Propositional Logic
Part 2

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Method 4: chaining with Horn clauses

- Resolution is too powerful for many practical situations.
- A weaker form: Horn clauses
  - Disjunction of literals with at most one positive

$$\neg R \vee P \vee Q \qquad \text{no}$$

$$\neg R \vee \neg P \vee Q \qquad \text{yes}$$

  - What's the big deal?

    $\neg R \vee \neg P \vee Q$

    $\neg (R \wedge P) \vee Q$

    ?

Horn clauses

$\neg R \lor \neg P \lor Q$

$\neg(R \land P) \lor Q$

$(R \land P) \Rightarrow Q$ Every rule in KB is in this form

P   (special case, no negative literals): fact

- The big deal:
    - KB easy for human to read
    - Natural forward chaining and backward chaining algorithm, proof easy for human to read
    - Deciding entailment with Horn clauses in time linear to KB size
- But…
    - Can only ask atomic queries

Forward chaining

- Fire any rule whose premises are satisfied in the KB
- Add its conclusion to the KB until query is found

KB:

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
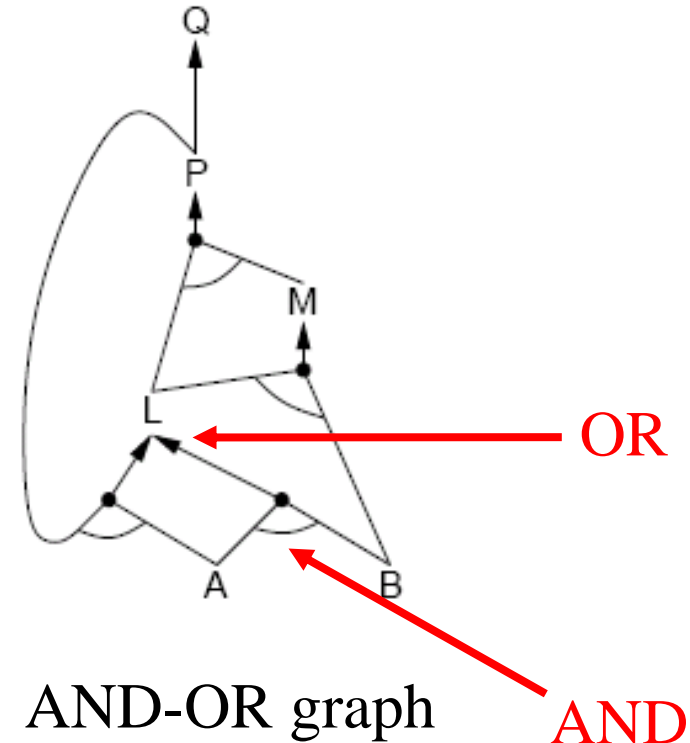$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
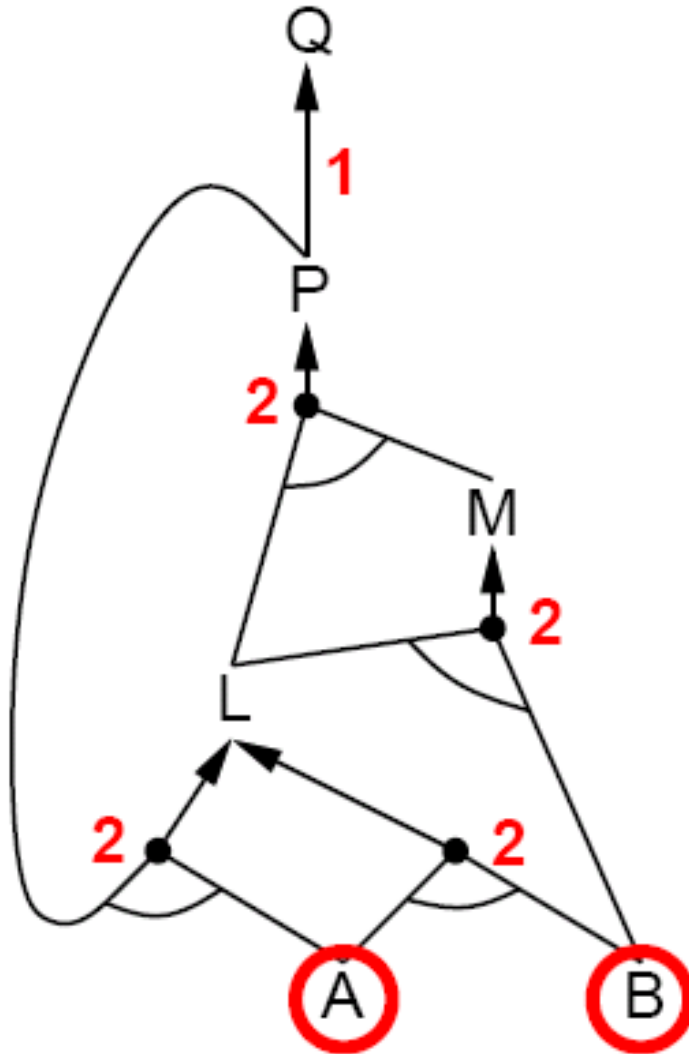$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

query: Q

AND-OR graph

OR

AND

Forward chaining

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Forward chaining

$$P \Rightarrow Q$$
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Forward chaining

