

Propositional Logic

Part 2

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Method 4: chaining with Horn clauses

- Resolution is too powerful for many practical situations.
- A weaker form: **Horn clauses**
 - Disjunction of literals with at most one positive

$$\neg R \vee P \vee Q \quad \text{no}$$

$$\neg R \vee \neg P \vee Q \quad \text{yes}$$

- What's the big deal?

$$\neg R \vee \neg P \vee Q$$

$$\neg(R \wedge P) \vee Q$$

?

Horn clauses

$$\neg R \vee \neg P \vee Q$$

$$\neg(R \wedge P) \vee Q$$

$$(R \wedge P) \Rightarrow Q$$

$$P$$

Every rule in KB is in this form

(special case, no negative literals): fact

- The big deal:
 - KB easy for human to read
 - Natural forward chaining and backward chaining algorithm, proof easy for human to read
 - Deciding entailment with Horn clauses in time **linear** to KB size
- But...
 - Can only ask atomic queries

Forward chaining

- Fire any rule whose premises are satisfied in the KB
- Add its conclusion to the KB until query is found

KB:

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

$$B \wedge L \Rightarrow M$$

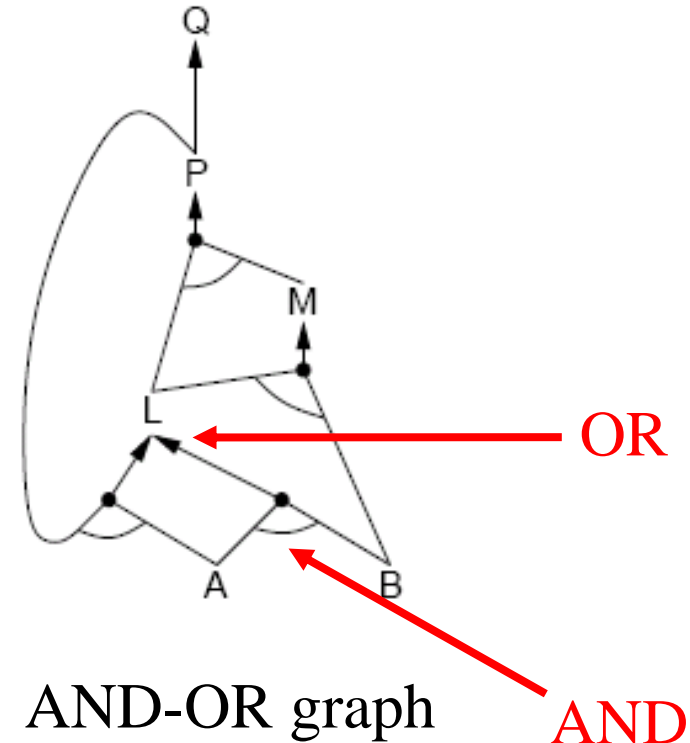
$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

