Game Playing
Part 1 Minimax Search

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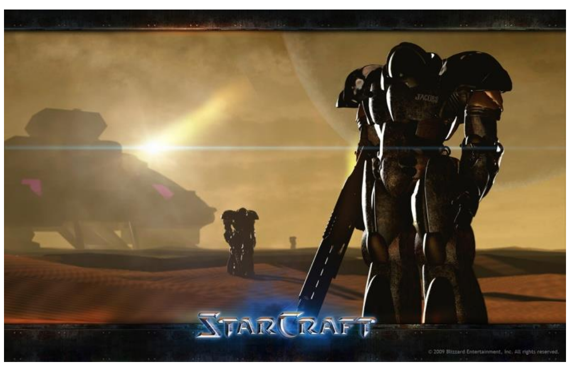
[based on slides from A. Moore http://www.cs.cmu.edu/~awm/tutorials , C. Dyer, J. Skrentny, Jerry Zhu]

Sadly, not these games (not in this course) …



Overview

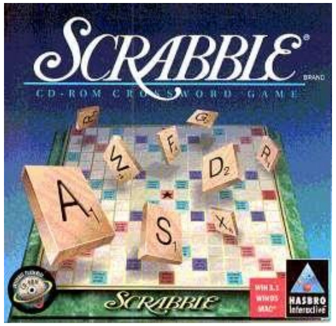
- two-player zero-sum discrete finite deterministic game of perfect information
- Minimax search
- Alpha-beta pruning
- Large games
- two-player zero-sum discrete finite NON-deterministic game of perfect information

Two-player zero-sum discrete finite deterministic games of perfect information

Definitions:

- Zero-sum: one player's gain is the other player's loss. Does not mean *fair*.
- Discrete: states and decisions have discrete values
- Finite: finite number of states and decisions
- Deterministic: no coin flips, die rolls – no chance
- Perfect information: each player can see the complete game state. No simultaneous decisions.

Which of these are: Two-player zero-sum discrete finite deterministic games of perfect information?



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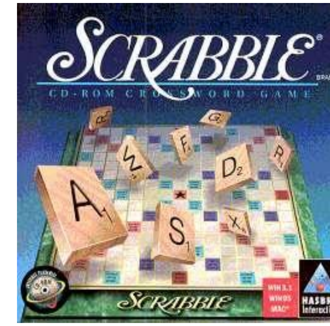

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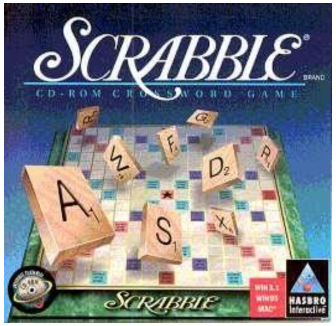

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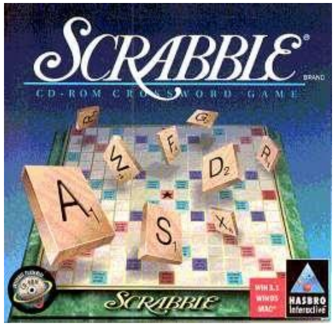

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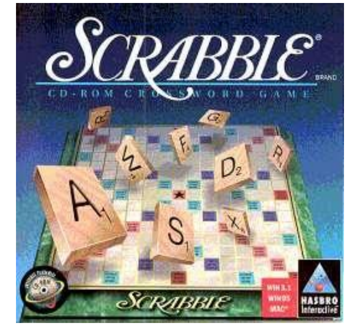

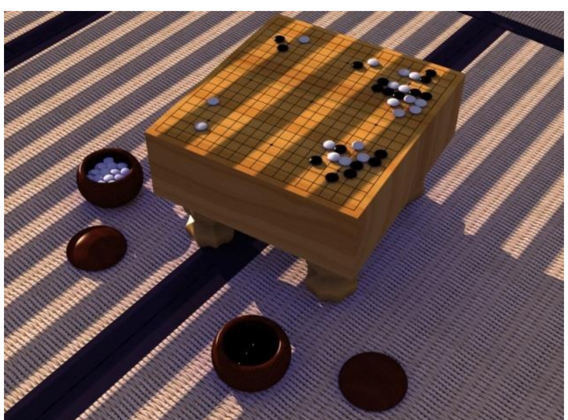

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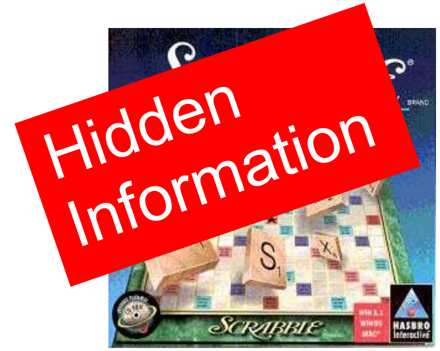

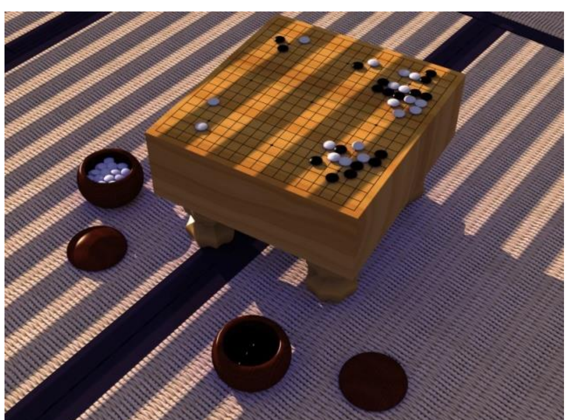

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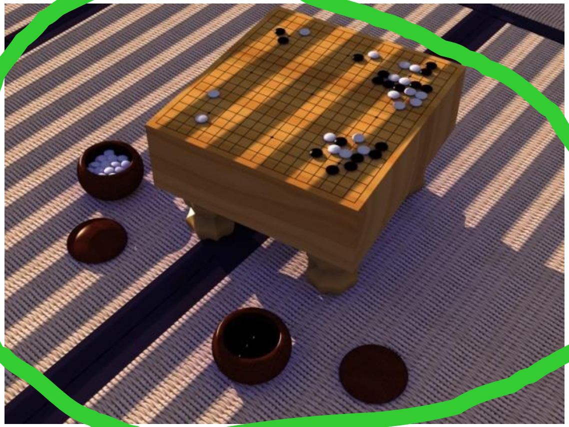

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II-Nim: Max simple game

- There are 2 piles of sticks. Each pile has 2 sticks.
- Each player takes one or more sticks from one pile.
- The player who takes the last stick loses.

(ii, ii)

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(ii, ii)

- Two players: Max and Min
- If Max wins, the score is **+1**; otherwise **-1**
- Min's score is –Max's
- Use Max's as the score of the game

