Multi-Property-Preserving Hash Domain Extension and the EMD Transform (Enveloped Merkle-Damgård)

Mihir Bellare and Thomas Ristenpart
University of California at San Diego
Security and Cryptography Lab

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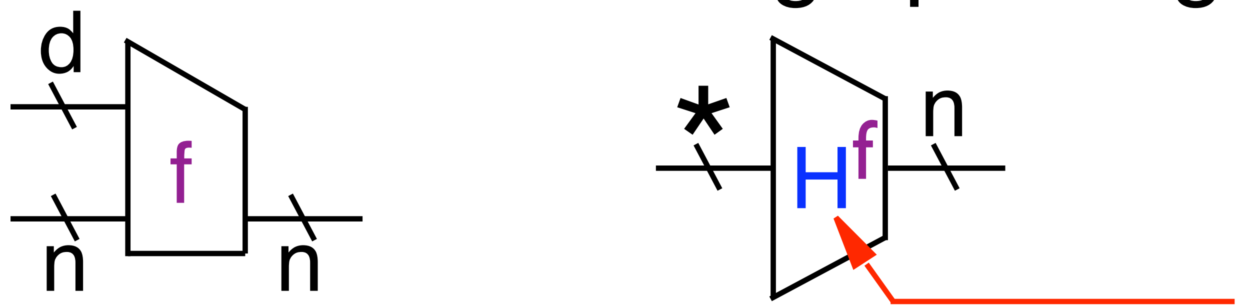
Current hash function design paradigm



Compression function

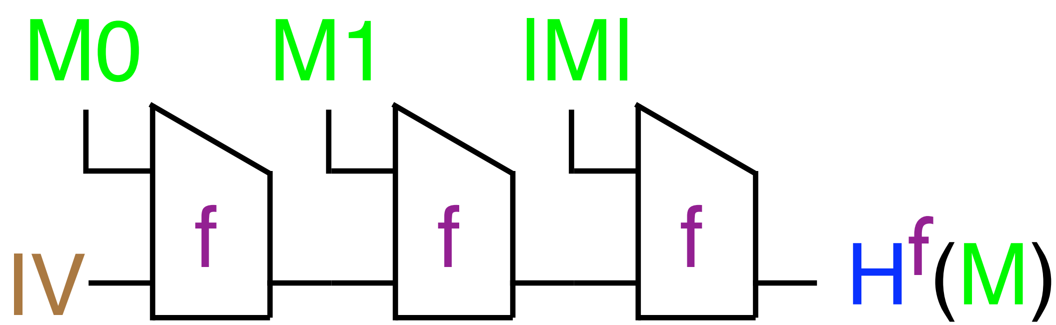
Hash function

Domain extension transform

One wants a transform H that is collision-resistance preserving (**CR-Pr**):

f is **CR** $\Rightarrow$ H^f is **CR**

E.g. H = MD_+ (Merkle-Damgård w/str)



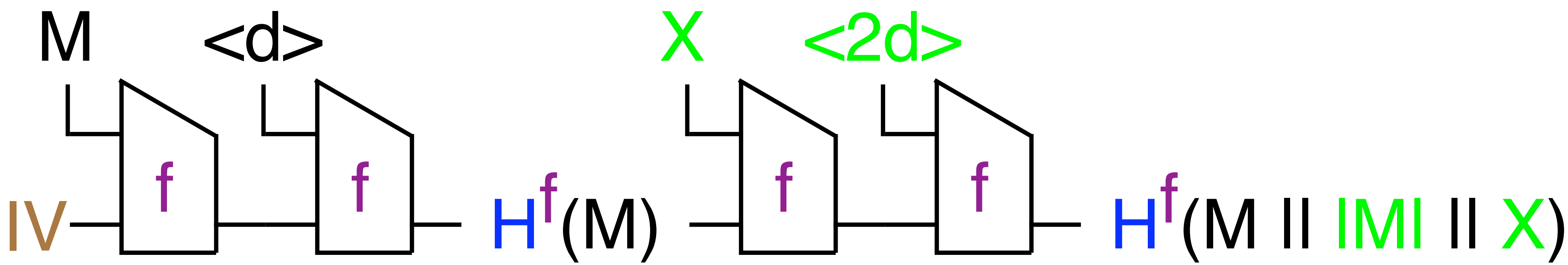
Used in MD4, MD5, SHA-1, SHA-256, etc.