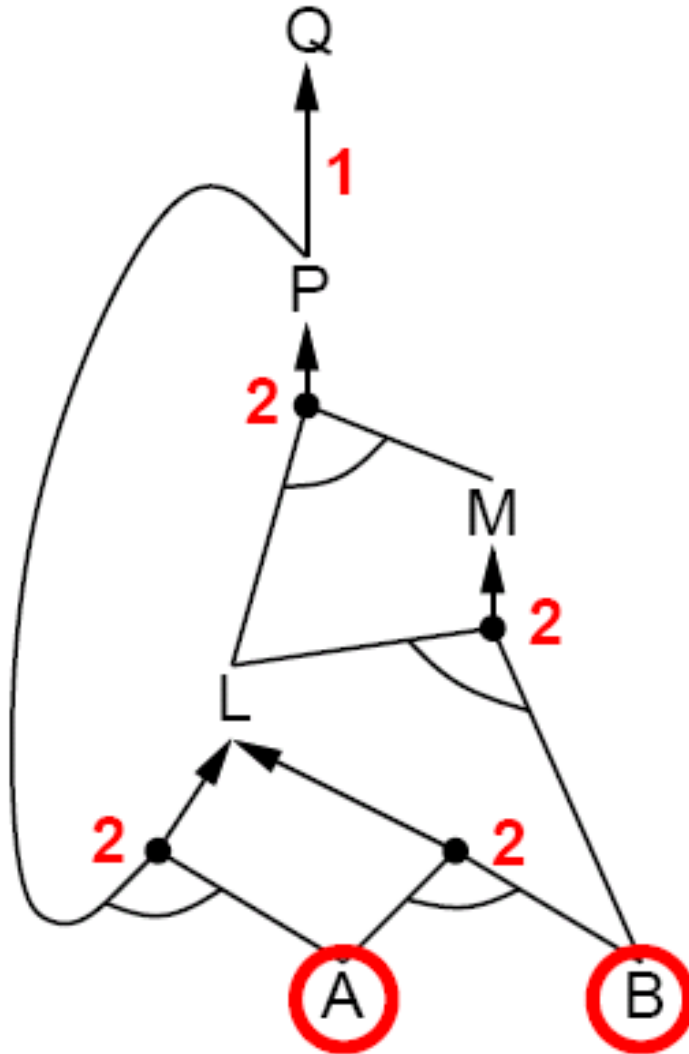
A

B

query: Q



Forward chaining



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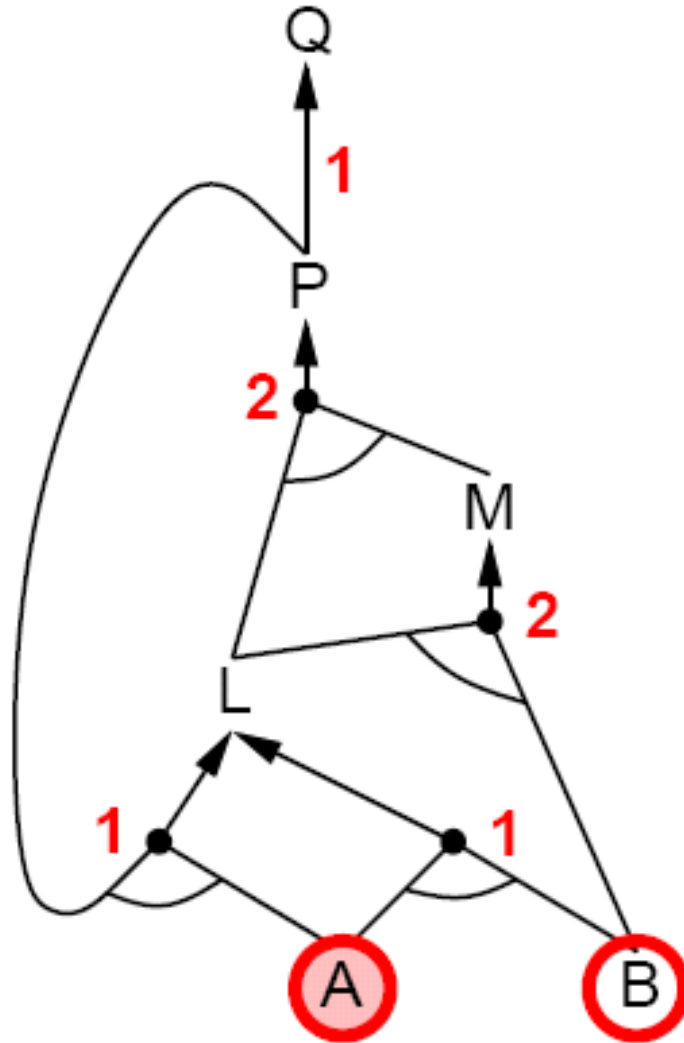
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Forward chaining



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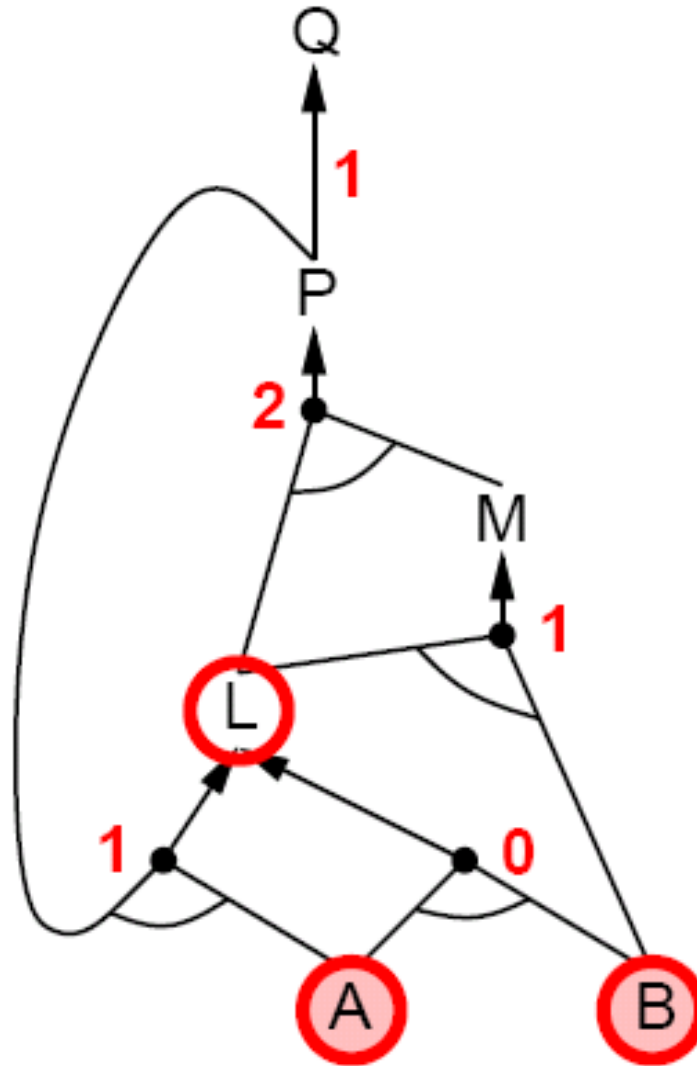
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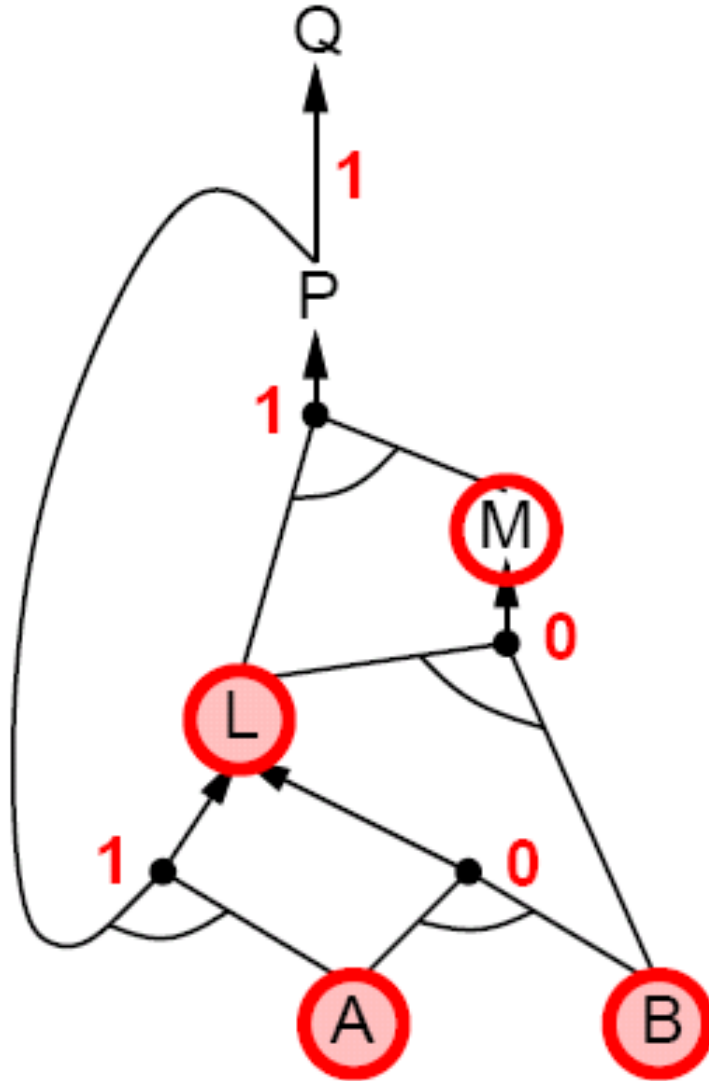
B