The game tree for II-Nim

Two players:
Max and Min

(ii ii) Max

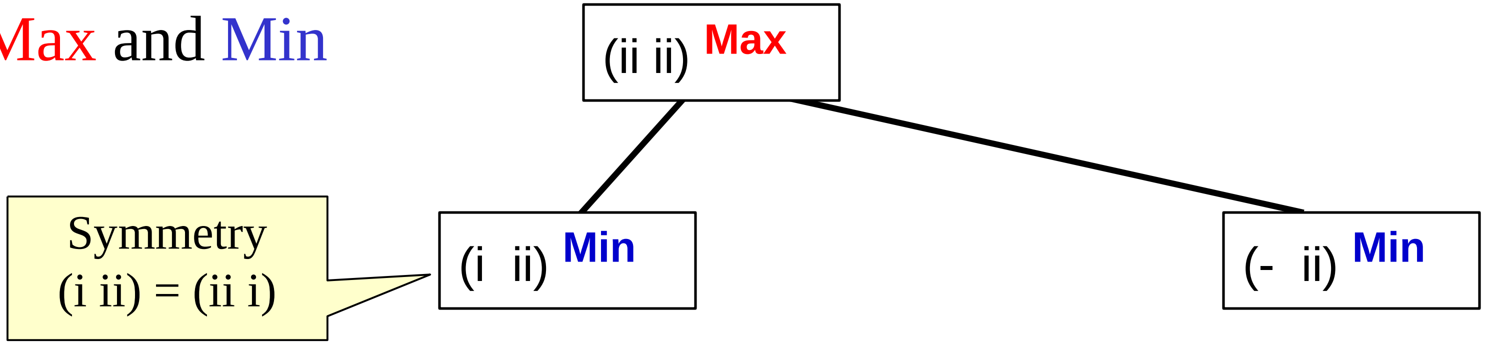
who is to move at this state

Max wants the largest score
Min wants the smallest score

Convention: score is w.r.t. the first player Max. Min’s score = – Max

The game tree for II-Nim

Two players:
Max and Min



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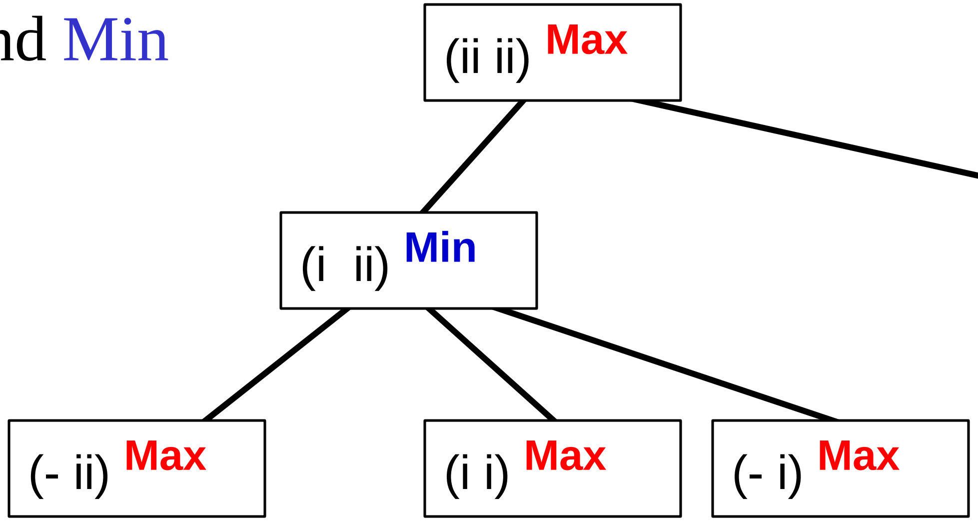

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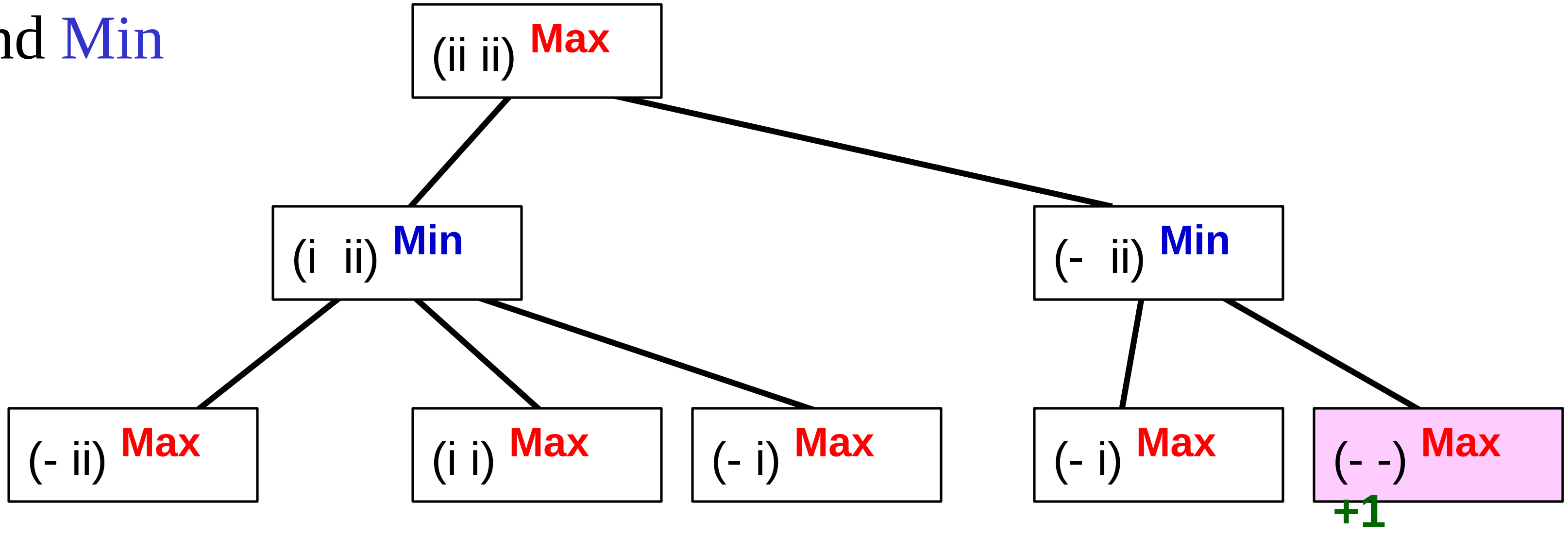

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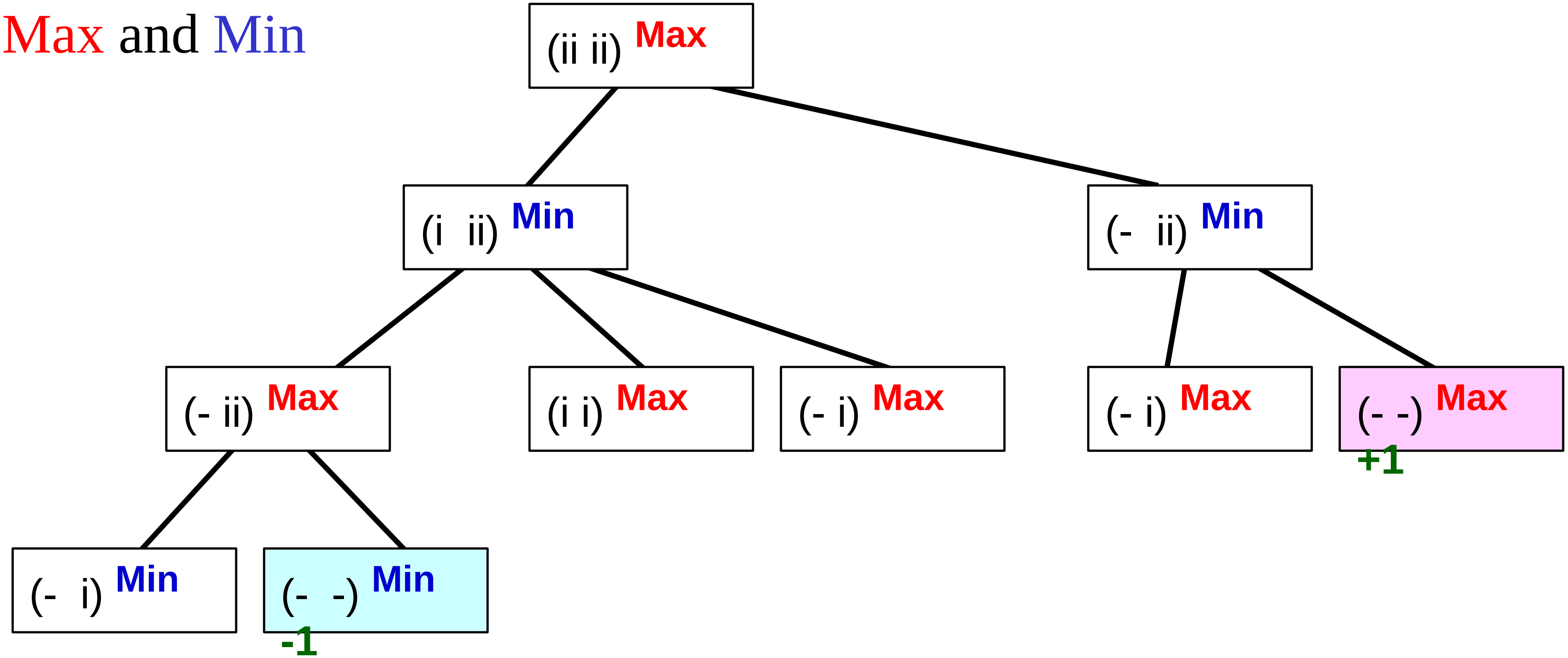