$$P \Rightarrow Q$$
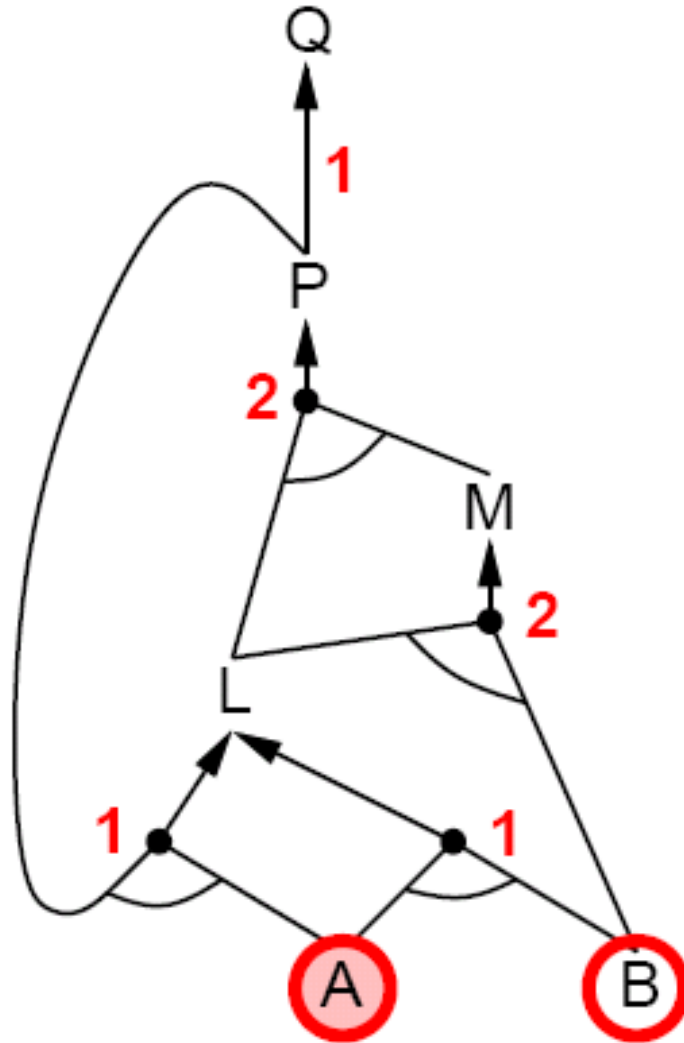$$L \wedge M \Rightarrow P$$
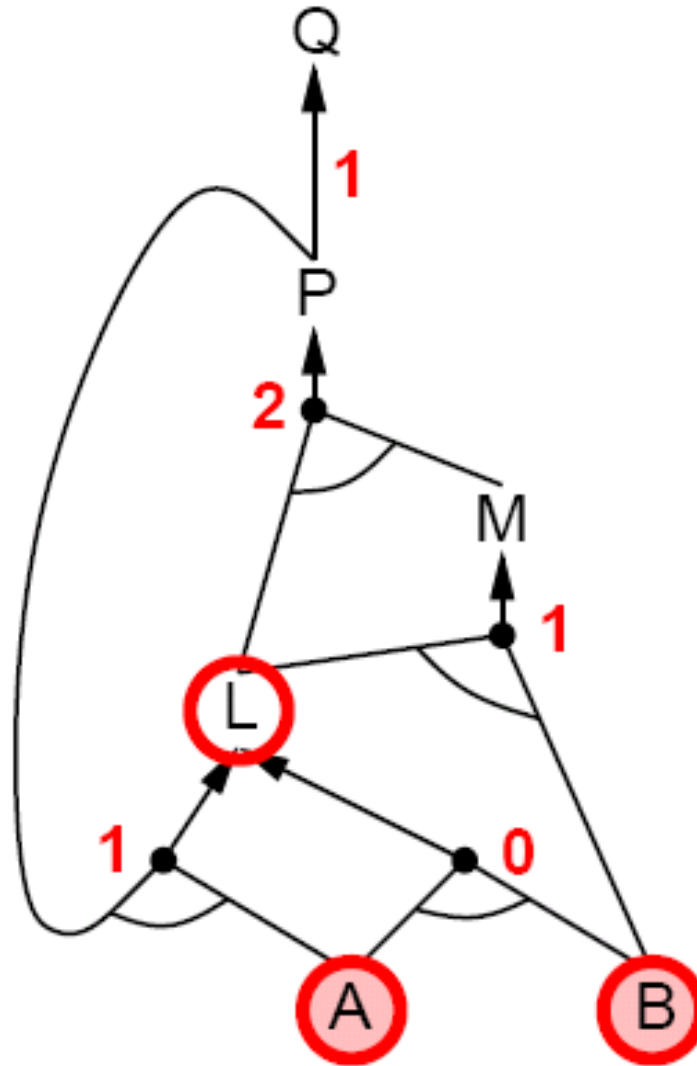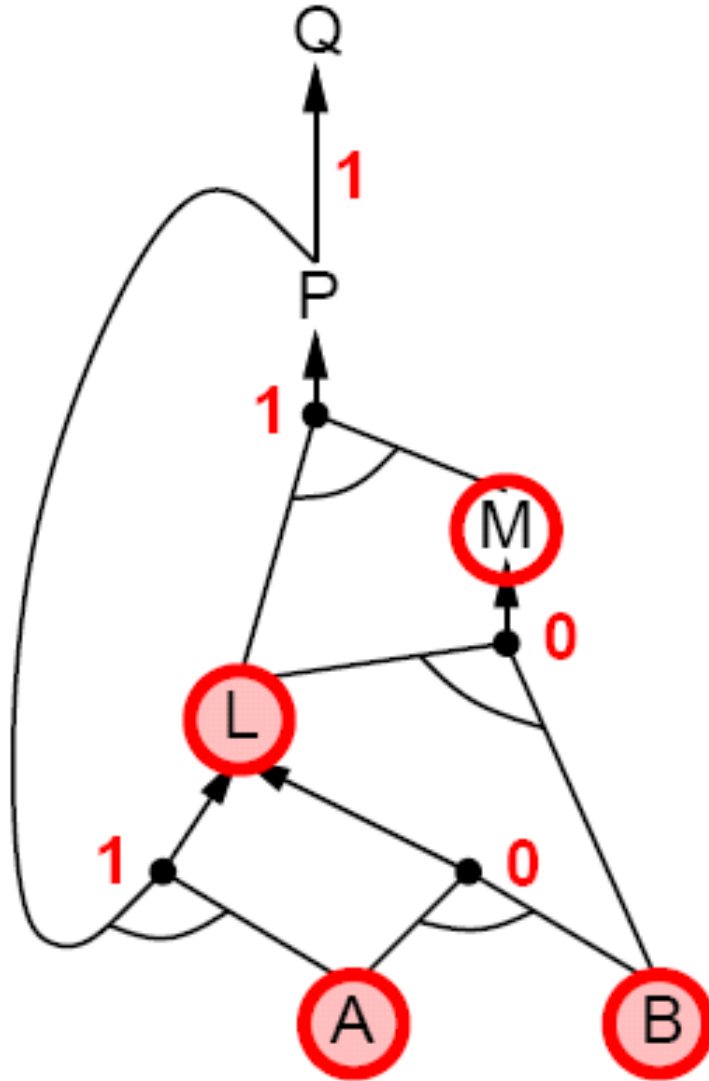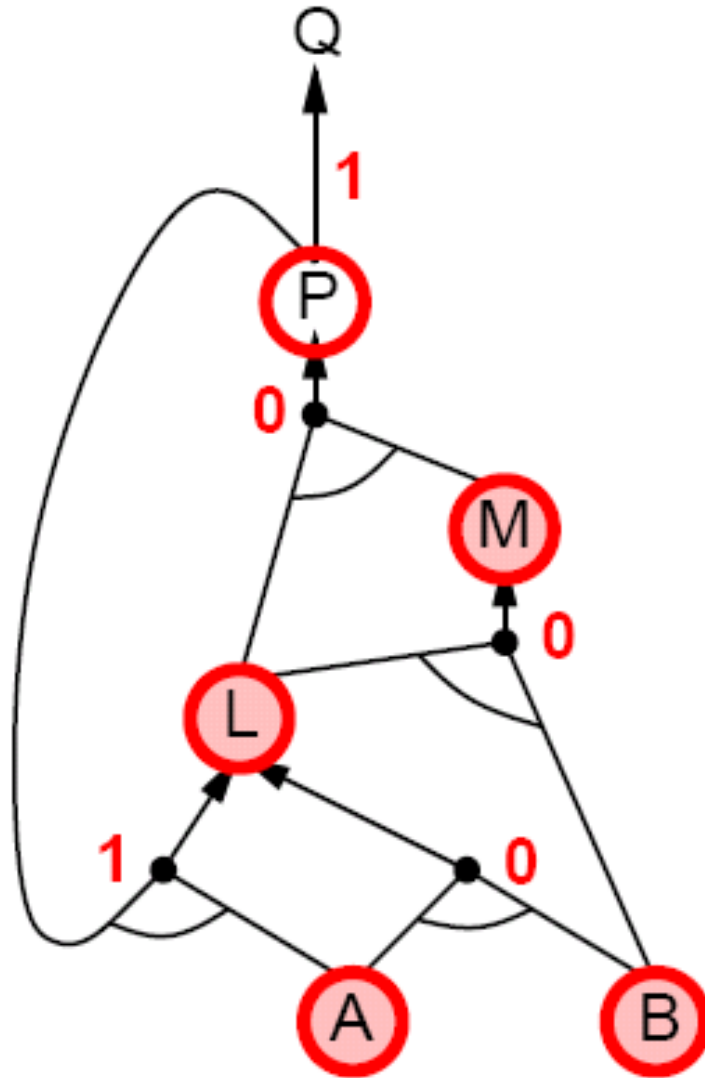$$B \wedge L \Rightarrow M$$