Forward chaining



$P \Rightarrow Q$
 $L \wedge M \Rightarrow P$
 $B \wedge L \Rightarrow M$
 $A \wedge P \Rightarrow L$
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Forward chaining



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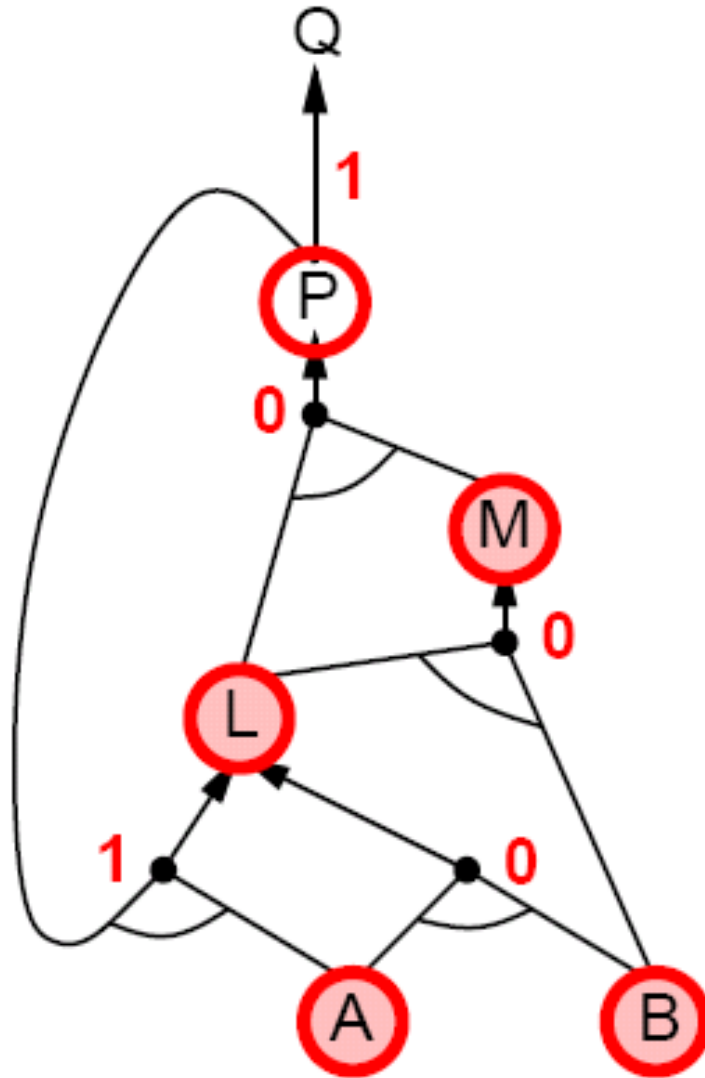