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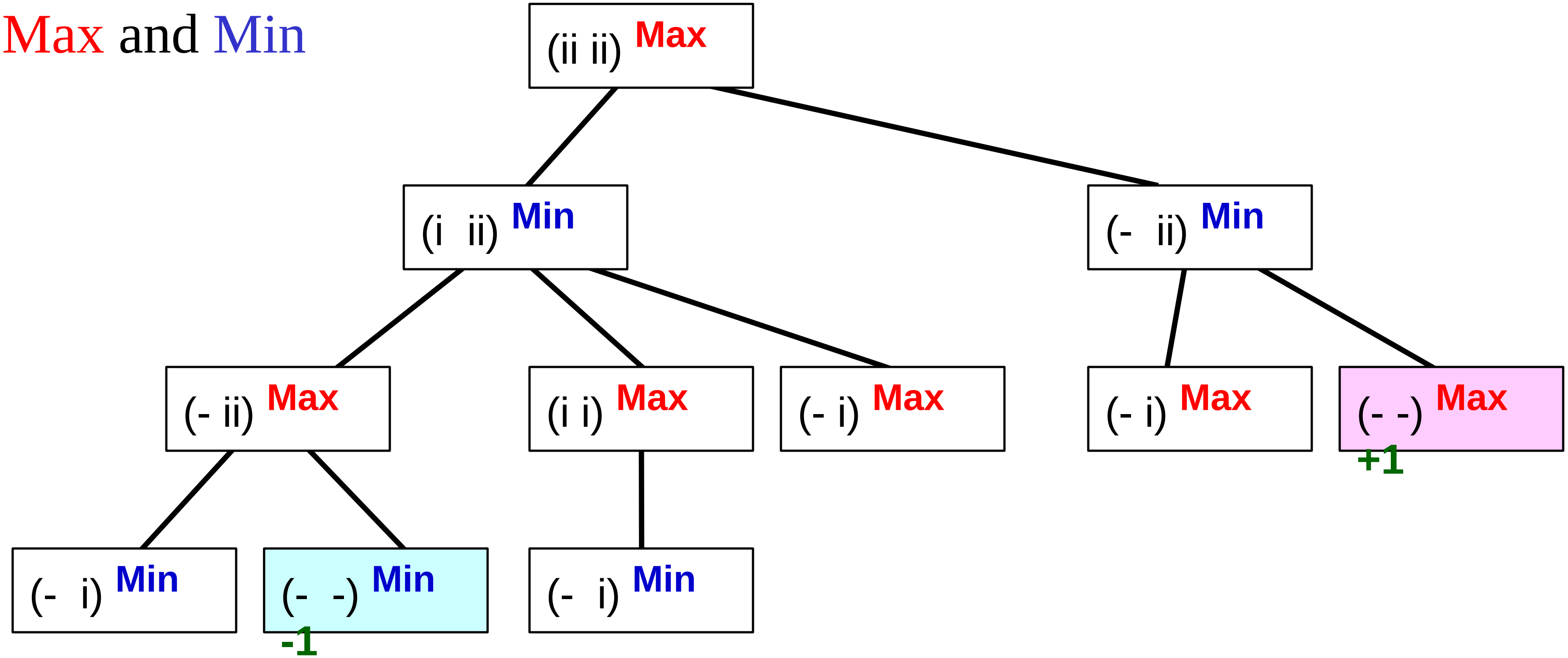

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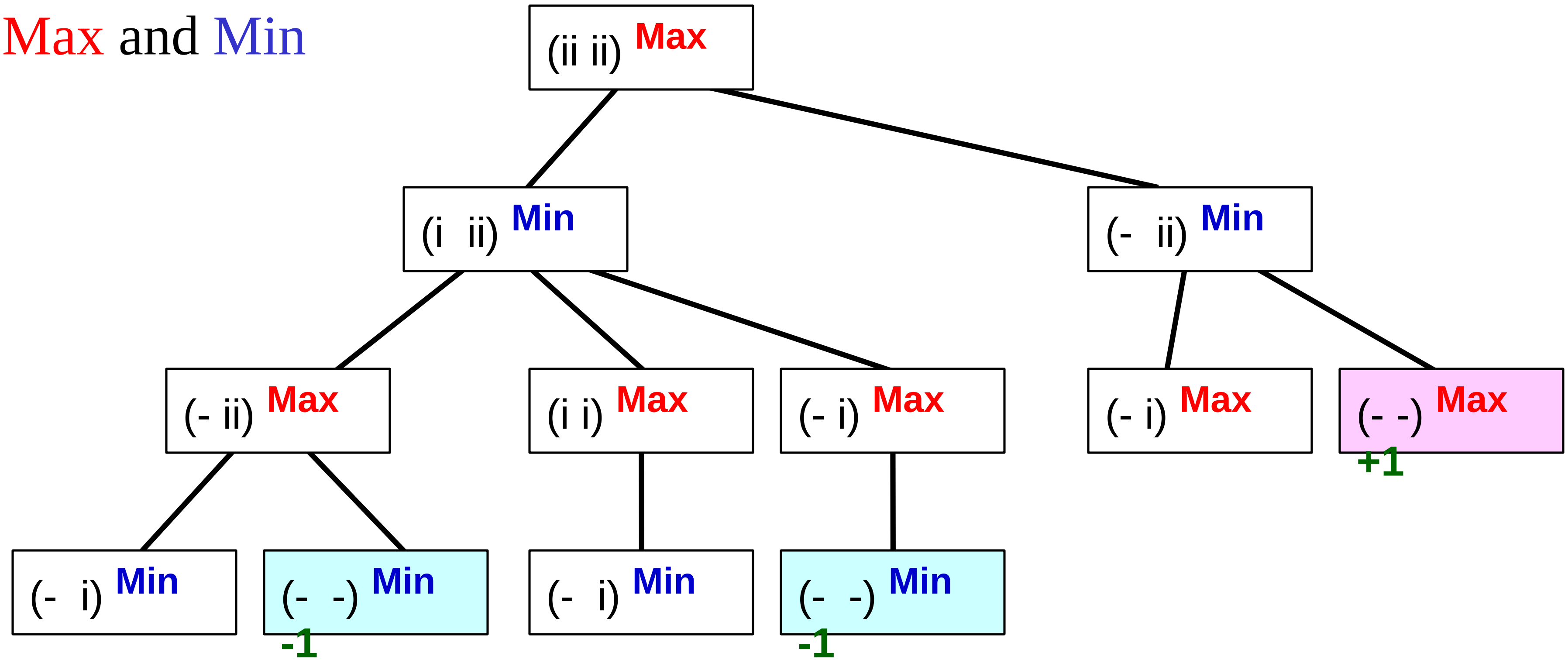