Extension attack

Let H = MD_+ and message M unknown to adversary

$$X, |M|, H^f(M) \xrightarrow{\text{easy}} H^f(M \,||\, |M| \,||\, X)$$

e.g. if $|X| = |M| = d$, then:



So what?

- Does not affect **CR**
- But means that H^f does not "behave like" a RO

Extension attack

Let $H = MD_+$ and message M unknown to adversary

$$X, |M|, H^f(M) \xrightarrow{\text{easy}} H^f(M \,\|\, |M| \,\|\, X)$$

$$X, |M|, RO(M) \xrightarrow{\text{hard}} RO(M \,\|\, |M| \,\|\, X)$$

So what?

- Does not affect **CR**
- But means that H^f does not “behave like” a RO
  This is true even if f is a RO.

[CDMP05]:

- Hash functions widely used as ROs
  e.g. RSA-OAEP [BR94], RSA-PSS [BR96]
  used in PKCS#1 v2.1

- Should (minimally) validate this use
  assuming compression function f is a RO

To that end they ask for domain extension transforms H which are (what we call) pseudo-random-oracle preserving (**PRO-Pr**):

$$f \approx \text{RO} \Rightarrow H^f \approx \text{RO}$$

indifferentiable
[MRH04]

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indifferentiable
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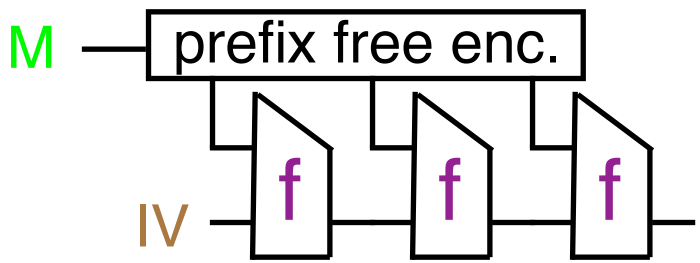
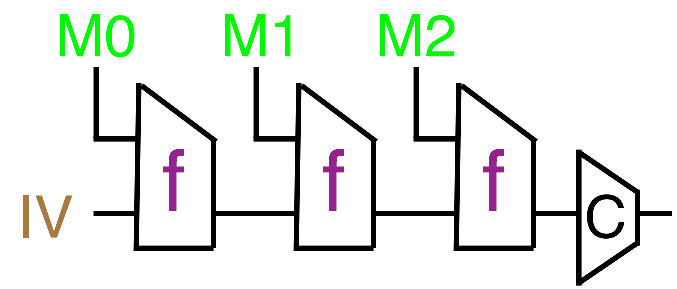
PRO's only exist in the random oracle model

$H = MD_+$ is <u>not</u> **PRO-Pr** (due to extension attack)

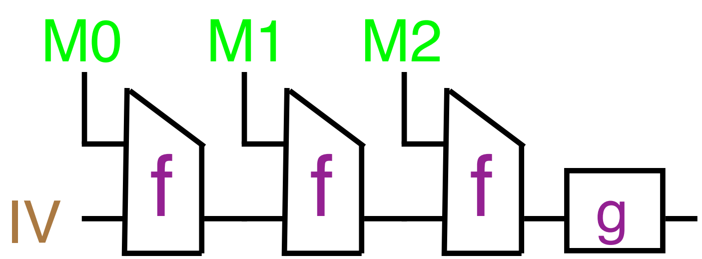
[CDMP05] present several new **PRO-Pr** transforms:



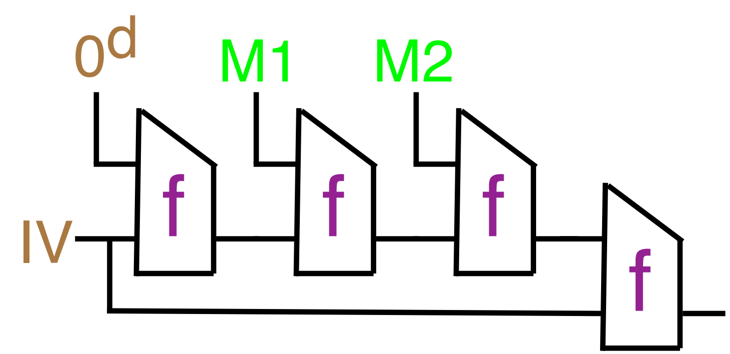
Prefix-free MD



Chop transform



NMAC construction



HMAC construction

PRO-Pr is a desirable property: Important for usage of hash functions as ROs.

