CS 540 Introduction to Artificial Intelligence
Logic

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Based on slides by Fred Sala

Logic & AI

Why are we studying logic?

- **Traditional** approach to AI ('50s-'80s)
  - "Symbolic AI"
  - The Logic Theorist – 1956
    - Proved a bunch of theorems!
- Logic also the language of:
  - Knowledge rep., databases, etc.

Two main approaches to AI in the early stage of AI: Symbolism using logic, and Connectionism using models in particular artificial neural networks.

Symbolic Techniques in AI

Lots of systems based on symbolic approach:

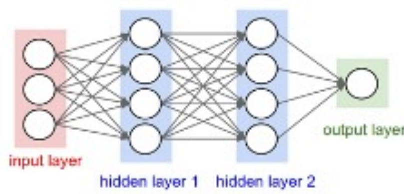
- Ex: expert systems, planning, more
- Playing great chess
- **Less popular recently**!

J. Gardner

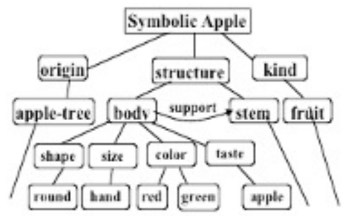
Symbolic vs Connectionist

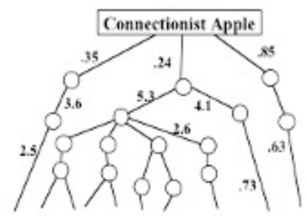
Rival approach: **connectionist**

- Probabilistic models
- Neural networks
- **Extremely popular** last 20 years



Stanford CS231n



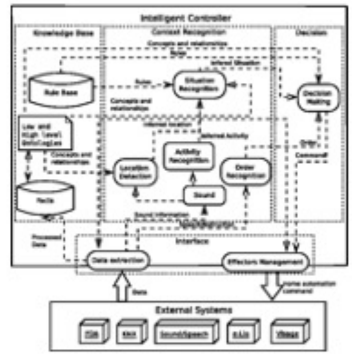


M. Minsky

Symbolic vs Connectionist

Analogy: Logic versus probability

- Which is better?
- Future: combination; best-of-both-worlds
  - Actually been worked on:
  - **Example**: Markov Logic Networks

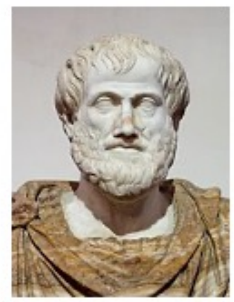


Outline

- Introduction to logic
  - Arguments, validity, soundness
- Propositional logic
  - Sentences, semantics, inference
- First order logic (FOL)
  - Predicates, objects, formulas, quantifiers

Basic Logic

- Arguments, premises, conclusions
  - Argument: a set of sentences (premises) + a sentence (a conclusion)
  - **Validity:** argument is valid iff it's necessary that if all premises are true, the conclusion is true
  - **Soundness:** argument is sound iff valid & premises true
  - **Entailment:** when valid arg., premises entail conclusion



Propositional Logic Basics

Logic Vocabulary:

- Sentences, symbols, connectives, parentheses
  - Symbols: P, Q, R, … (**atomic** sentences)
  - Connectives:

$\wedge$	and	[conjunction]
$\vee$	or	[disjunction]
$\Rightarrow$	implies	[implication]
$\Leftrightarrow$	is equivalent	[biconditional]
$\neg$	not	[negation]

  - Literal: P or negation $\neg$P

There are various logic systems. Propositional Logic is a standard one.

Its sentences are constructed using symbols, connectives, and parentheses, following some grammar.

Propositional Logic Basics

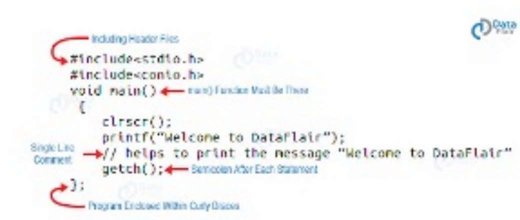
Examples:

- $(P \vee Q) \Rightarrow S$
  - "If it is cold or it is raining, then I need a jacket"
- $Q \Rightarrow P$
  - "If it is raining, then it is cold"
- $\neg R$
  - "It is not hot"

Propositional Logic Basics

Several rules in place

- Precedence: $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- Use parentheses when needed
- Sentences: **well-formed** or not well-formed:
  - $P \Rightarrow Q \Rightarrow S$ **X (not associative!)**



Simple grammar for building sentences in propositional logic.