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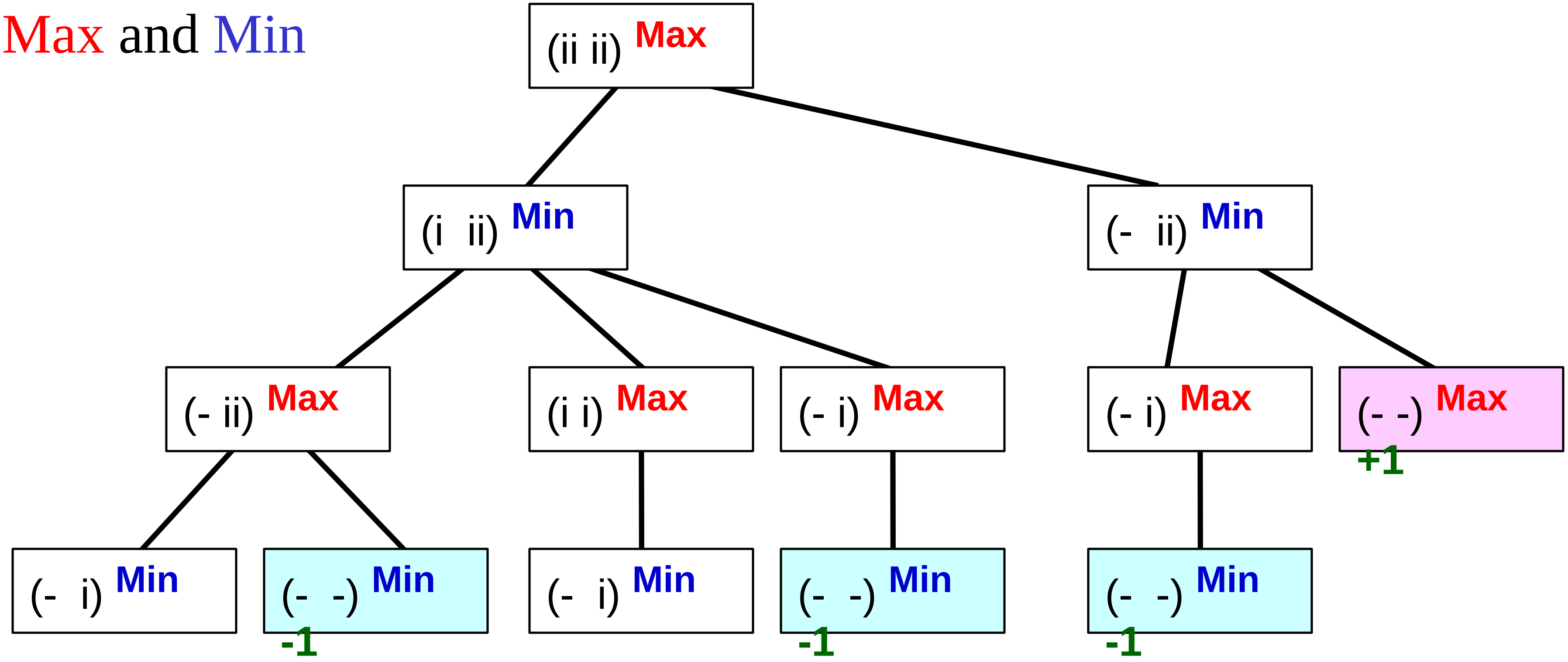

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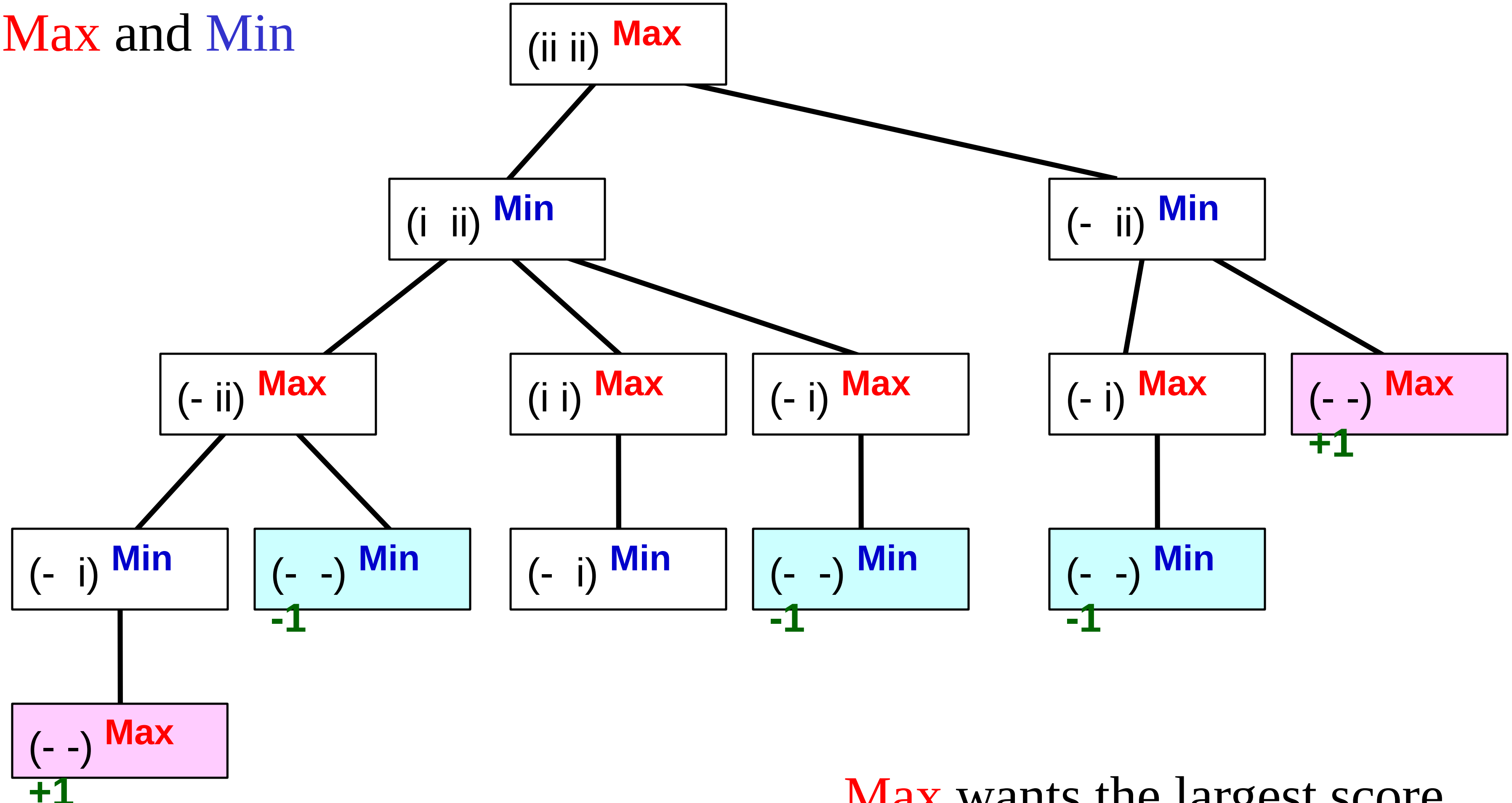