But, there is also danger in using **PRO-Pr** transforms...

The same hash functions will be used both as ROs and (just) as **CR** functions.

Will **PRO-Pr** transforms yield CR hash functions?

It might *seem* so:

$$f = \mathrm{RO} \;\Rightarrow\; H^f \approx \mathrm{RO} \;\Rightarrow\; H^f \text{ is } \mathbf{CR}$$

Problem!

PRO-Pr

When f is a real compression function, then

- $f \neq \mathrm{RO}$
- so above does not justify that H^f is **CR**

The problem is real

For each of 4 **PRO-Pr** transforms H proposed in [CDMP05] we show that:

$\exists$f such that

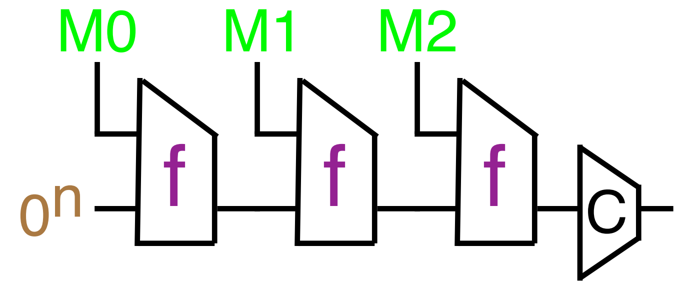
f is **CR** but H^f is not **CR**

In other words

$$\textbf{PRO-Pr} \not\Rightarrow \textbf{CR-Pr}$$

Example: H = chop transform

C outputs first n-s bits of its n bit input

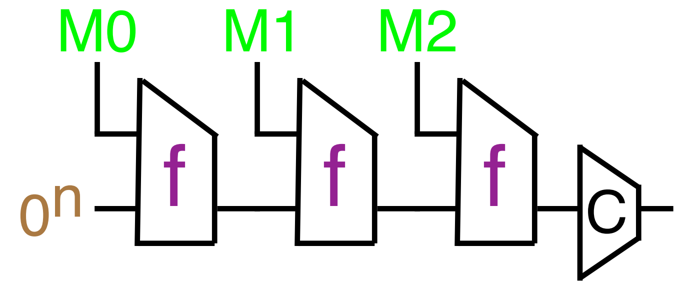


We build a **CR** compression function f for which H^f is not **CR**.

$$\text{Let } f(x) = \begin{cases} 0^n & \text{if } x = 0^{n+d} \\ h(x) \,||\, 1 & \text{otherwise} \end{cases}$$

Claim 1: f is **CR** (assuming h is **CR**)

Example: H = chop transform



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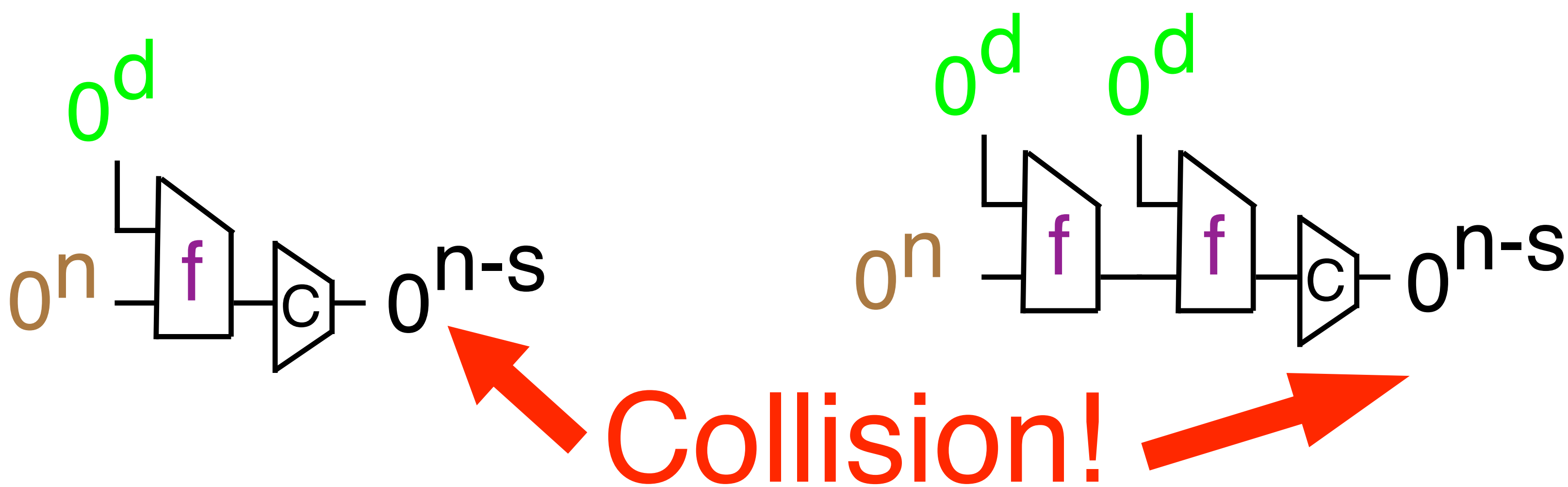