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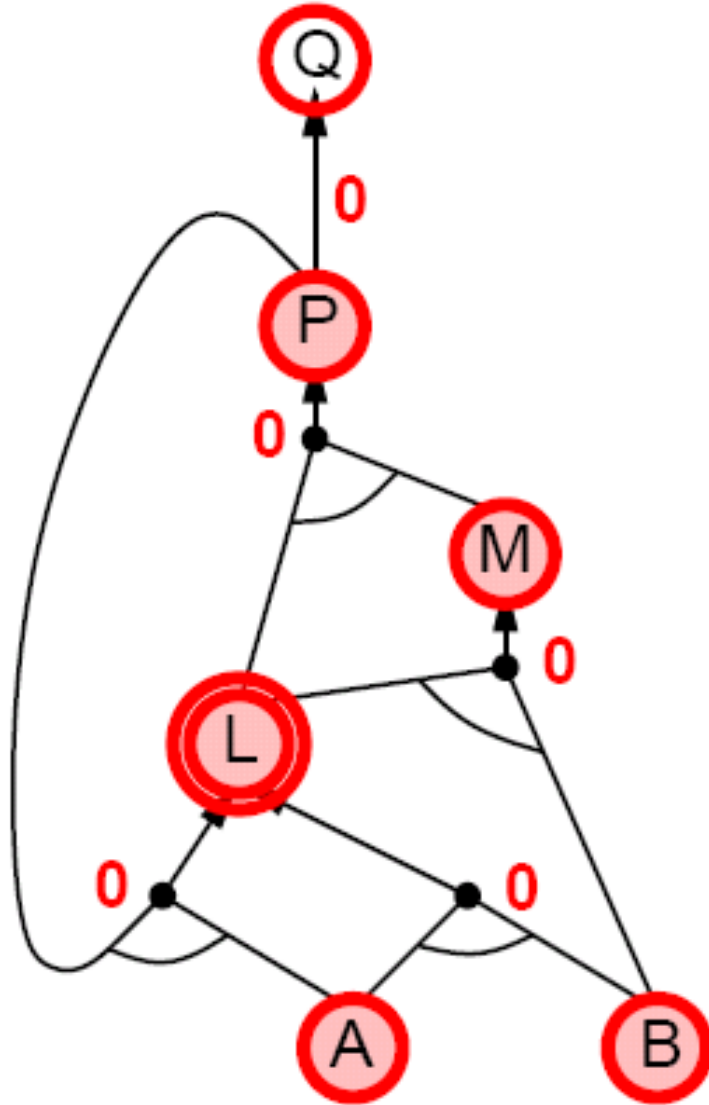
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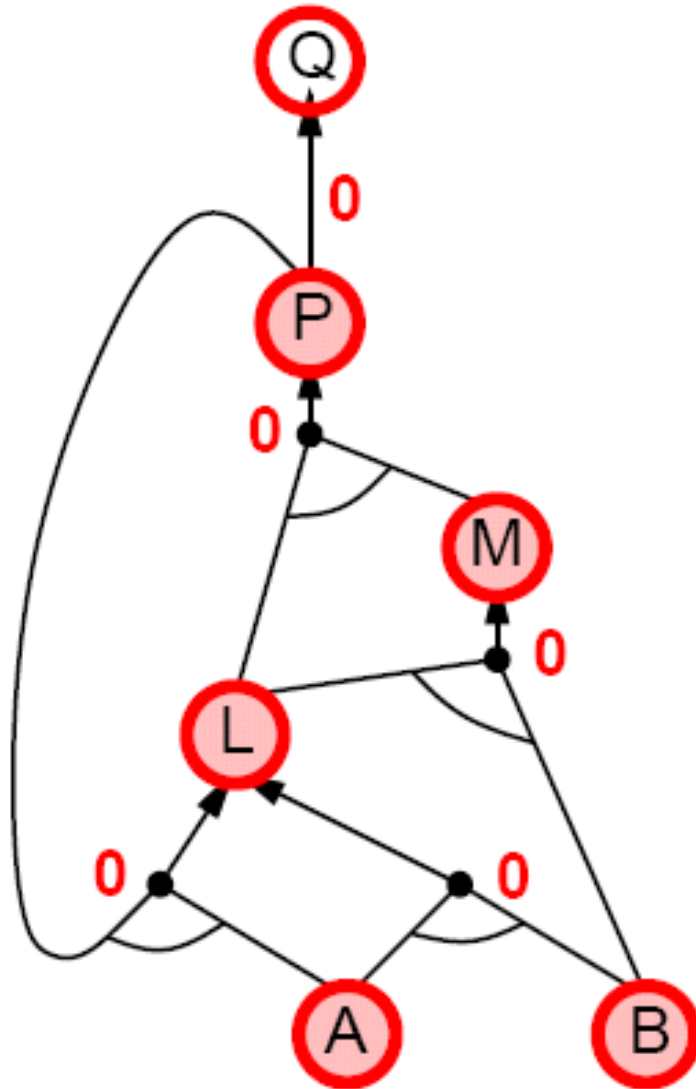
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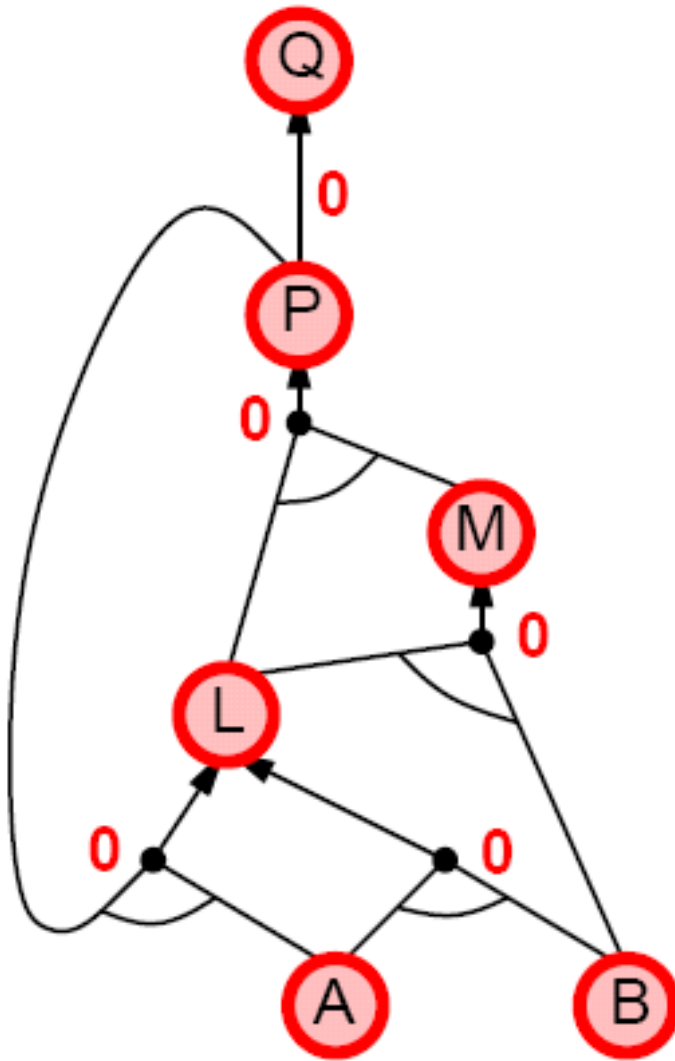
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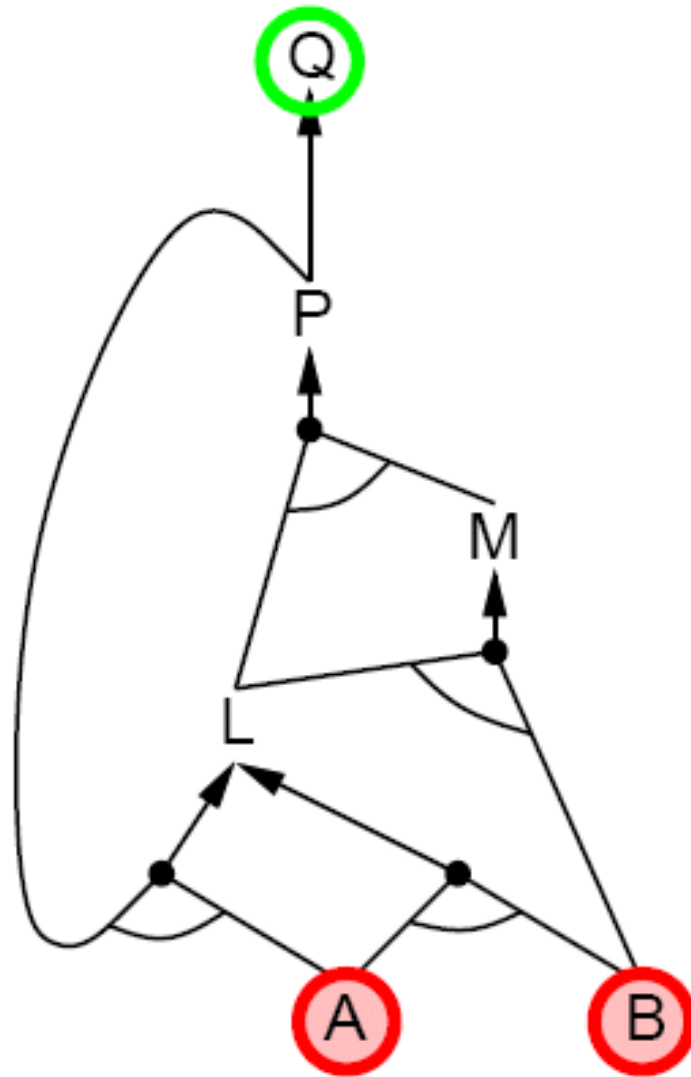
A