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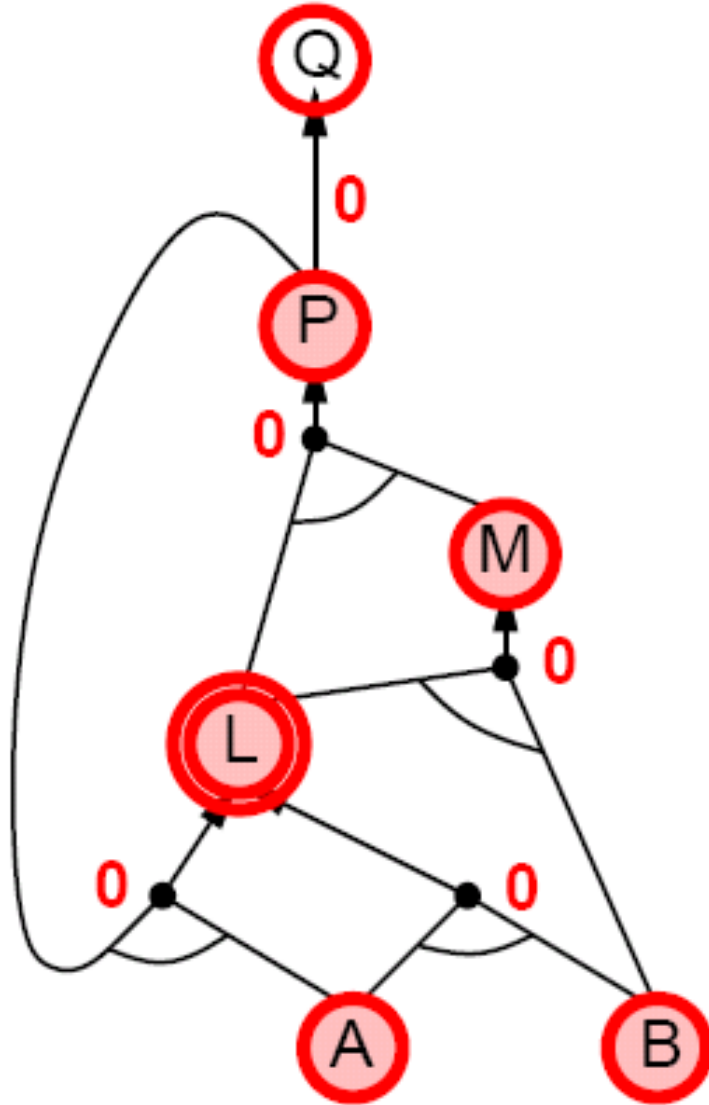$$A \wedge P \Rightarrow L$$
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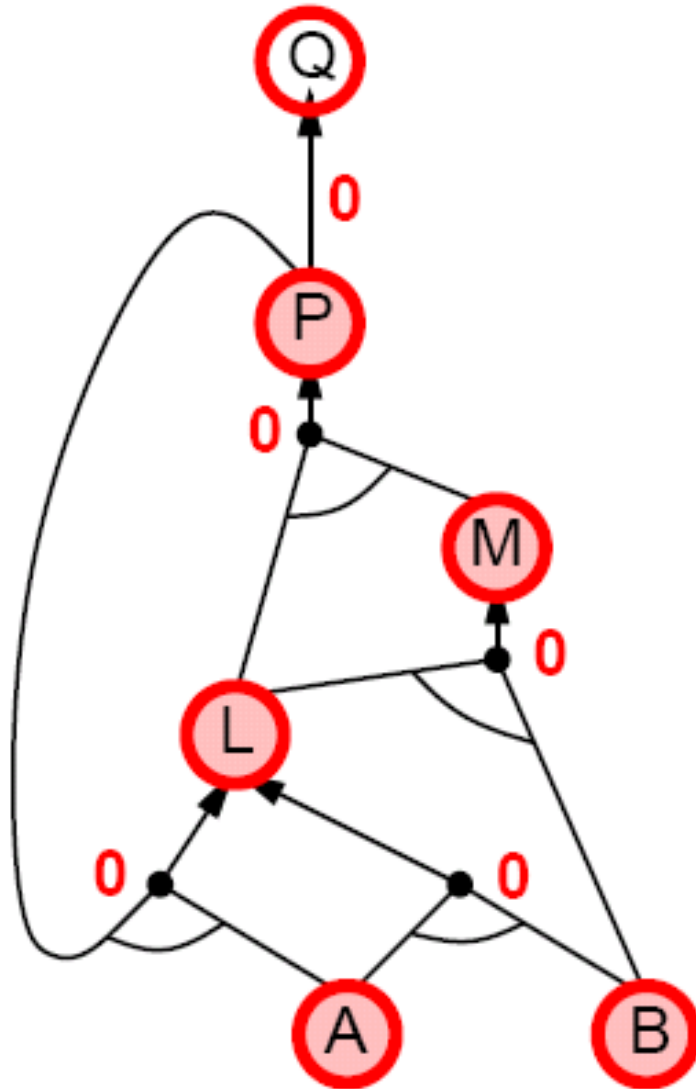$$B \wedge L \Rightarrow M$$
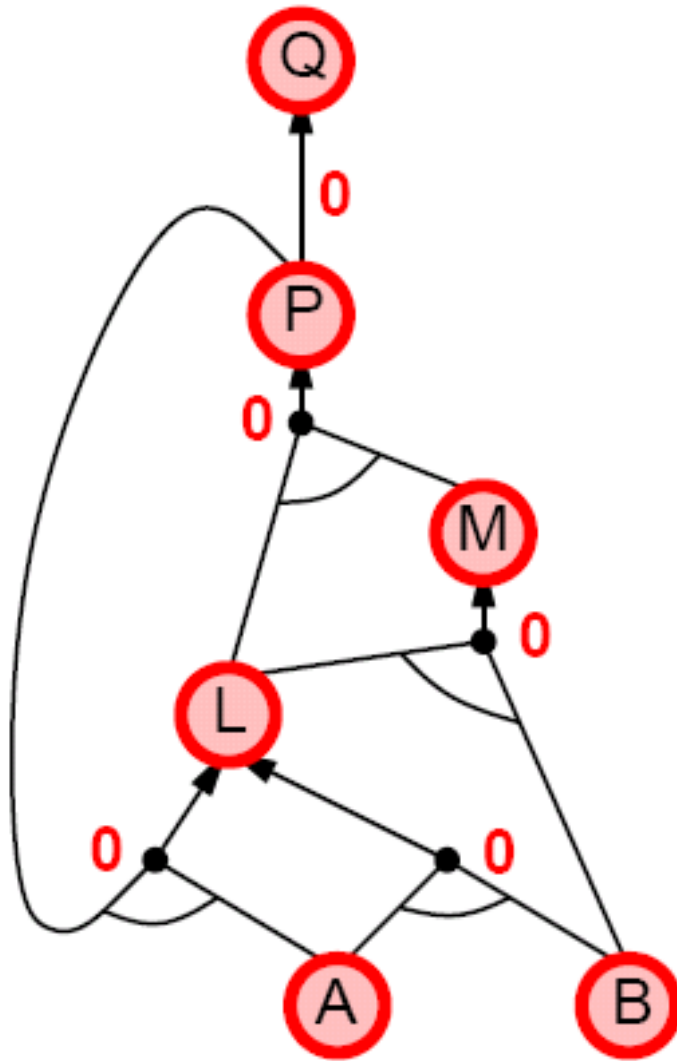$$A \wedge P \Rightarrow L$$
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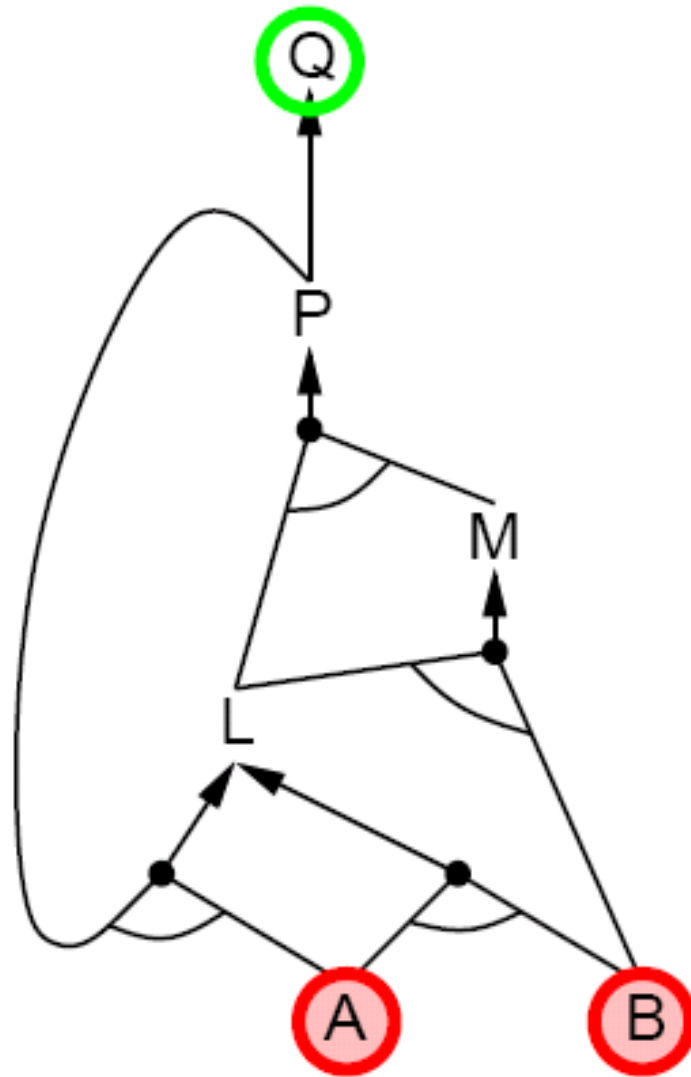
Backward chaining