Claim 2: H^f is not **CR**

What this means

For **CR**, guarantee of transforms from [CDMP05] is worse than that of MD_+

Root of problem:

PRO-Pr provides guarantee of security *only in the model* where f = RO.
No guarantee in the standard model!

This speaks against standardizing any of the [CDMP05] transforms

PRO-Pr in review...

+ Important for building hash functions used as ROs

– Does not guarantee H^f is **CR** when f is **CR**

So what types of transforms should we use?

Preserve both **CR** and **PRO**

Natural solution is to require H to be both

1. **CR-Pr** f is **CR** $\Rightarrow H^f$ is **CR**
2. **PRO-Pr** $f = \mathrm{RO} \Rightarrow H^f \approx \mathrm{RO}$

Solves the previous problems with (only) **PRO-Pr** transforms: single hash function good for both uses.

	Random oracles	*Digital signatures*
H is **PRO-Pr**, **CR-Pr**	Alice $\rightleftarrows$ $H^f(.)$ H^f secure if f = RO	Sign($H^f(M)$) H^f secure if f is **CR**
H is just **PRO-Pr**	Alice $\rightleftarrows$ $H^f(.)$ H^f secure if f = RO	Sign($H^f(M)$) H^f secure if f = RO

One can “patch” the [CDMP05] transforms to get them to be both **CR-Pr** and **PRO-Pr**: add strengthening!

but...

Hash functions have all kinds of applications:

CR functions

random oracles

message authentication

key derivation

near-collision resistant functions

one-way functions

others...

Want security guarantees for as many settings as possible

Two very important uses:

message authentication codes (MACs)

key derivation

These require that hash functions be keyed and are good **PRF**s. Does a **CR-Pr**, **PRO-Pr** H suffice?

PRO-Pr transforms again *seem* sufficient:

secret key K

$$f = RO \Rightarrow H^f \approx RO \Rightarrow H^f(K \,||\, .) \text{ is } \mathbf{PRF}$$

Problem!

PRO-Pr

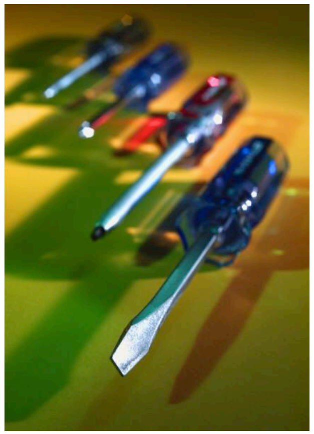
But as before, no guarantee for a real f.

Solution: use multi-property-preserving (MPP) transforms, which simultaneously preserve all properties of interest.

