CS 540 Introduction to Artificial Intelligence
Reinforcement Learning II

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Based on slides by Fred Sala

Announcements (details on Piazza)

- Final Exam information
  - On Canvas/Quizzes as midterm; but no one-day window
  - Main: Dec 20 2:45-4:45pm
  - Makeup: Dec 23 2:45-4:45pm

- Course Evaluation
  - Dec 1 to Dec 15
  - Explicit incentive: some details about the final exam if the participation rate reaches 50%/75%/95%

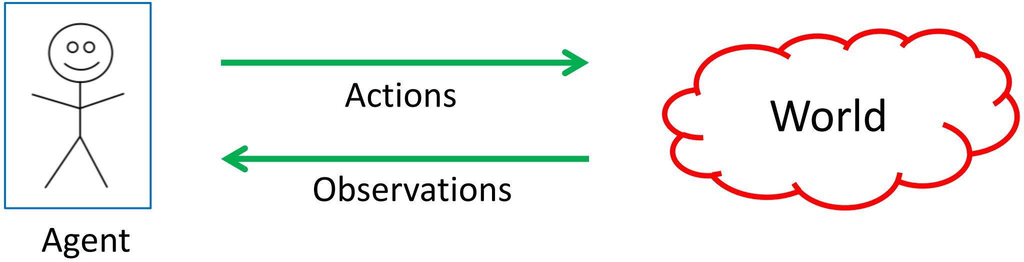
Outline

- Review of reinforcement learning
  - MDPs, value functions, Bellman Equation, value iteration
- Q-learning
  - Q function, Q-learning, epsilon-greedy, SARSA

Building The Theoretical Model

Basic setup:

- Set of states, S
- Set of actions A



- Information: at time t, observe state $s_t \in$ S. Get reward r_t
- Agent makes choice $a_t \in$ A. State changes to $s_{t+1,}$ continue

Goal: find a map from **states to actions** maximize rewards.

A "policy"

Markov Decision Process (MDP)

The formal mathematical model:

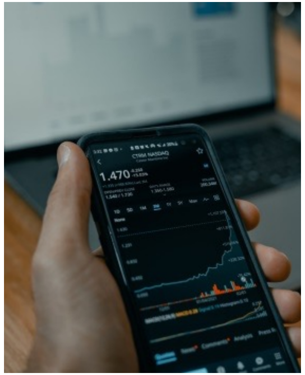
- **State set** S. Initial state s_0. **Action set** A
- **State transition model:** $P(s_{t+1} | s_t, a_t)$
  - Markov assumption: transition probability only depends on s_t and a_t, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state.

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

Defining the Optimal Policy

For policy π, **expected utility** over all possible state sequences from s_0 produced by following that policy:

$$V^{\pi}(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$



Called the **value function** (for π, s_0)

Discounting Rewards

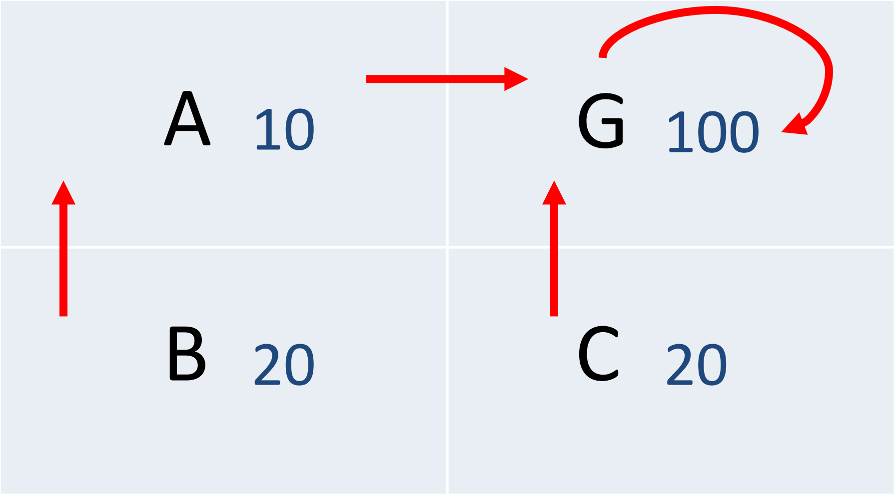
One issue: these are infinite series. **Convergence**?