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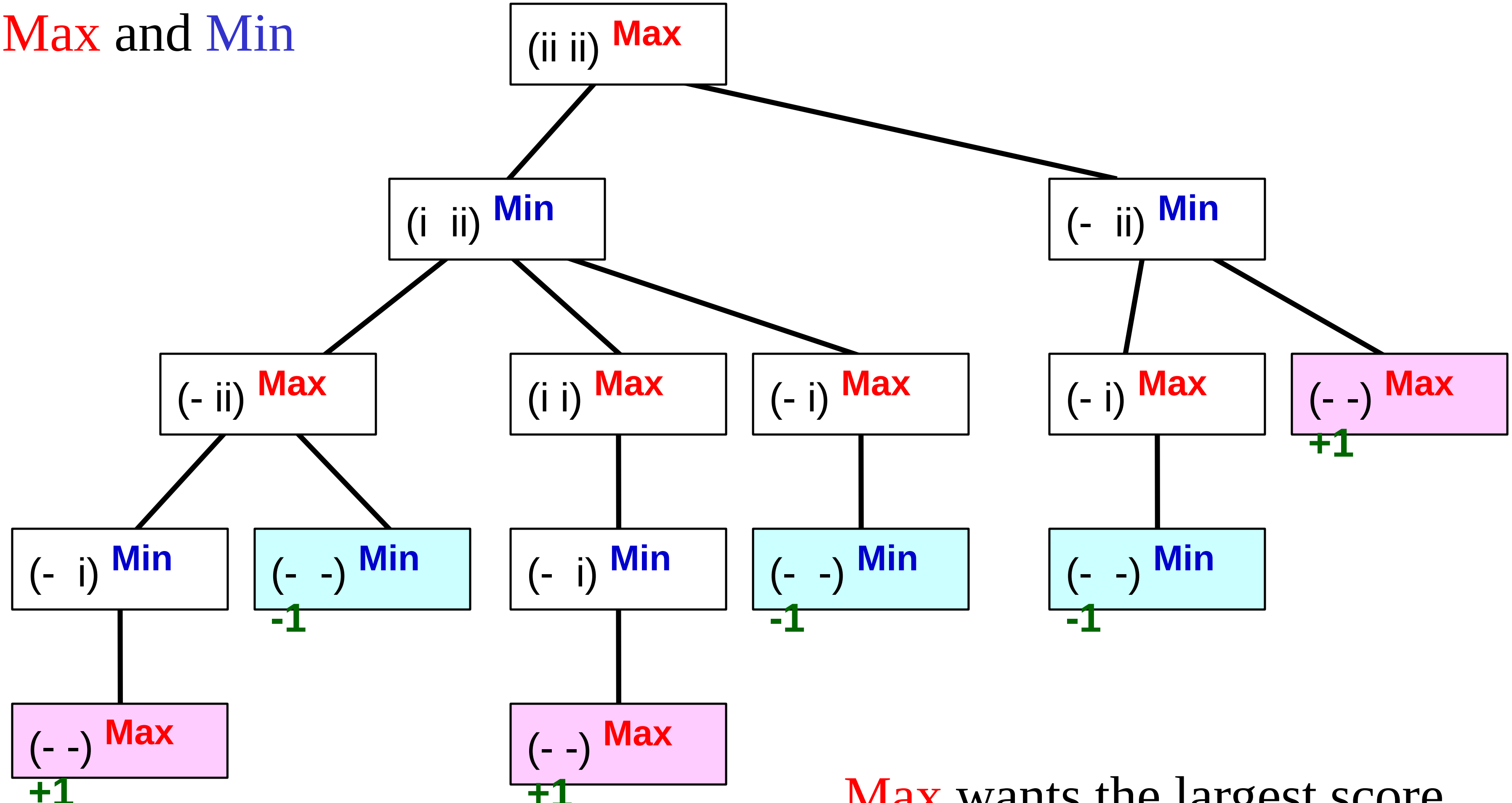

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Game theoretic value

- Game theoretic value (a.k.a. minimax value) of a node = the score of the terminal node that will be reached if both players play optimally.

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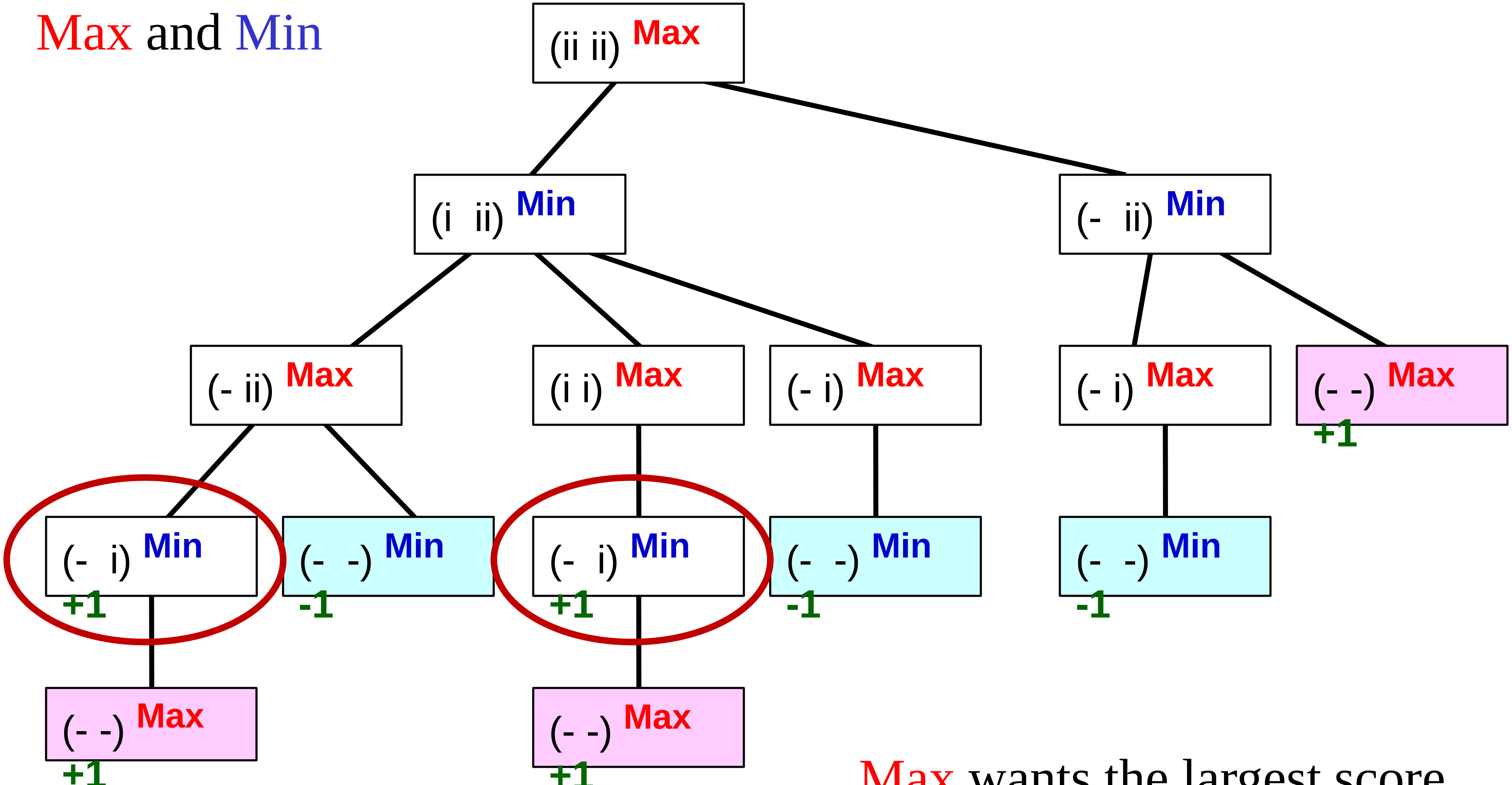

