals

$X_j = X_k$

$X_j \neq X_k$

both $X_j$ and $X_k$ either appear in the LHS of the rule, or were introduced by a previous literal

$Q(V_1, V_2, \ldots V_a)$

$\neg Q(V_1, V_2, \ldots V_a)$

at least one of the $V_i$'s has to be in the LHS of the rule, or was introduced by a previous literal

where $Q$ is a background relation

# Literals in FOIL (continued)

$X_j = c$

where $c$ is a constant

$X_j \neq c$

$X_j > a$

$X_j \leq a$

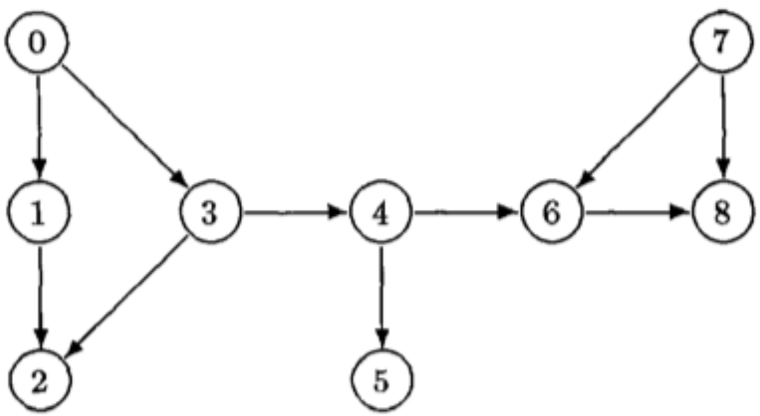
$X_j > X_k$

$X_j \leq X_k$

where $X_j$ and $X_k$ are numeric variables and $a$ is a numeric constant

# FOIL example

- suppose we want to learn rules for the target relation can-reach($X_1, X_2$)
- we're given instances of the target relation from the following graph




$\oplus$: $\langle 0,1\rangle$ $\langle 0,2\rangle$ $\langle 0,3\rangle$ $\langle 0,4\rangle$ $\langle 0,5\rangle$ $\langle 0,6\rangle$ $\langle 0,8\rangle$ $\langle 1,2\rangle$ $\langle 3,2\rangle$ $\langle 3,4\rangle$
$\langle 3,5\rangle$ $\langle 3,6\rangle$ $\langle 3,8\rangle$ $\langle 4,5\rangle$ $\langle 4,6\rangle$ $\langle 4,8\rangle$ $\langle 6,8\rangle$ $\langle 7,6\rangle$ $\langle 7,8\rangle$

$\ominus$: $\langle 0,0\rangle$ $\langle 0,7\rangle$ $\langle 1,0\rangle$ $\langle 1,1\rangle$ $\langle 1,3\rangle$ $\langle 1,4\rangle$ $\langle 1,5\rangle$ $\langle 1,6\rangle$ $\langle 1,7\rangle$ $\langle 1,8\rangle$
$\langle 2,0\rangle$ $\langle 2,1\rangle$ $\langle 2,2\rangle$ $\langle 2,3\rangle$ $\langle 2,4\rangle$ $\langle 2,5\rangle$ $\langle 2,6\rangle$ $\langle 2,7\rangle$ $\langle 2,8\rangle$ $\langle 3,0\rangle$
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$\langle 7,4\rangle$ $\langle 7,5\rangle$ $\langle 7,7\rangle$ $\langle 8,0\rangle$ $\langle 8,1\rangle$ $\langle 8,2\rangle$ $\langle 8,3\rangle$ $\langle 8,4\rangle$ $\langle 8,5\rangle$ $\langle 8,6\rangle$
$\langle 8,7\rangle$ $\langle 8,8\rangle$

- and instances of the background relation linked-to

$$\textit{linked-to} = \{\langle 0,1\rangle, \langle 0,3\rangle, \langle 1,2\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,5\rangle, \langle 4,6\rangle, \langle 6,8\rangle, \langle 7,6\rangle, \langle 7,8\rangle\}$$