Minimally, we suggest building a single transform H that is simultaneously

1) **CR-Pr** f is **CR** $\Rightarrow$ H^f is **CR**
2) **PRO-Pr** $f = RO \Rightarrow H^f \approx RO$
3) **PRF-Pr** f is **PRF** $\Rightarrow$ H^f is **PRF**

Current situation



Transform	Security	Example Applications
MD w/str	**CR-Pr**	digital signatures
[CDMP05]	**PRO-Pr**	ROs
HMAC/NMAC	**PRF-Pr**	PRF/MAC

Even if one f, must build many hash functions:

- Standardize many hash functions
- Complicates implementations

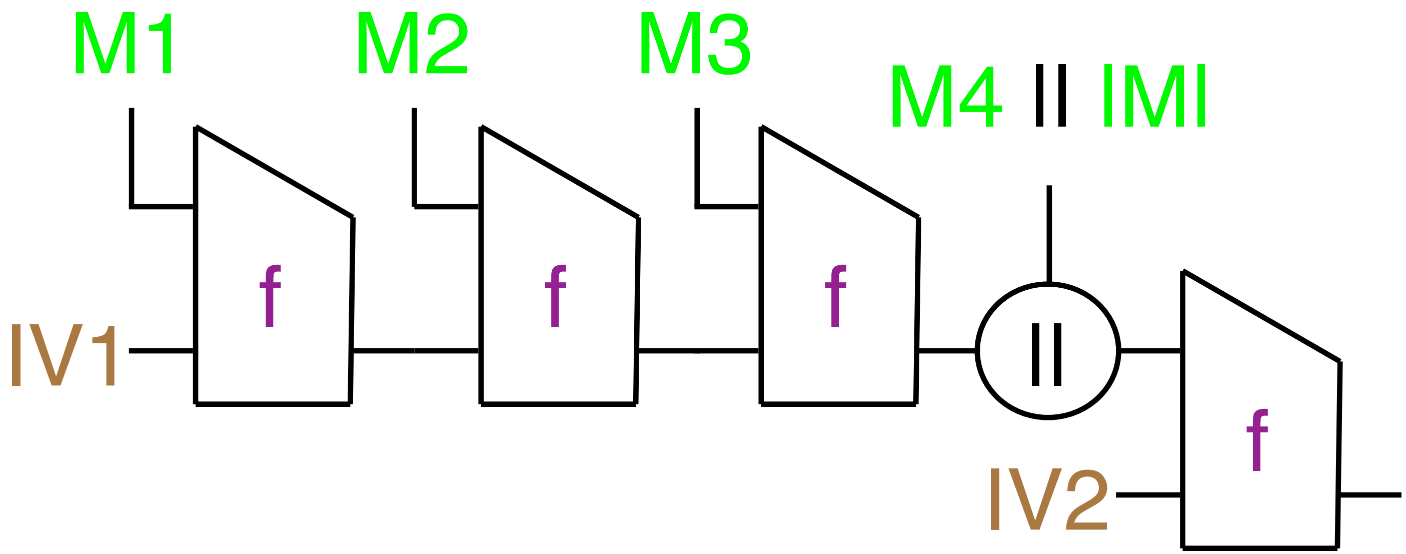
Using MPP approach

Transform	Security	Example Applications
H	**CR-Pr**	digital signatures
	PRO-Pr	ROs
	PRF-Pr	PRF/MAC

Apply H to a single f to build one hash function good for many tasks.