- Forward chaining problem: can generate a lot of irrelevant conclusions
    - Search forward, start state = KB, goal test = state contains query
- Backward chaining
    - Reverse search from goal
    - Find all implications of the form
        $$(\dots) \Rightarrow query$$
    - Prove all the premises of one of these implications

Backward chaining

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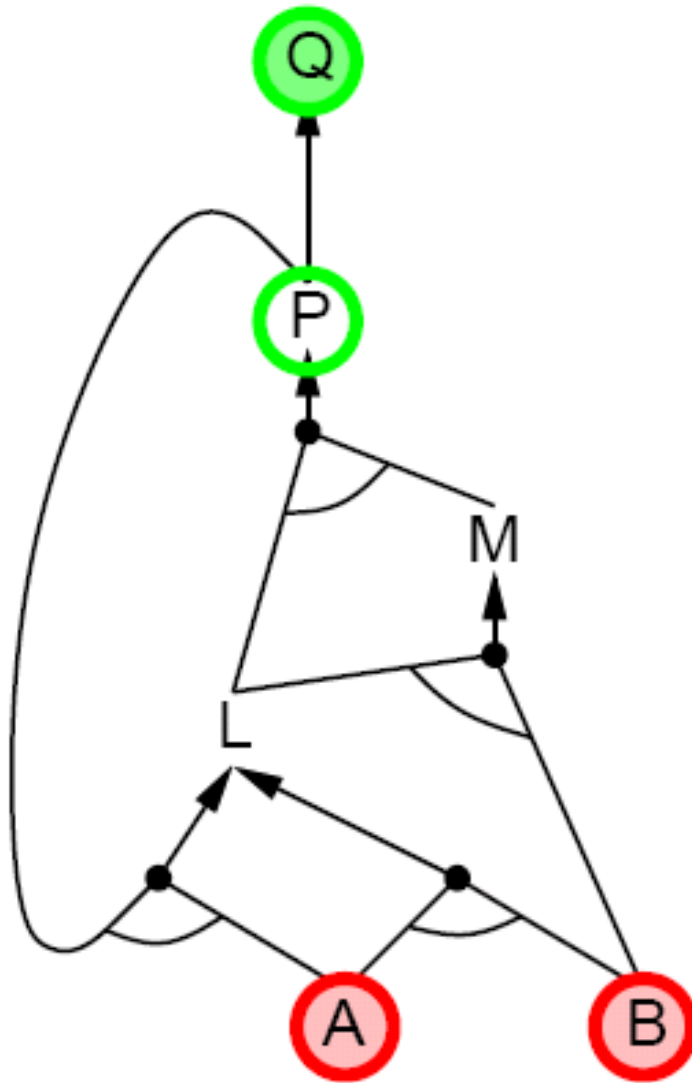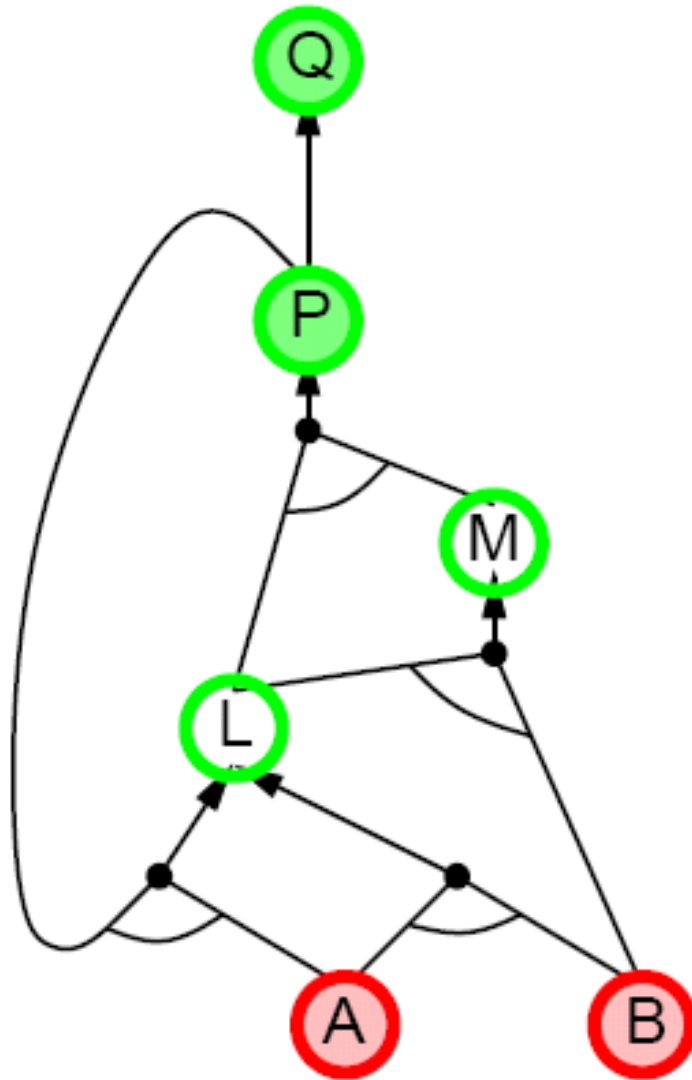$$L \wedge M \Rightarrow P$$
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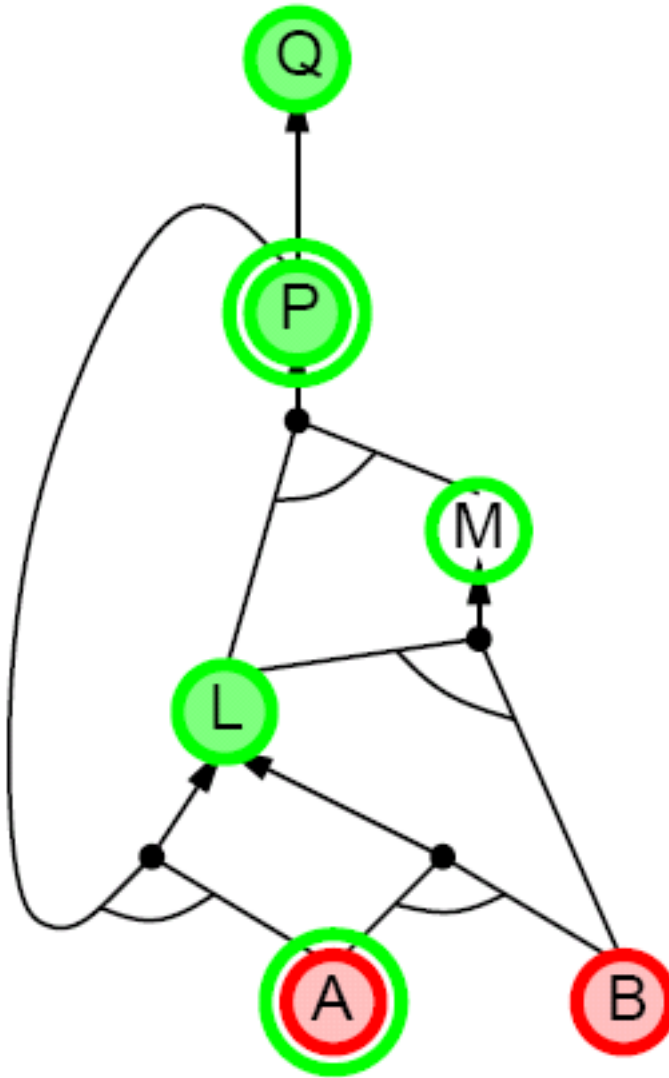$$A \wedge B \Rightarrow L$$
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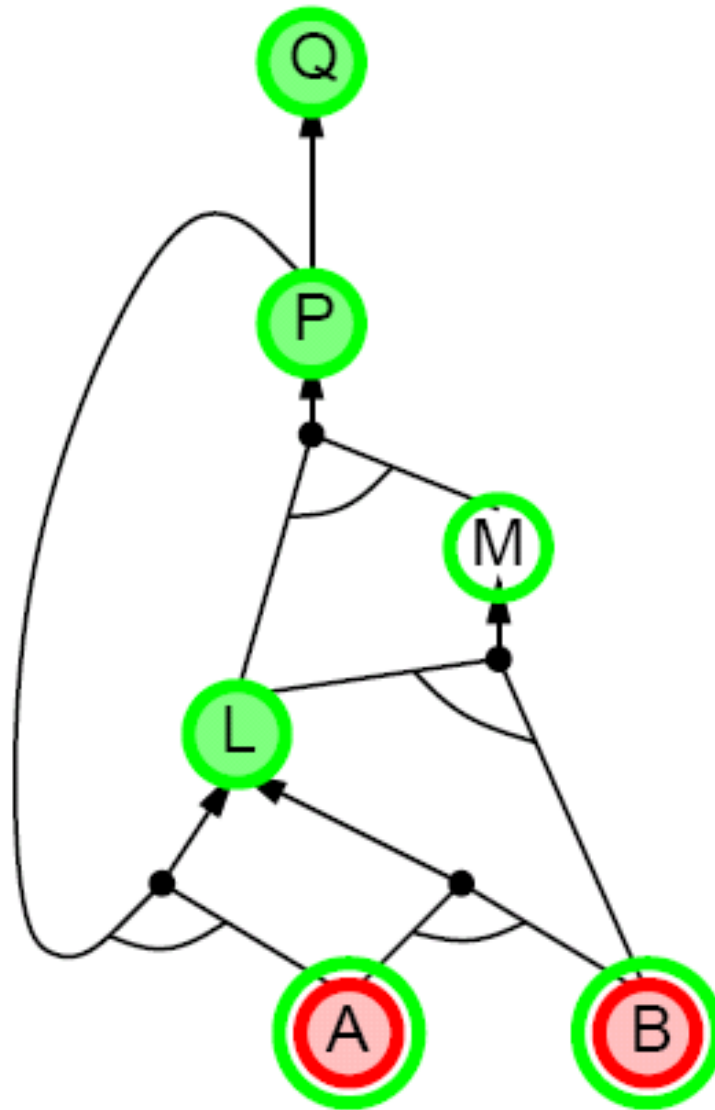$$A \wedge P \Rightarrow L$$
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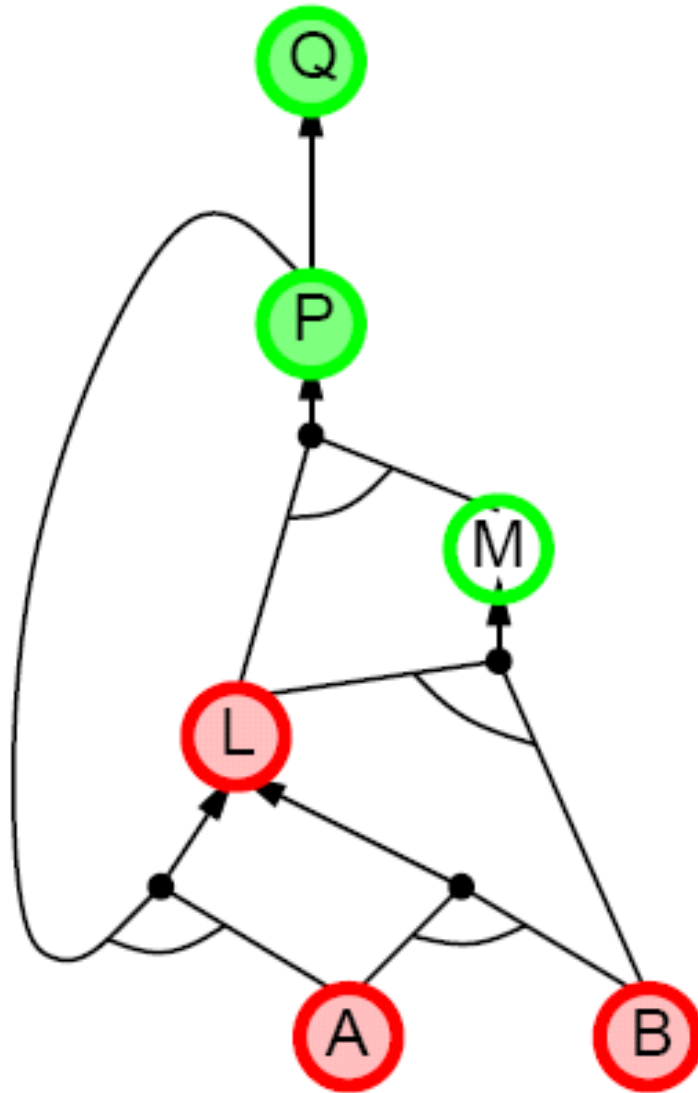$$B \wedge L \Rightarrow M$$