- Solution

$$U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is VS **future**
  - Note: has to be less than 1 for convergence

Example



Deterministic transition. $\gamma = 0.8$, policy shown in red arrow.

Values and Policies

Now that $V^{\pi}(s_0)$ is defined what a should we take?

- First, set V*(s) to be expected utility for **optimal** policy from s
- What's the expected utility of an action?
  - Specifically, action a in state s?

$$\sum_{s'} P(s'|s,a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

Obtaining the Optimal Policy

We know the expected utility of an action.

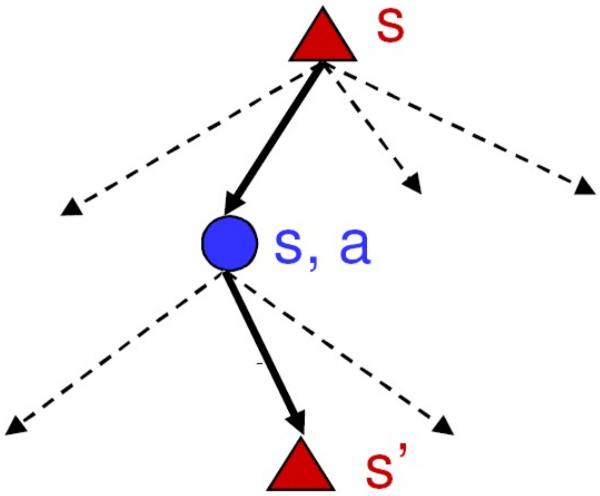
- So, to get the optimal policy, compute

$$\pi^*(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a) V^*(s')$$

All the states we could go to

Transition probability

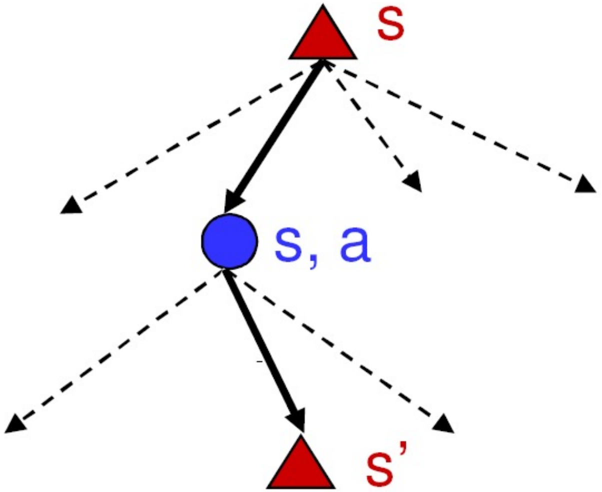
Expected rewards



Credit L. Lazbenik

Bellman Equation

Let's walk over one step for the value function:



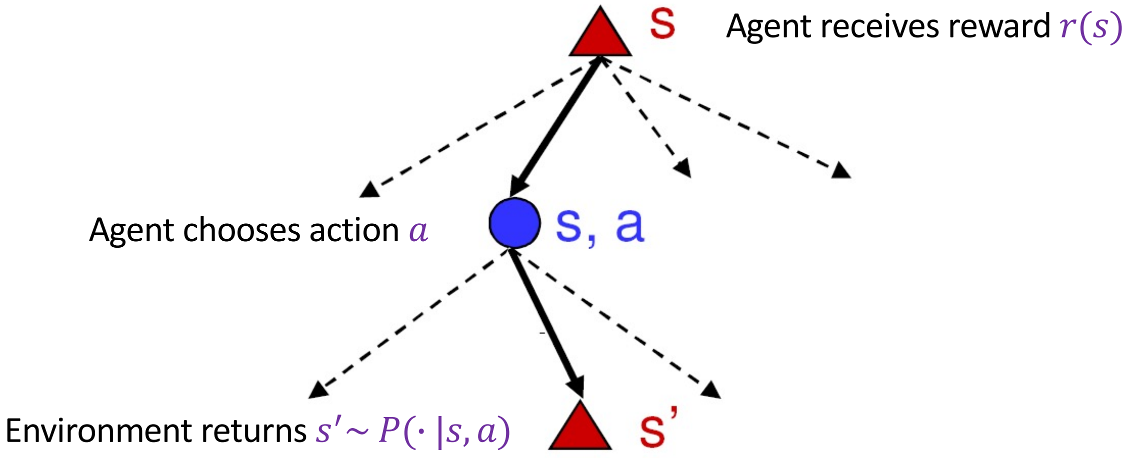
Credit L. Lazbenik

$$V^*(s) = r(s) + \gamma \underbrace{\max_a \sum_{s'} P(s'|s,a)V^*(s')}_{\text{Discounted expected future rewards}}$$

Current state reward

Discounted expected future **rewards**

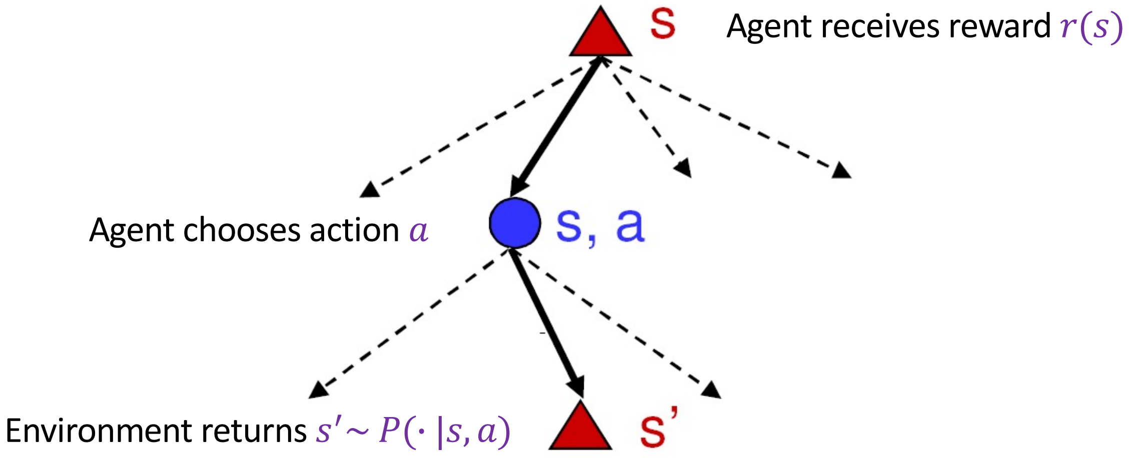
The Bellman equation



- Define state utility $V^*(s)$ as the expected sum of discounted rewards if the agent executes an *optimal* policy starting in state s

Image source: L. Lazbenik

The Bellman equation



- What is the expected utility of taking action **a** in state **s**?