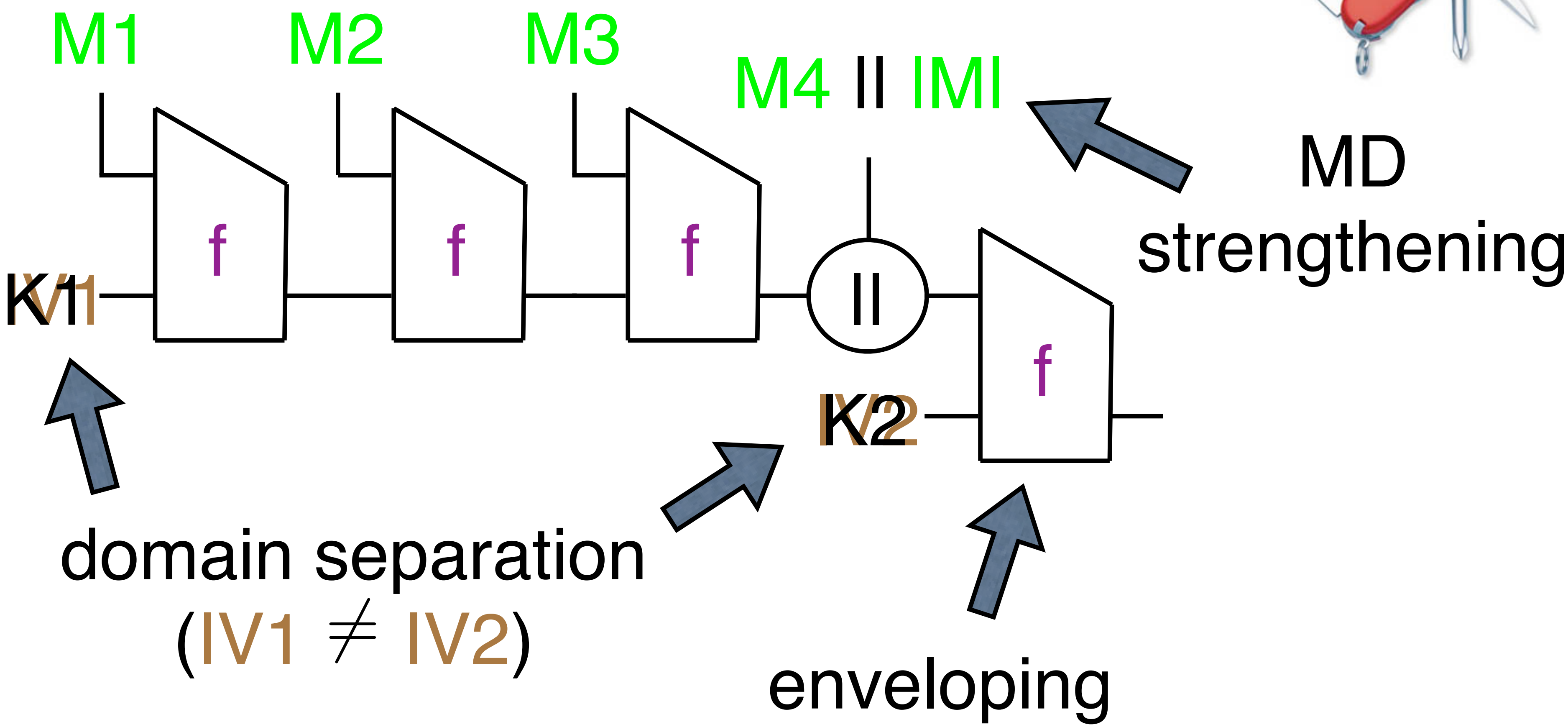
- \+ Standardize just one hash function
- \+ Simplifies implementation choices, one hardware implementation needed

The EMD transform



- Similar in design to NMAC [BCK96], Chain shift construction [MS05].
- Combines several techniques for preserving individual properties.

The EMD transform



EMD is **CR-Pr**

EMD is **PRO-Pr**

EMD is **PRF-Pr**

Theorem **[EMD is CR-Pr]** *Fix n, d, and let $IV1, IV2 \in \{0,1\}^n$ with $IV1 \neq IV2$. Let $f\colon \{0,1\}^{n+d} \to \{0,1\}^n$. Let A be a CR adversary that runs in time t_A. Then there exists an adversary B such that*

$$\mathbf{Adv}^{\mathrm{cr}}_{\mathrm{EMD}}(A) \leq \mathbf{Adv}^{\mathrm{cr}}_{f}(B)$$

where B runs in time $t \leq t_A + \mathcal{O}(l)$ where l is the number of blocks in the longer message output by A.

Theorem 5.2 **[EMD is PRO-Pr]** *Fix n, d, and let $IV1, IV2 \in \{0,1\}^n$ with $IV1 \neq IV2$. Let $f = \mathsf{RF}_{d+n,n}$ be a random oracle. Let A be an adversary that asks at most q_L left queries (each of length no larger than ld bits), q_1 right queries with lowest n bits not equal to $IV2$, q_2 right queries with lowest n bits equal to $IV2$, and runs in time t. Then*

$$\mathbf{Adv}^{\mathrm{pro}}_{\mathrm{EMD},\,SA}(A) \leq \frac{(q_L + q_2)^2 + q_1^2 + q_2 q_1}{2^n} + \frac{l q_L^2}{2^n}\,.$$

where the simulator SA, defined in Fig. 4, makes $q_{SA} \leq q_2$ queries and runs in time $\mathcal{O}(q_1^2 + q_2 q_1)$.