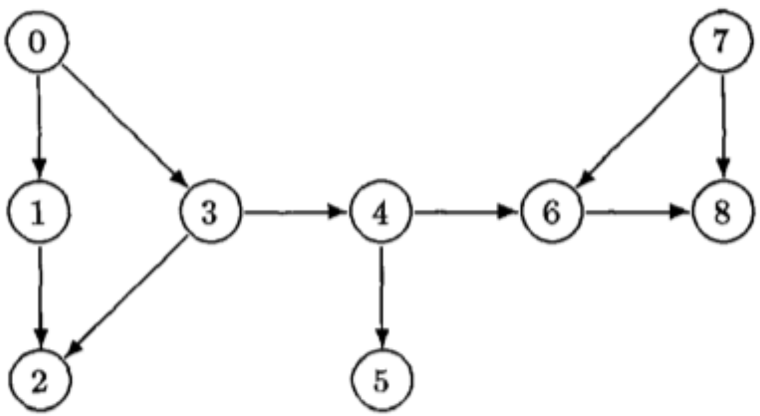
# FOIL example

- the first rule learned covers 10 of the positive instances

  can-reach($X_1$, $X_2$) ← linked-to($X_1$, $X_2$)

- the second rule learned covers the other 9 positive instances

  can-reach($X_1$, $X_2$) ← linked-to($X_1$, $X_3$) ∧ can-reach($X_3$, $X_2$)




⊕: ⟨0,1⟩ ⟨0,2⟩ ⟨0,3⟩ ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ ⟨1,2⟩ ⟨3,2⟩ ⟨3,4⟩ ⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ ⟨4,5⟩ ⟨4,6⟩ ⟨4,8⟩ ⟨6,8⟩ ⟨7,6⟩ ⟨7,8⟩

⊖: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩ ⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩ ⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩ ⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩ ⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩ ⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩ ⟨8,7⟩ ⟨8,8⟩

- note that these rules generalize to other graphs

# Evaluating literals in FOIL

- FOIL evaluates the addition of a literal $L$ to a rule $R$ by

$$FOIL\_Gain(L,R) = t\left(\log_2 \frac{p_1}{p_1+n_1} - \log_2 \frac{p_0}{p_0+n_0}\right)$$

- where

  $p_0$ = # of positive tuples covered by $R$

  $n_0$ = # of negative tuples covered by $R$

  $p_1$ = # of positive tuples covered by $R \wedge L$

  $n_1$ = # of negative tuples covered by $R \wedge L$

  $t$ = # of positive of tuples of $R$ also covered by $R \wedge L$

- like information gain, but takes into account
  - we want to cover positives, not just get a more “pure” set of tuples
  - the size of the tuple set grows as we add new variables

# Evaluating literals in FOIL

$$FOIL\_Gain(L,R) = t\left(Info\left(R_0\right) - Info\left(R_1\right)\right)$$

- where $R_0$ represents the rule without $L$ and $R_1$ is the rule with $L$ added
- $Info(R_i)$ is the number of bits required to encode a positive in the set of tuples covered by $R_i$

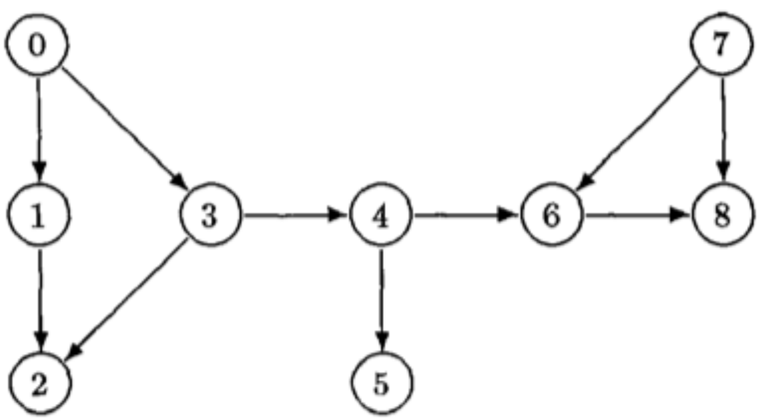
$$Info\left(R_i\right) = -\log_2\left(\frac{p_i}{p_i + n_i}\right)$$

# Recall this example

- Definition of can-reach:

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_2)$$

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_3) \wedge \text{can-reach}(X_3, X_2)$$




$\oplus$: ⟨0,1⟩ ⟨0,2⟩ ⟨0,3⟩ ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ ⟨1,2⟩ ⟨3,2⟩ ⟨3,4⟩ ⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ ⟨4,5⟩ ⟨4,6⟩ ⟨4,8⟩ ⟨6,8⟩ ⟨7,6⟩ ⟨7,8⟩