$$\sum_{s'} P(s'|s,a)V^*(s')$$

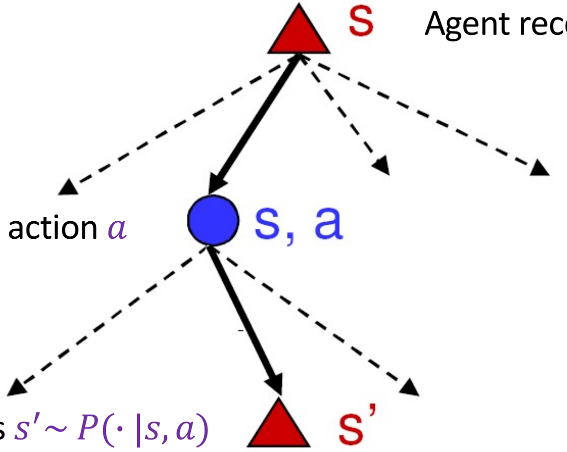
Image source: L. Lazbenik

The Bellman equation

Agent receives reward $r(s)$

Agent chooses action a

Environment returns $s' \sim P(\cdot\,|s,a)$

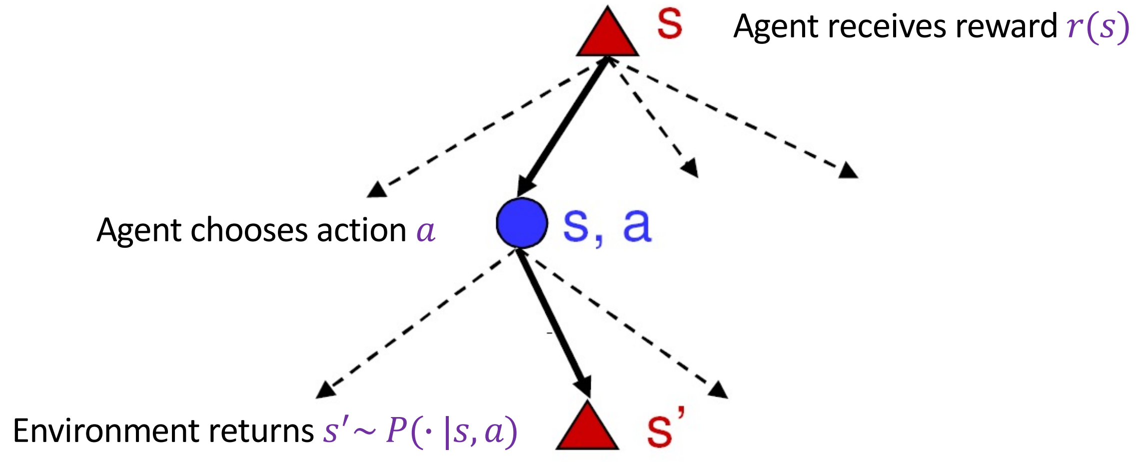


- What is the recursive expression for $V^*(s)$ in terms of $V^*(s')$ - the utilities of its successors?

$$V^*(s) = r(s) + \gamma \sum_{s'} P(s'|s,\pi^*(s)\,)V^*(s')$$

Image source: L. Lazbenik

The Bellman equation



- How do we choose the action?

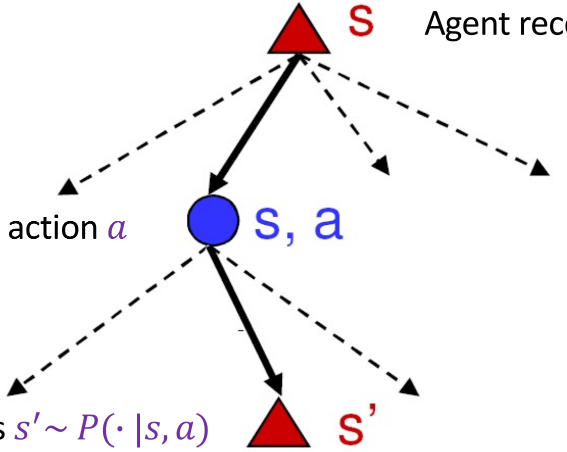
$$\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s,a)V^*(s')$$