Sentences & Semantics

- Think of symbols as defined by user
- Sentences: built up from symbols with connectives

  - **Interpretation**: assigning True / False to symbols
  - **Semantics**: interpretations for which sentence evaluates to True
  - **Model**: (of a set of sentences) interpretation for which all sentences are True

Once we have the vocabulary and the grammar, then we can build sentences, and form arguments. This is the syntactic part of the logic system.

The other part of the logic system is the sematic part.

Evaluating a Sentence

- Example:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

- Note:
  - If P is false, P⇒Q is true regardless of Q (“5 is even implies 6 is odd” is True!)
  - Causality unneeded: “5 is odd implies the Sun is a star” is True!)

Evaluating a Sentence: Truth Table

- **Ex**:

P	Q	R	¬P	Q∧R	¬P∨Q∧R	¬P∨Q∧R⇒Q
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	0	0	0	1
1	1	0	0	0	0	1
1	1	1	0	1	1	1

- **Satisfiable**
  - There exists some interpretation where sentence true

Break & Quiz

Q 1.1: Suppose P is false, Q is true, and R is true. Does this assignment satisfy

(i) $\neg(\neg P \rightarrow \neg Q) \wedge R$

(ii) $(\neg P \vee \neg Q) \rightarrow (P \vee \neg R)$

- A. Both
- B. Neither
- C. Just (i)
- D. Just (ii)

Break & Quiz

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- **C. Just (i)**
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Break & Quiz

Q 1.2: Let A =“Aldo is Italian” and B =“Bob is English”. Formalize “Aldo is Italian or if Aldo isn’t Italian then Bob is English”.

- a. $A \lor (\neg A \rightarrow B)$
- b. $A \lor \neg B$
- c. $A \lor (A \rightarrow B)$
- d. $A \rightarrow B$

Break & Quiz

Q 1.2: Let A =“Aldo is Italian” and B =“Bob is English”. Formalize “Aldo is Italian or if Aldo isn’t Italian then Bob is English”.

- **a. $A \lor (\neg A \rightarrow B)$**
- b. $A \lor \neg B$
- c. $A \lor (A \rightarrow B)$
- d. $A \rightarrow B$

Break & Quiz

Q 1.3: How many different assignments can there be to

$(x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \ldots \vee (x_n \wedge y_n)$

- A. 2
- B. 2^n
- C. 2^{2n}
- D. 2n

Break & Quiz

Q 1.3: How many different assignments can there be to
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- A. 2
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- **C. 2^{2n}**
- D. 2n

Knowledge Bases

- **Knowledge Base (KB)**: A set of sentences
  - Like a long sentence, connect with conjunction

Model of a KB: interpretations where all sentences are True

Goal: inference to discover new sentences

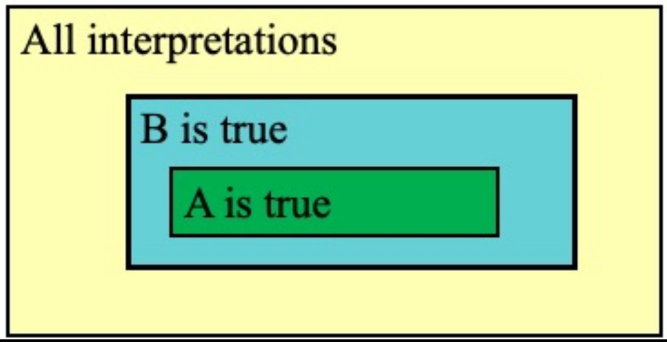
Once we learn the syntax and semantics of the logic system, we think of making use of it.

We would like to discover new sentences from a given sentences, by doing syntactic operations but ensuring semantics of the discovered sentences.