B

Backward chaining

- Forward chaining problem: can generate a lot of irrelevant conclusions
 - Search forward, start state = KB, goal test = state contains query
- Backward chaining
 - Reverse search from goal
 - Find all implications of the form
 $(\dots) \Rightarrow \text{query}$
 - Prove all the premises of one of these implications

Backward chaining



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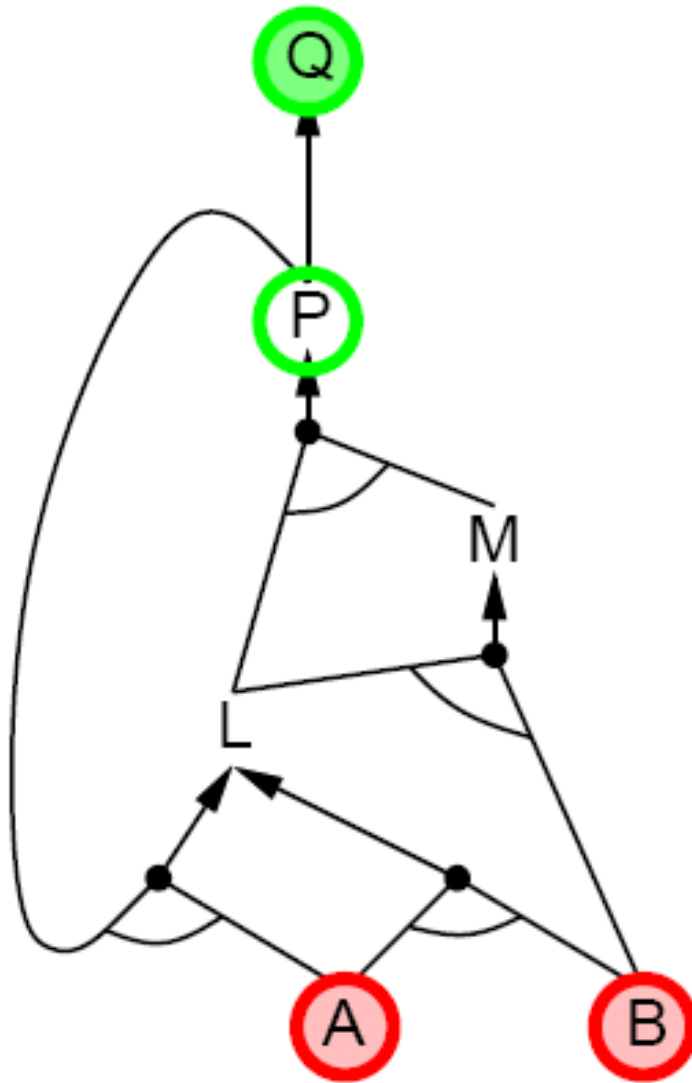
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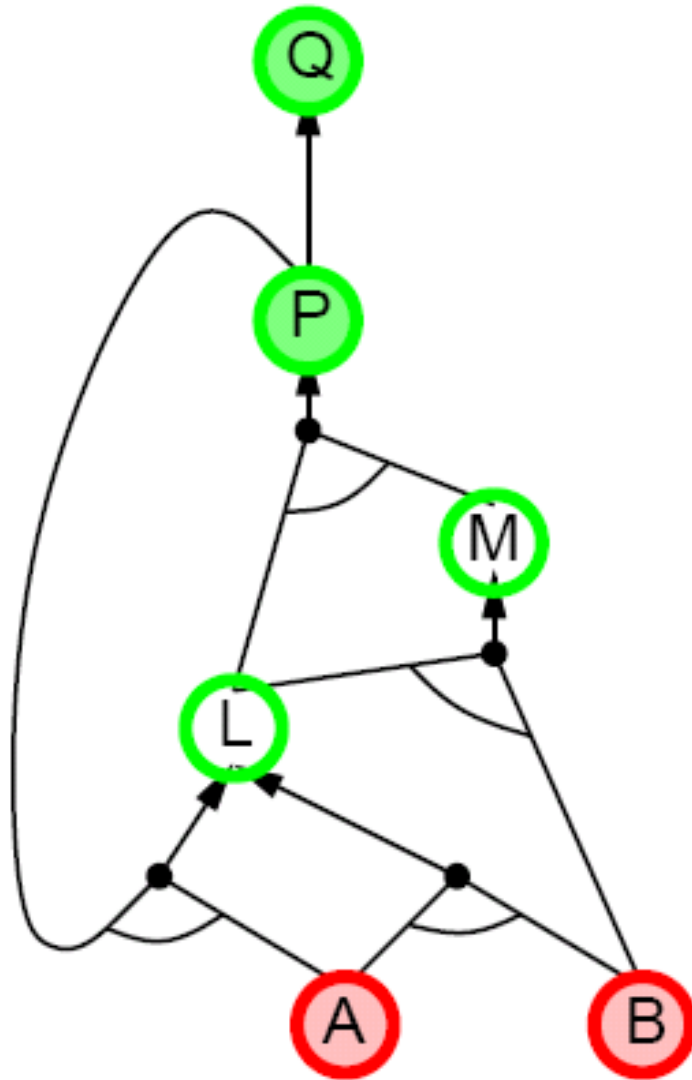
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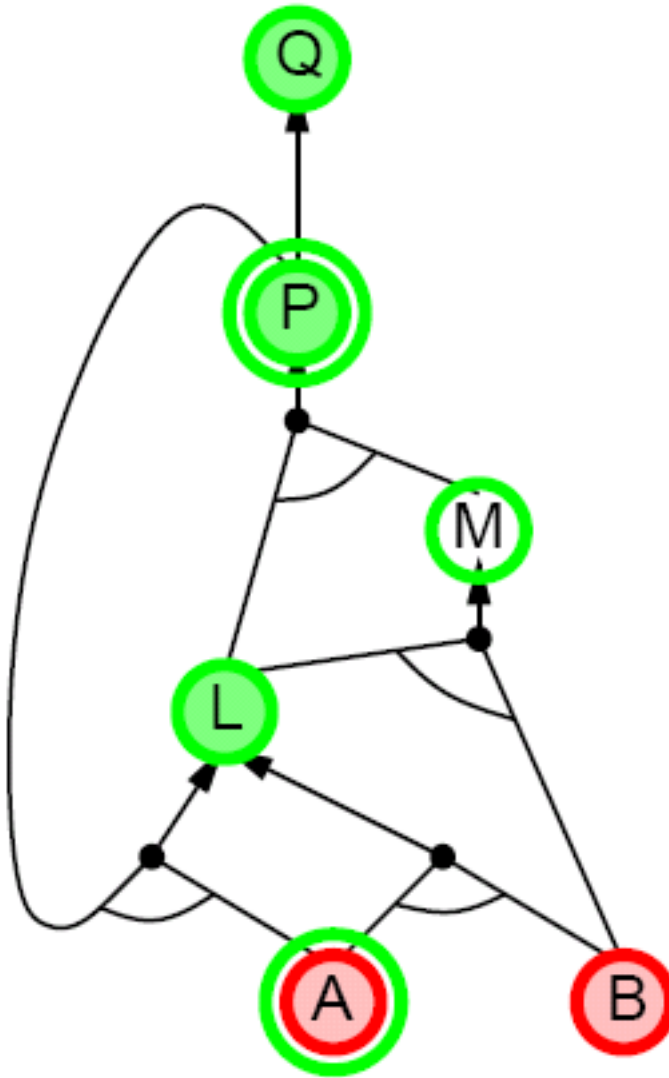