Image source: L. Lazbenik

The Bellman equation

Agent receives reward $r(s)$

Agent chooses action a



Environment returns $s' \sim P(\cdot\,|s,a)$

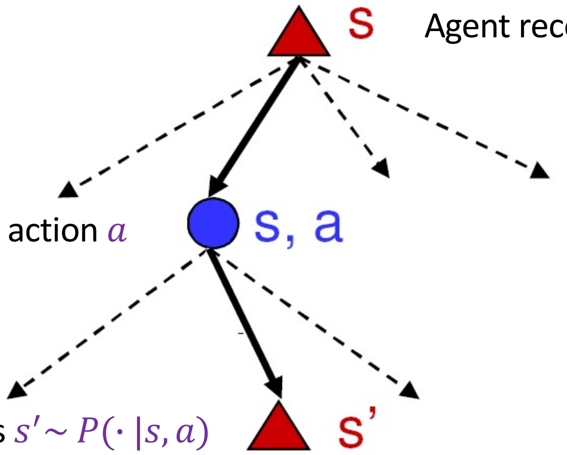
- What is the recursive expression for $V^*(s)$ in terms of $V^*(s')$ - the utilities of its successors?

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')$$

Image source: L. Lazbenik

The Bellman equation

Agent receives reward $r(s)$



Agent chooses action a

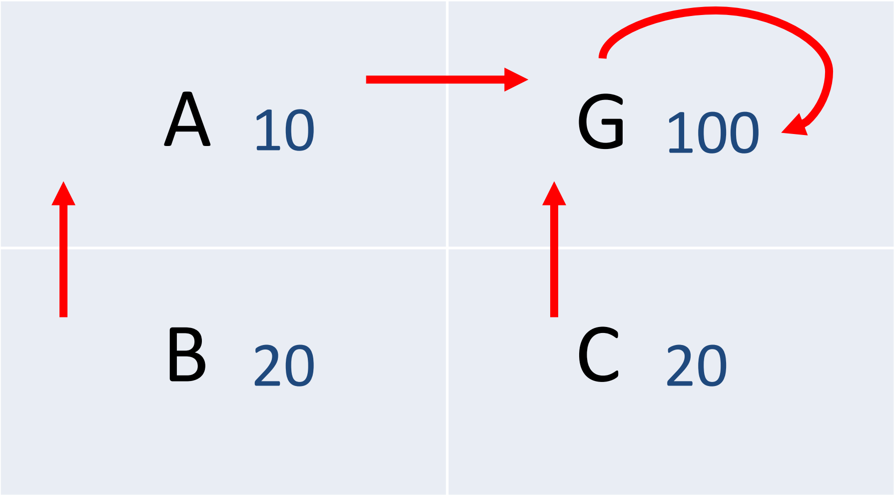
Environment returns $s' \sim P(\cdot\,|s,a)$

- The same reasoning gives the Bellman equation for a general policy:

$$V^{\pi}(s) = r(s) + \gamma \sum_{s'} P(s'|s, \pi(s)\,)V^{\pi}(s')$$

Image source: L. Lazbenik

Example



Deterministic transition. $\gamma = 0.8$, policy shown in red arrow.

Value Iteration

Q: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

A: Use the property. Start with $V_0(s)=0$. Then, update

$$V_{i+1}(s) = r(s) + \gamma \max_{a} \sum_{s'} P(s'|s,a) V_i(s')$$

Q-Learning

What if we don't know transition probability P(s'|s,a)?

- Need a way to learn to act without it
- **Q-learning**: get an action-utility function Q(s,a) that tells us the value of doing a in state s (including the reward in s)

$$Q(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a)V^*(s')$$

- Note: $V^*(s) = \max_a Q(s,a)$
- Now, we can just do $\pi^*(s) = \arg\max_a Q(s,a)$
  - But need to estimate Q!

Q-Learning Iteration

How do we get Q(*s*,*a*)?

- Similar iterative procedure

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Learning rate

Idea: combine old value and new estimate of future value.

Note: We are using a policy to take actions; based on the estimated Q!

Exploration Vs. Exploitation

General question!

- **Exploration:** take an action with unknown consequences
  - Pros:
    - Get a more accurate model of the environment
    - Discover higher-reward states than the ones found so far
  - Cons:
    - When exploring, not maximizing your utility
    - Something bad might happen
- **Exploitation:** go with the best strategy found so far
  - Pros:
    - Maximize reward as reflected in the current utility estimates
    - Avoid bad stuff
  - Cons:
    - Might also prevent you from discovering the true optimal strategy

Q-Learning: Epsilon-Greedy Policy

How to **explore**?