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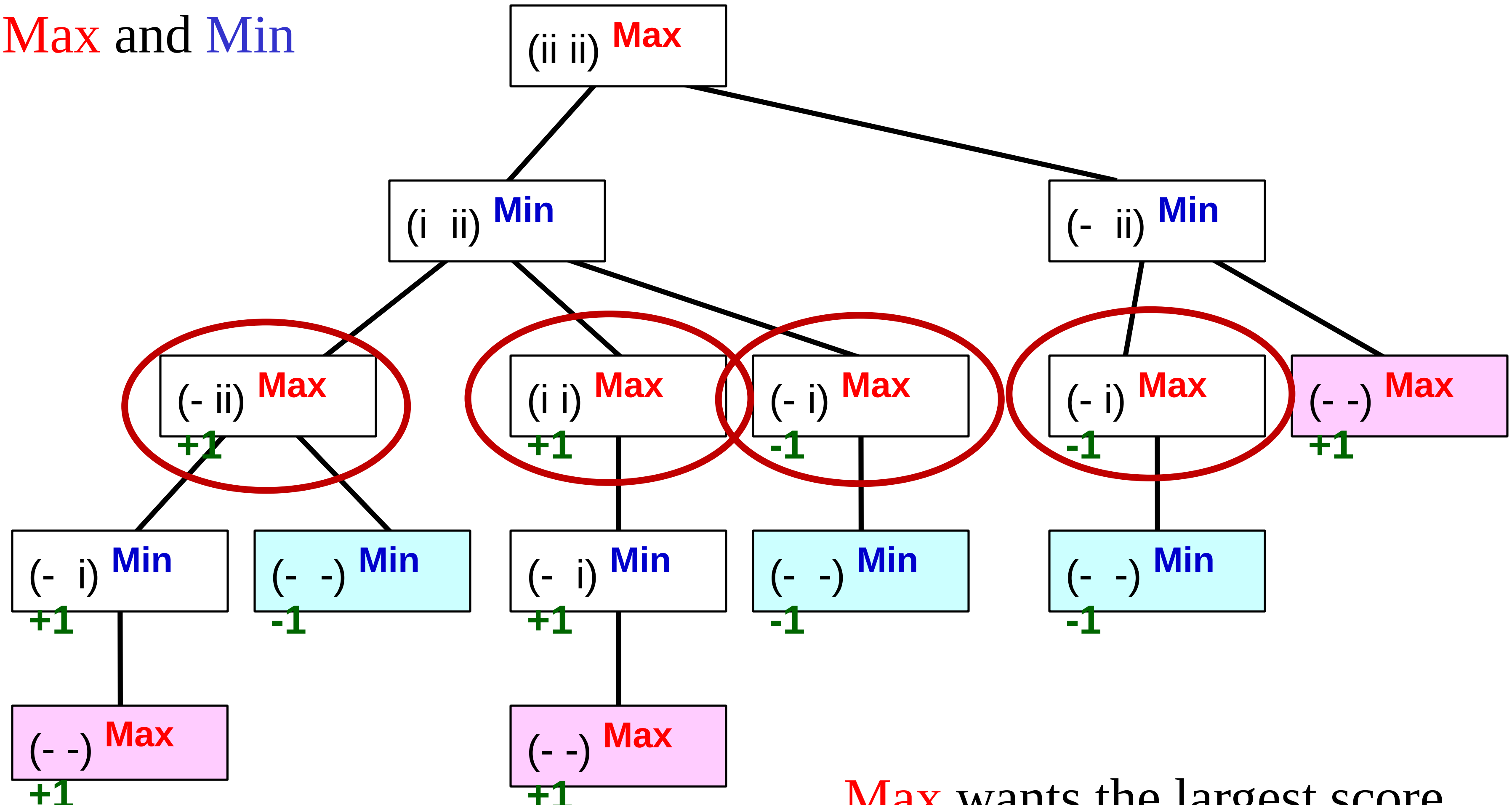

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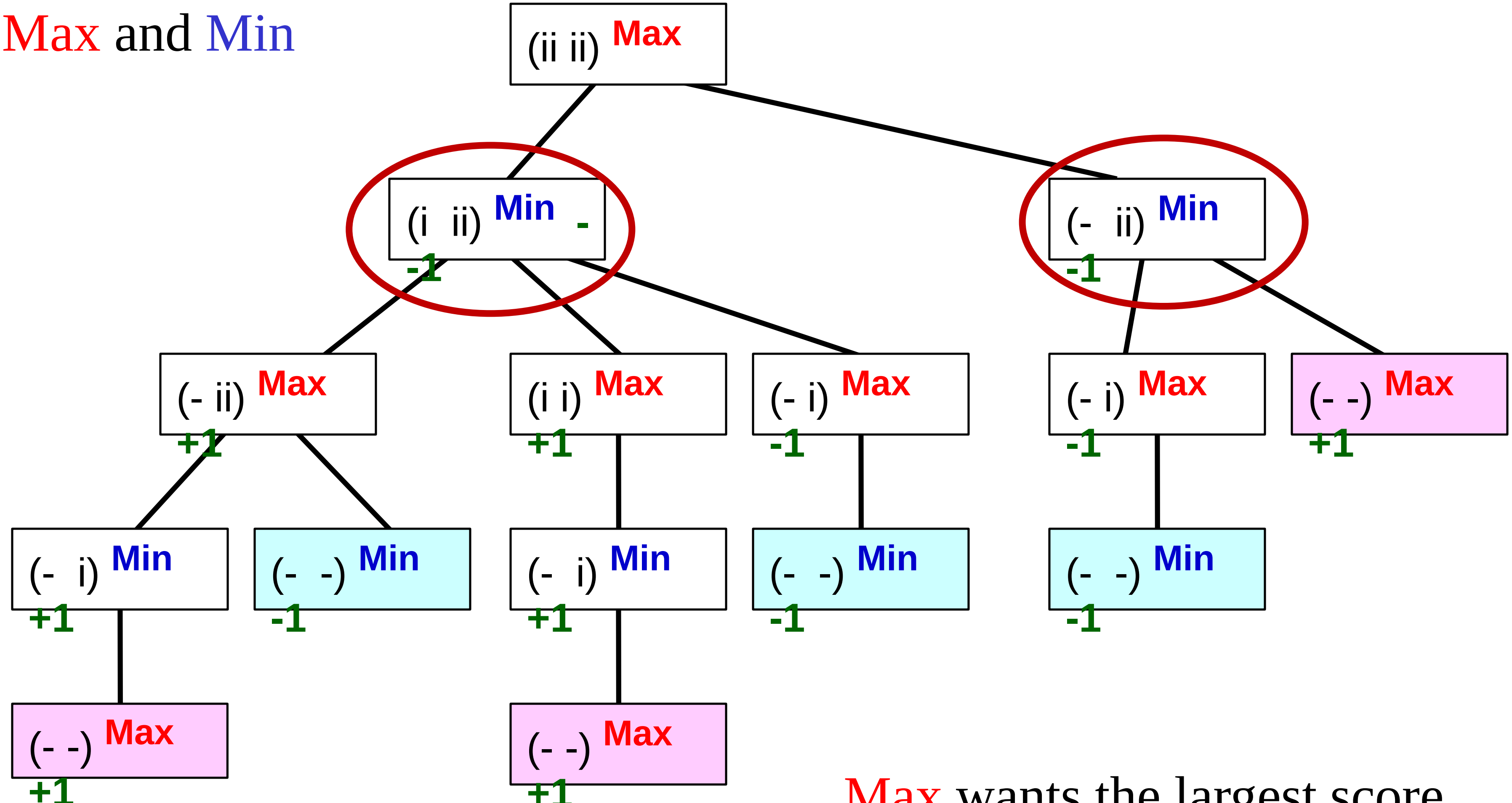