$\ominus$: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩ ⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩ ⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩ ⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩ ⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩ ⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩ ⟨8,7⟩ ⟨8,8⟩

# FOIL example

- consider the first step in learning the second clause

can-reach($X_1$, $X_2$) ← 

⬇

can-reach($X_1$, $X_2$) ← linked-to($X_1$, $X_3$)

$$FOIL\_Gain(L,R) = 9\left(\log_2 \frac{18}{18+54} - \log_2 \frac{9}{9+62}\right)$$
$$= 8.8$$

⊕: [redacted] ⟨0,2⟩ [redacted] ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ [redacted] [redacted] [redacted]
⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ [redacted] [redacted] ⟨4,8⟩ [redacted] [redacted] [redacted]

⊖: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩
⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩
⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩
⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩
⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩
⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩
⟨8,7⟩ ⟨8,8⟩

⊕: ⟨0,2,1⟩ ⟨0,2,3⟩ ⟨0,4,1⟩ ⟨0,4,3⟩ ⟨0,5,1⟩ ⟨0,5,3⟩ ⟨0,6,1⟩
⟨0,6,3⟩ ⟨0,8,1⟩ ⟨0,8,3⟩ ⟨3,5,2⟩ ⟨3,5,4⟩ ⟨3,6,2⟩ ⟨3,6,4⟩
⟨3,8,2⟩ ⟨3,8,4⟩ ⟨4,8,5⟩ ⟨4,8,6⟩

⊖: ⟨0,0,1⟩ ⟨0,0,3⟩ ⟨0,7,1⟩ ⟨0,7,3⟩ ⟨1,0,2⟩ ⟨1,1,2⟩ ⟨1,3,2⟩
⟨1,4,2⟩ ⟨1,5,2⟩ ⟨1,6,2⟩ ⟨1,7,2⟩ ⟨1,8,2⟩ ⟨3,0,2⟩ ⟨3,0,4⟩
⟨3,1,2⟩ ⟨3,1,4⟩ ⟨3,3,2⟩ ⟨3,3,4⟩ ⟨3,7,2⟩ ⟨3,7,4⟩ ⟨4,0,5⟩
⟨4,0,6⟩ ⟨4,1,5⟩ ⟨4,1,6⟩ ⟨4,2,5⟩ ⟨4,2,6⟩ ⟨4,3,5⟩ ⟨4,3,6⟩
⟨4,4,5⟩ ⟨4,4,6⟩ ⟨4,7,5⟩ ⟨4,7,6⟩ ⟨6,0,8⟩ ⟨6,1,8⟩ ⟨6,2,8⟩
⟨6,3,8⟩ ⟨6,4,8⟩ ⟨6,5,8⟩ ⟨6,6,8⟩ ⟨6,7,8⟩ ⟨7,0,6⟩ ⟨7,0,8⟩
⟨7,1,6⟩ ⟨7,1,8⟩ ⟨7,2,6⟩ ⟨7,2,8⟩ ⟨7,3,6⟩ ⟨7,3,8⟩ ⟨7,4,6⟩
⟨7,4,8⟩ ⟨7,5,6⟩ ⟨7,5,8⟩ ⟨7,7,6⟩ ⟨7,7,8⟩

# Additional refinements of FOIL

- early stopping to prevent overfitting
- using *m*-estimates of rule precision to guide search
  [Džeroski & Bratko, *ILP* 1992]
- type constraints on variables
- *relational pathfinding* to guide search for binary target relations
  [Craven, Slattery & Nigam, *ECML* 1998]
- using *intensional* background relations
  [Pazzani & Kibler, *Machine Learning* 1992]

$$\text{between}(X, Y, Z) \leftarrow \text{less-than}(X, Y) \wedge \text{less-than}(Y, Z)$$

$$\text{between}(X, Y, Z) \leftarrow \text{less-than}(Z, Y) \wedge \text{less-than}(Y, X)$$

# Comments on relational learning

- enables learning with more expressive hypothesis spaces
- but this comes at the cost of having large hypothesis spaces
  - harder to search
  - easier to overfit
- can take advantage of background knowledge represented as extensional relations or logical clauses (rules)
- one branch of research in this area – *inductive logic programming* – focuses on learning hypotheses in a logic programming framework
- search can be top-down (like FOIL) or bottom-up
- many relational learning methods not well suited to handling noisy data, representing uncertainty
  - but in the next lecture we'll discuss *statistical relational learning* methods which are designed to address these limitations