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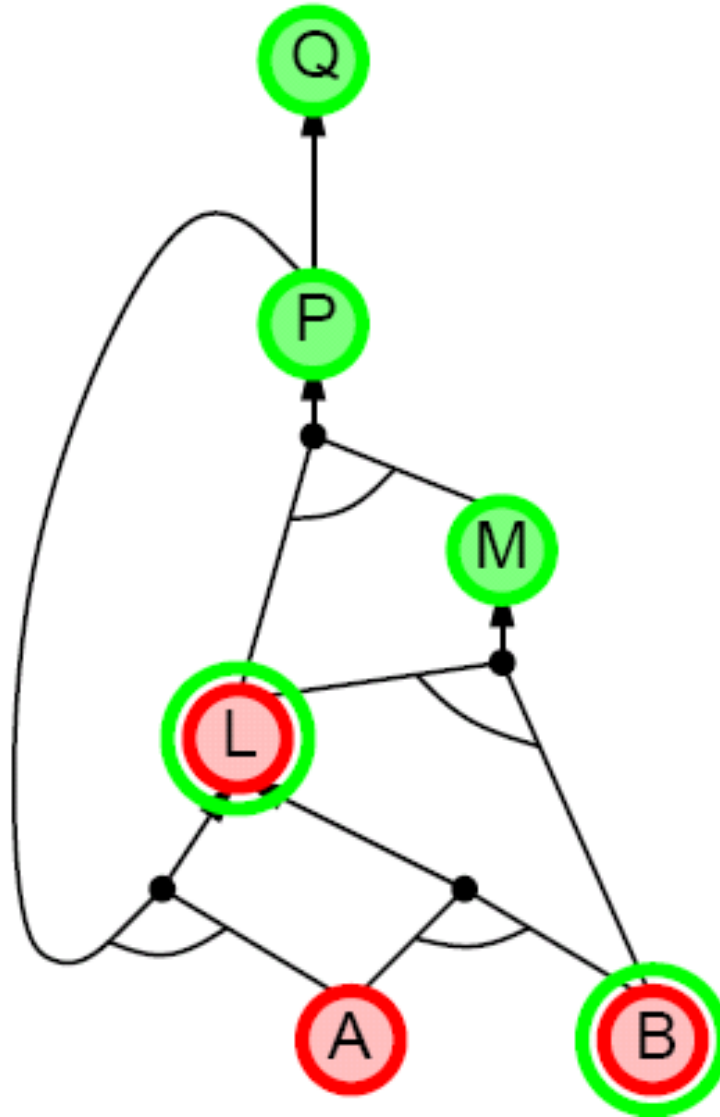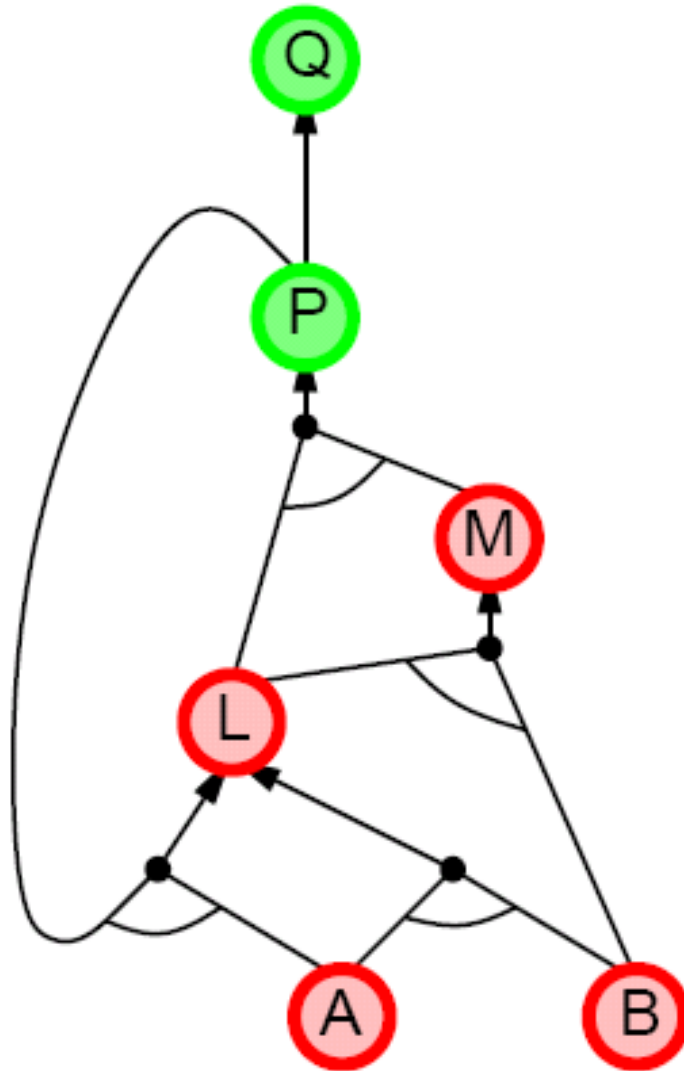$$B \wedge L \Rightarrow M$$
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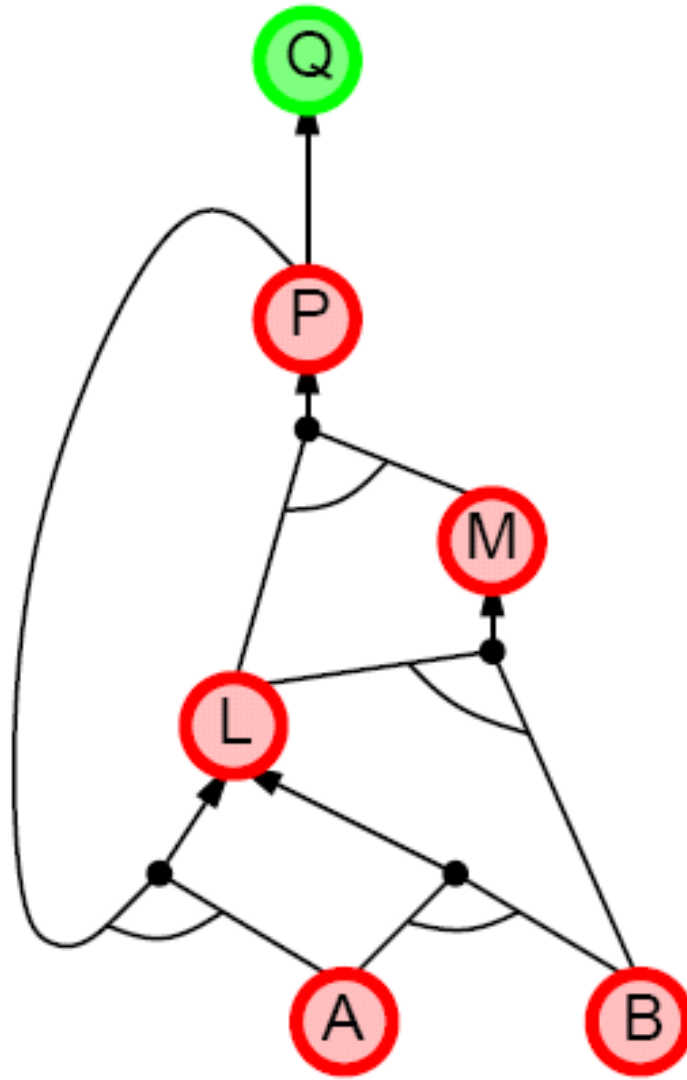$$L \wedge M \Rightarrow P$$
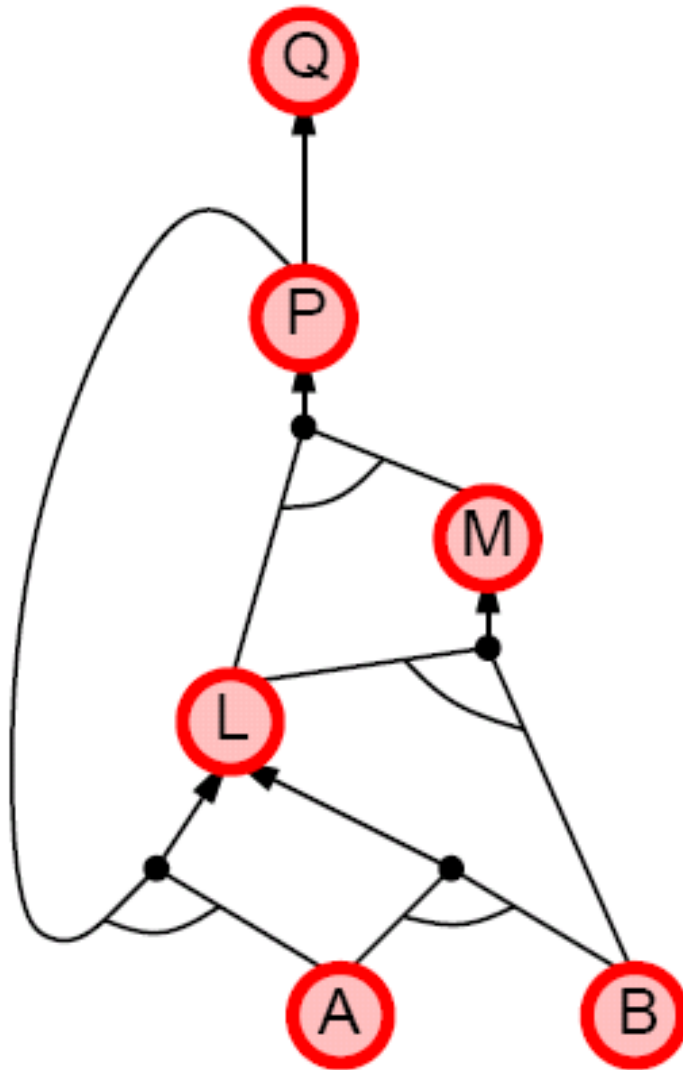$$B \wedge L \Rightarrow M$$
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Forward vs. backward chaining

- Forward chaining is data-driven
  - May perform lots of work irrelevant to the goal
- Backward chaining is goal-driven
  - Appropriate for problem solving
- Some form of bi-directional search is even better

What you should know

- A lot of terms
- Use truth tables
- Proofs
- Conjuctive Normal Form
- Proofs with resolution
- Horn clauses
- Forward chaining algorithm
- Backward chaining algorithm