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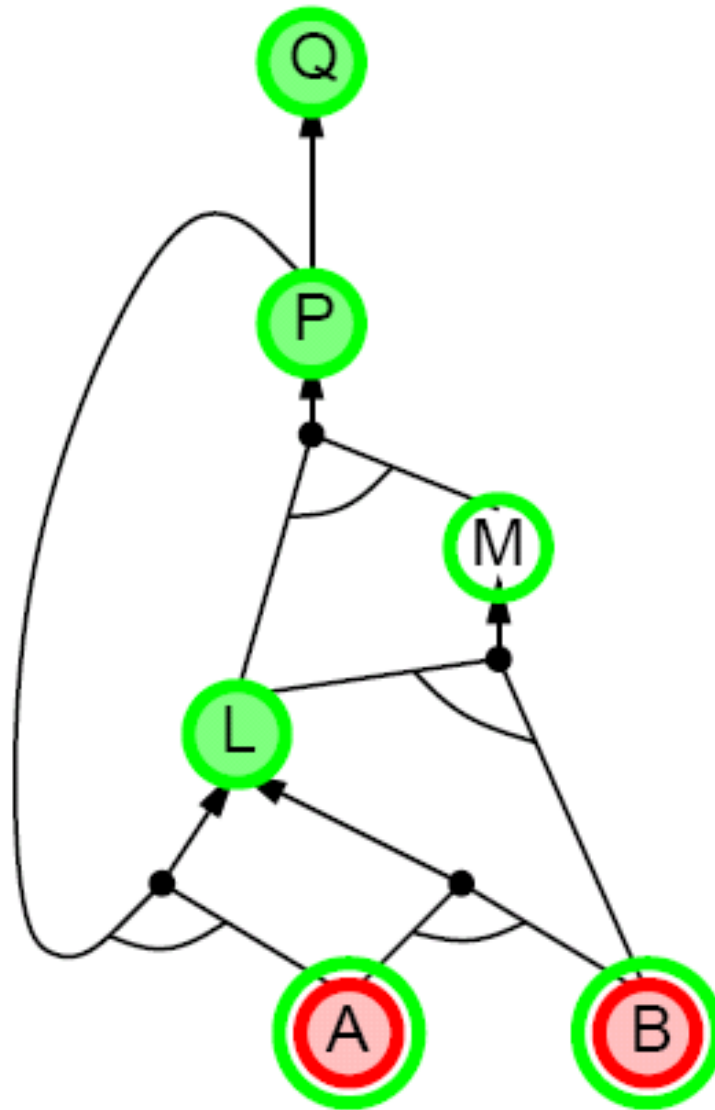
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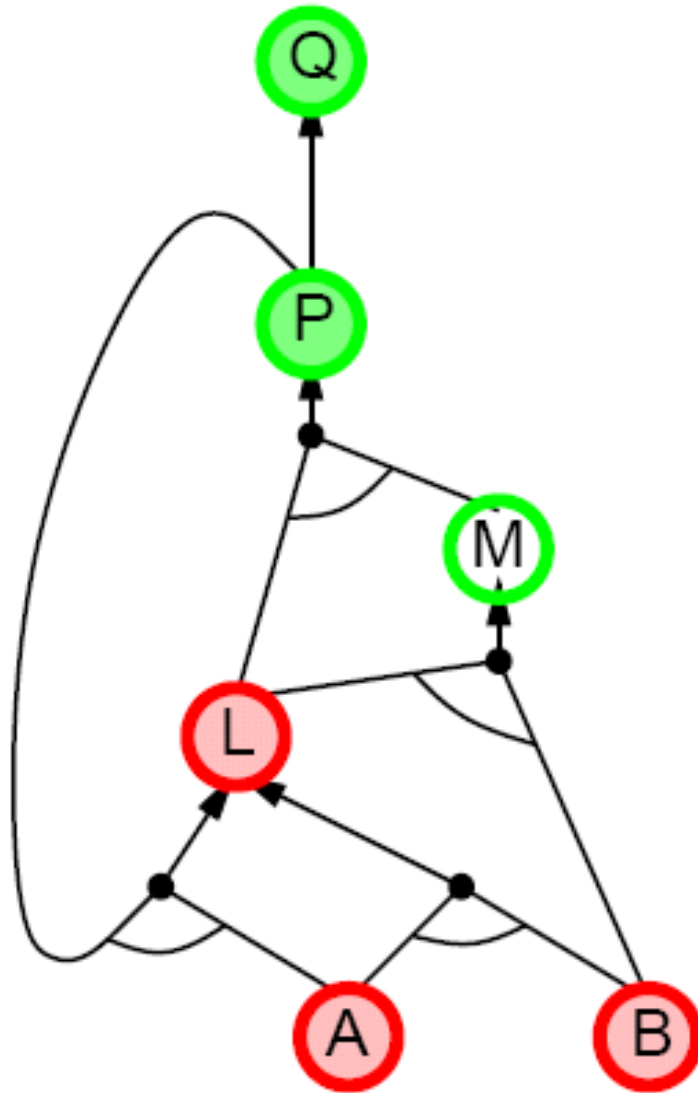
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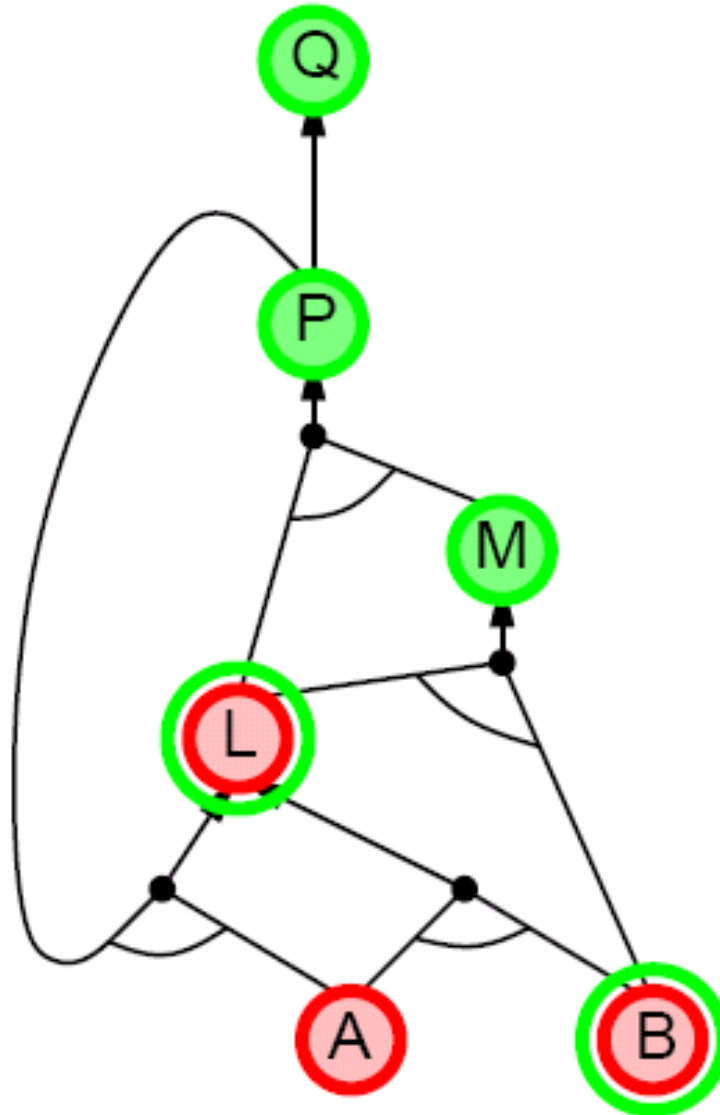
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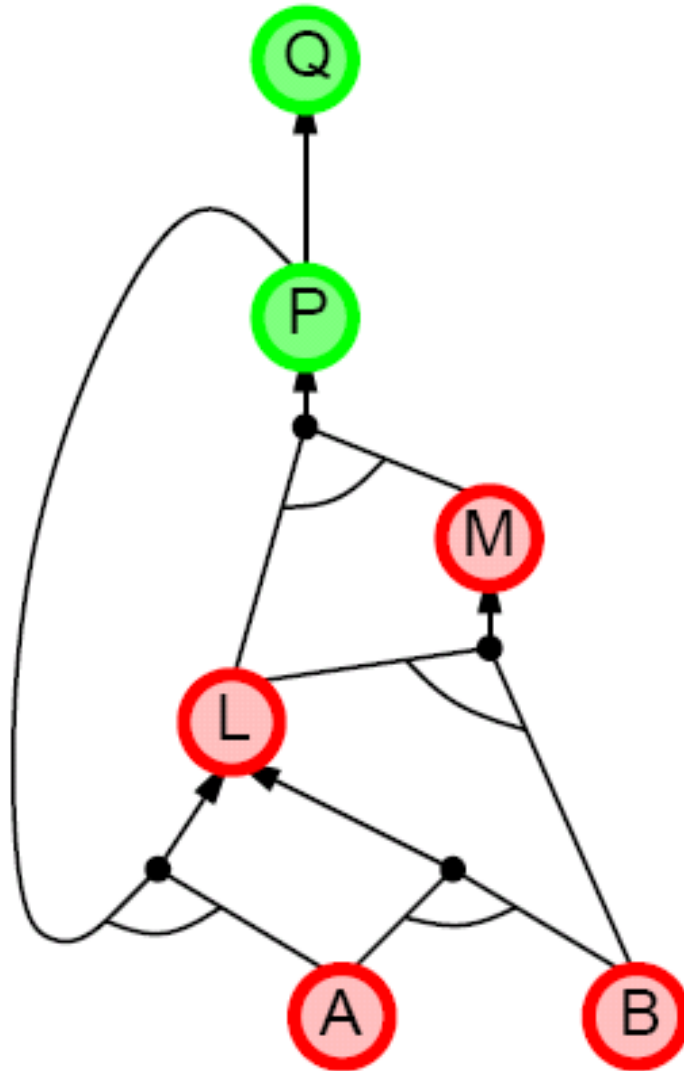