- With some 0<ε<1 probability, take a random action at each state, or else the action with highest Q(*s*,*a*) value.

$$a = \begin{cases} \text{argmax}_{a \in A} Q(s, a) & \text{uniform}(0,1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$

Q-Learning: SARSA

An alternative:

- Just use the next action, no max over actions:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

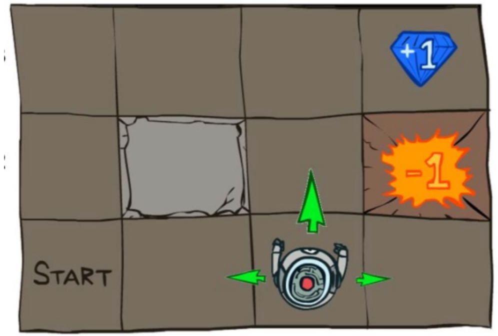
Learning rate

- Called state–action–reward–state–action (**SARSA**)
- Can use with epsilon-greedy policy

Q-Learning Details

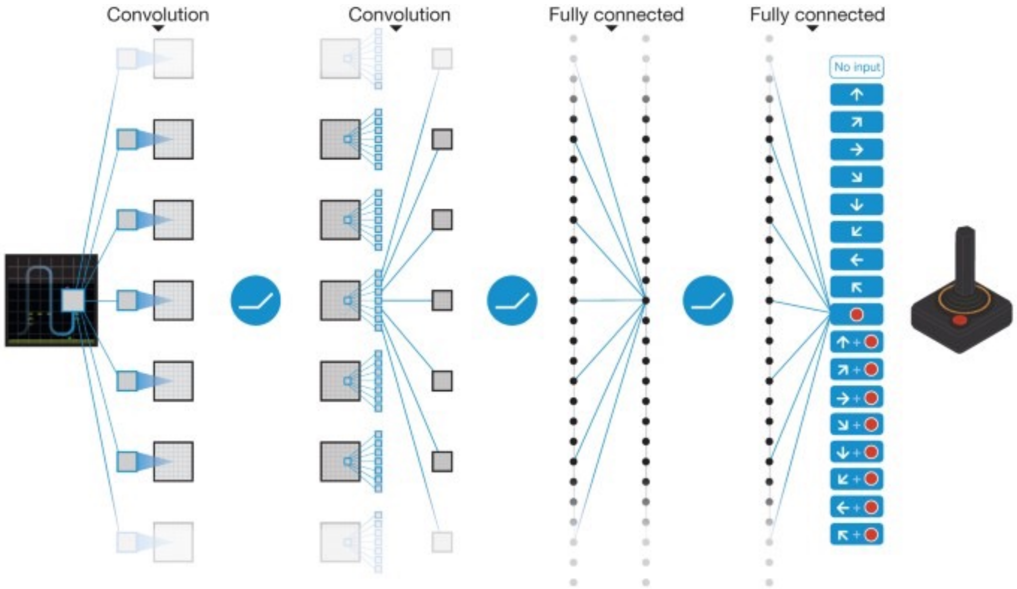
Note: if we have a **terminal** state, the process ends

- An **episode**: a sequence of states ending at a terminal state
- Want to run on many episodes
- Slightly different Q-update for terminal states (see homework!)



Deep Q-Learning

How do we get Q(s,a)?



Mnih et al, "Human-level control through deep reinforcement learning"

Summary of RL

- Reinforcement learning setup
- Mathematical formulation: MDP
- Value functions & the Bellman equation
- Value iteration
- Q-learning

Acknowledgements: Based on slides from Yin Li, Jerry Zhu, Svetlana Lazebnik, Yingyu Liang, David Page, Mark Craven, Pieter Abbeel, Dan Klein