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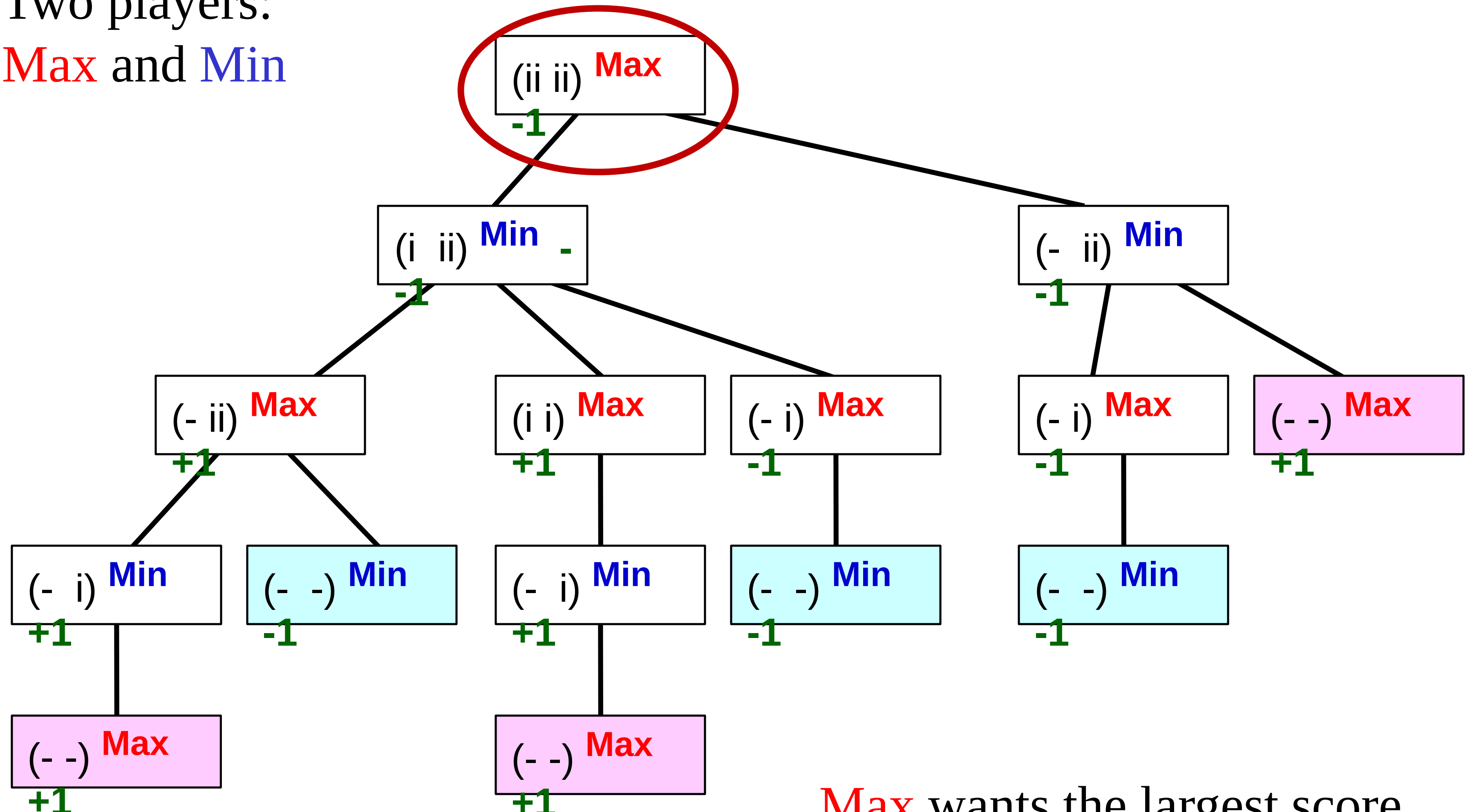

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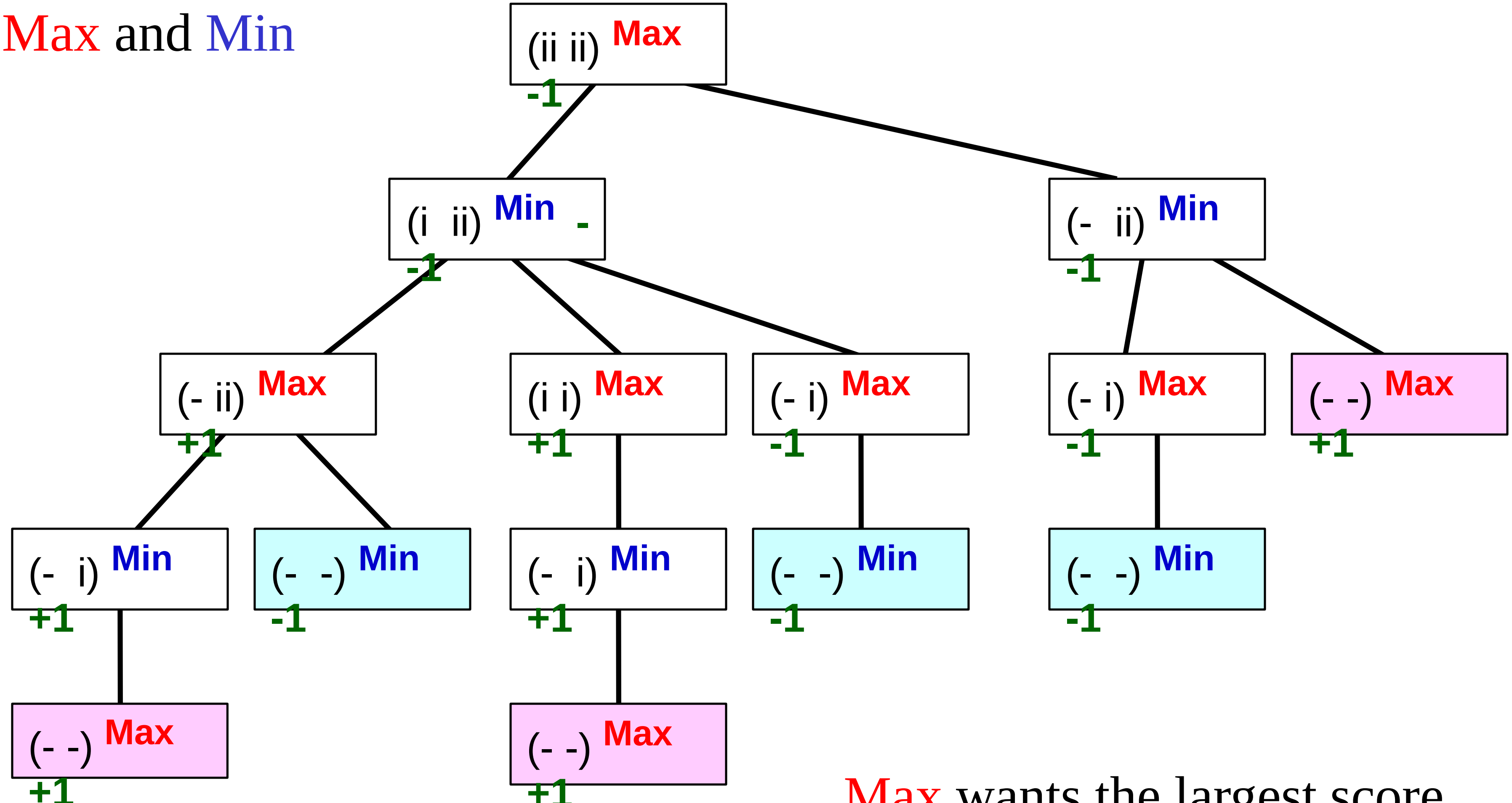