Theorem 5.3 **[EMD is PRF-Pr]** *Fix n, d and let $e\colon \{0,1\}^{d+n} \to \{0,1\}^n$ be a function family keyed via the low n bits of its input. Let A be a prf-adversary against keyed* EMD *using q queries of length at most m blocks and running in time t. Then there exists prf-adversaries A_1 and A_2 against e such that*

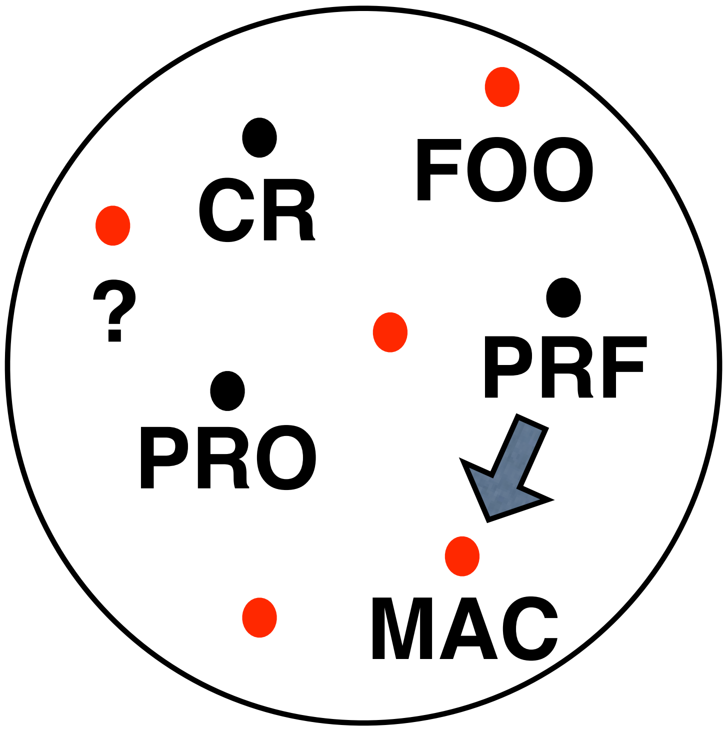
$$\mathbf{Adv}^{\mathrm{prf}}_{\mathrm{EMD}^e_{K_1,K_2}}(A) \leq \mathbf{Adv}^{\mathrm{prf}}_{e}(A_1) + \binom{q}{2}\left[2m \cdot \mathbf{Adv}^{\mathrm{prf}}_{e}(A_2) + \frac{1}{2^n}\right]$$

where A_1 utilizes q queries and runs in time at most t and A_2 utilizes at most two oracle queries and runs in time $\mathcal{O}(mT_e)$ where T_e is the time for one computation of e.

Transform	**CR-Pr**	**PRO-Pr**	**PRF-Pr**	Efficiency $\lvert M \rvert$ = b >= d
EMD	✓ [BR06]	✓ [BR06]	✓ [BR06]	[(b+1+64+n) / d]
Plain MD	✗	✗	✗	[(b+1) / d]
MD w/str	✓ [D89,M89]	✗	✗	[(b+1+64) / d]
Prefix-free MD	✗	✓ [CDMP05]	✓ [BCK96]	[(b+1) / (d-1)]
Chop solution	✗	✓ [CDMP05]	?	[(b+1) / d]
NMAC construction	✗	✓ [CDMP05]	?	1 + [(b+1) / d]
HMAC construction	✗	✓ [CDMP05]	?	2 + [(b+1) / d]

What about other properties?

Choices to make...



Some properties implied by others (e.g., PRF => MAC)

Should only worry about *useful* properties

Design trade-offs: security versus efficiency

Summary

We propose multi-property-preserving transforms for building the next generation of hash functions

- Minimally a transform H should be **CR-Pr**, **PRO-Pr**, and **PRF-Pr**
- Enables building a single hash function that is good for a variety of applications

We point out that previous **PRO-Pr** transforms are not **CR-Pr** and thus give worse guarantees than MD_+

We describe an efficient MPP transform EMD (Enveloped Merkle-Damgård)

Multi-Property-Preserving Hash Domain Extension
and
the EMD Transform
(Enveloped Merkle-Damgård)