Entailment

Entailment: a sentence logically follows from others

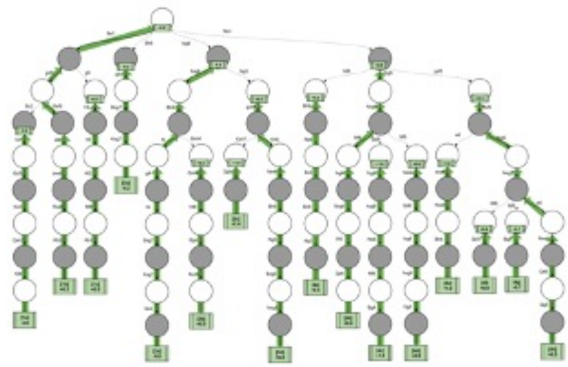
- Like from a KB. Write A ⊨ B
- A ⊨ B iff in every interpretation where A is true, B is also true



ensuring semantics of the discovered sentences: entailment

Inference

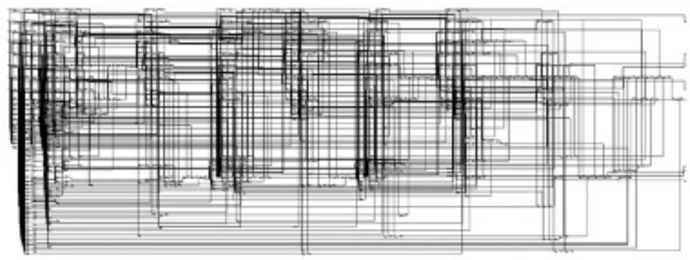
- Given a set of sentences (a KB), **logical inference** creates new sentences
  - Compare to prob. inference!
- **Challenges**:
  - Soundness
  - Completeness
  - Efficiency



doing syntactic operations: inference methods. There are many inference methods, here we only consider 3 examples.

Methods of Inference: **1. Enumeration**

- Enumerate all interpretations; look at the truth table
  - "Model checking"
- Downside: 2^n interpretations for n symbols



S. Leadley

Methods of Inference: **2. Using Rules**

- *Modus Ponens*: $(A \Rightarrow B, A) \vDash B$
- And-elimination
- Many other rules
  - Commutativity, associativity, de Morgan's laws, distribution for conjunction/disjunction

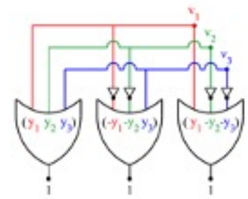
Methods of Inference: **3. Resolution**

- Convert to special form and use a single rule
- **Conjunctive Normal Form** (CNF)

$$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$$

a clause

Conjunction of clauses; each clause disjunction of literals



- Simple rules for converting to CNF

Methods of Inference: **3. Resolution**

Start with our KB and **query** B

- Add $\neg$B
- Show that this leads to a contradiction
- Take clauses with a symbol and its complement
  - Merge, throw away symbol: $P \vee Q \vee R$, $\neg Q \vee S \vee T$: $P \vee R \vee S \vee T$
  - If no symbol left, KB entails B
  - No new clauses, KB does not entail B

Break & Quiz

Q 2.1: Which is larger: the number of rows in a truth table on *n* symbols, or the number of entries in a joint distribution table on *n* binary random variables?

- A. Truth table
- B. Distribution
- C. Same size
- D. It depends

Break & Quiz

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Both are 2^n.

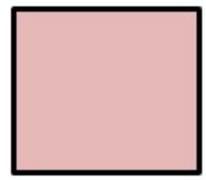
First Order Logic (FOL)

Propositional logic has some limitations

- Ex: how to say “all squares have four sides”
- No context, hard to generalize; express facts

FOL is a more expressive logic; works over

- Facts, Objects, Relations, Functions



FOL: more expressive by introducing quantifiers and allowing context.

First Order Logic (FOL)

Basics:

- Constants: “16”, “Green”, “Bob”
- Functions: map objects to objects
- Predicates: map objects to T/F:
  - Greater(5,3)
  - Color(grass, green)

These can be used to express context.

First Order Logic (FOL)

Basics:

- Variables: x, y, z
- Connectives: Same as propositional logic
- Quantifiers:
  - ∀ universal quantifier: $\forall x\ \mathbf{human(x)} \Rightarrow \mathbf{mammal(x)}$
  - ∃ existential quantifier: $\exists x\ \mathbf{mammal(x)}$

The quantifiers can be used to express “all” or “exist” kinds of statement.