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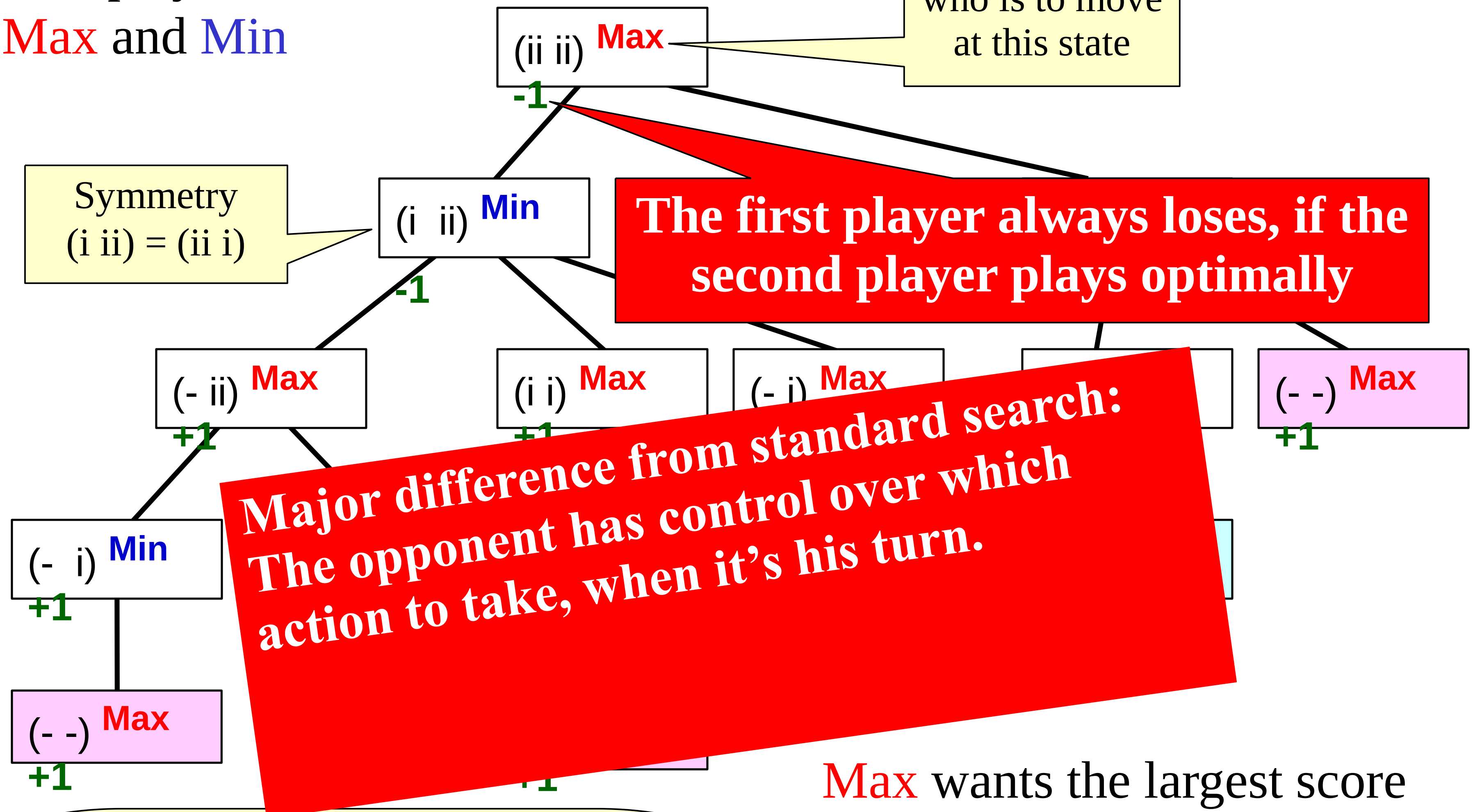

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Game theoretic value

- Game theoretic value (a.k.a. minimax value) of a node = the score of the terminal node that will be reached if both players play optimally.
- = The numbers we filled in.
- Computed bottom up
  - In Max's turn, take the max of the children (Max will pick that maximizing action)
  - In Min's turn, take the min of the children (Min will pick that minimizing action)
- Implemented as a modified version of DFS: minimax algorithm

Minimax algorithm

```
function Max-Value(s)
inputs:
    s: current state in game, Max about to play
output: best-score (for Max) available from s

    if ( s is a terminal state )
    then return ( terminal value of s )
    else
            α := – ∞
            for each s' in Succ(s)
                α := max( α , Min-value(s'))
    return α
```

```
function Min-Value(s)
output: best-score (for Min) available from s

    if ( s is a terminal state )
    then return ( terminal value of s)
    else
            β := ∞
            for each s' in Succs(s)
                β := min( β , Max-value(s'))
    return β
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- Time complexity?
- Space complexity?

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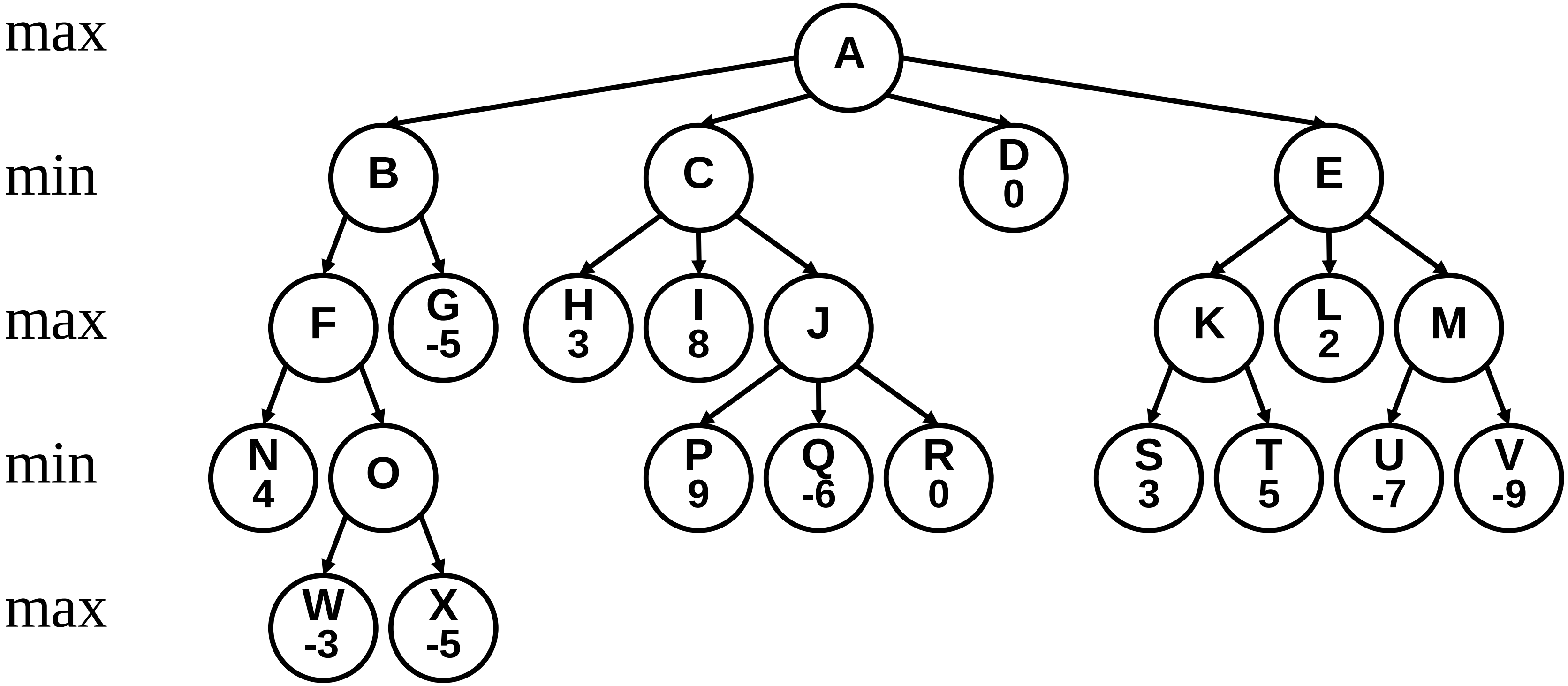
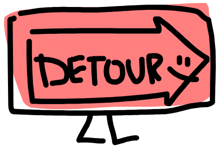
- Time complexity?
  O(b^m) ← bad
- Space complexity?
  O(bm)

Minimax example



What are the game theoretic values? In particular, A's



Against a dumber opponent?

- Max surely loses!
- If Min not optimal,
- Which way?
- Why?