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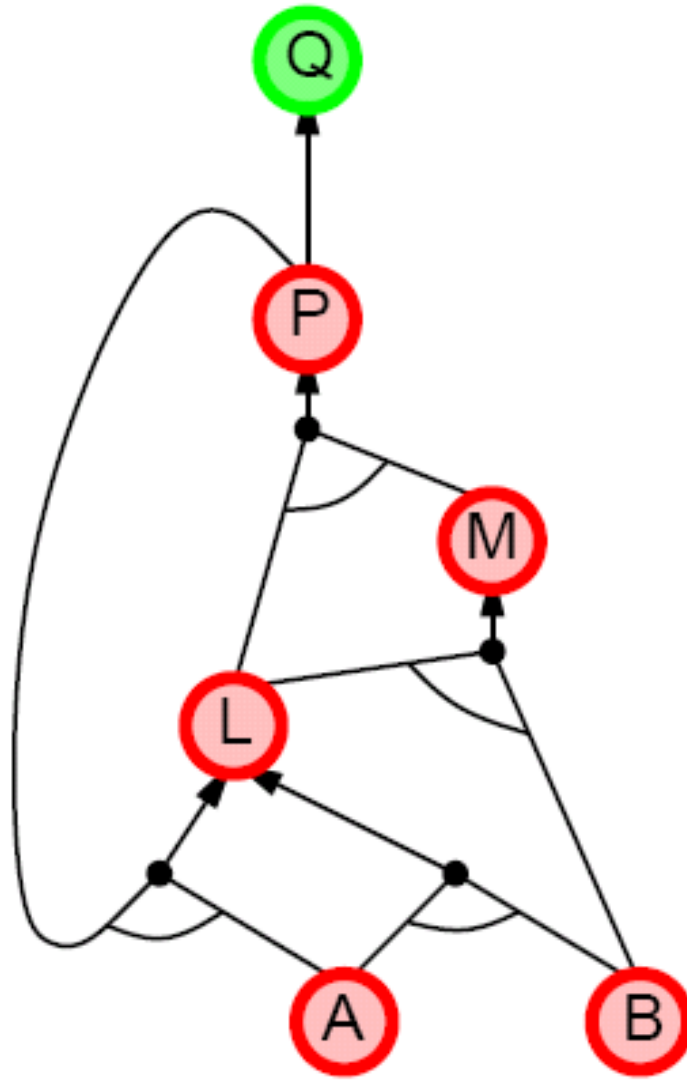
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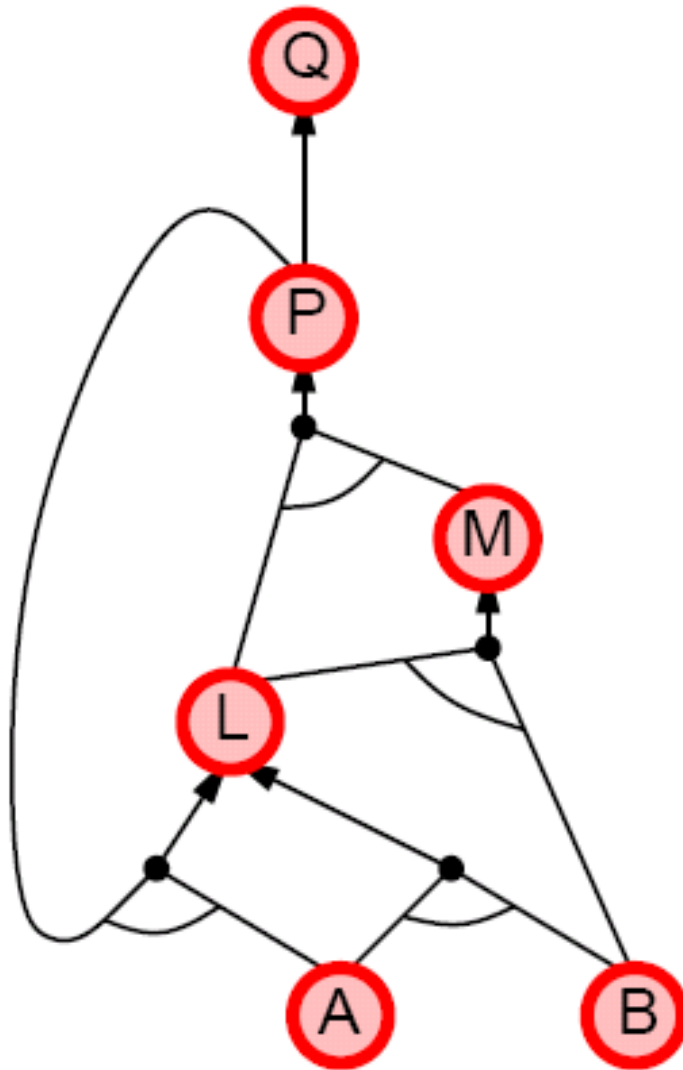
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Forward vs. backward chaining

- Forward chaining is data-driven
 - May perform lots of work irrelevant to the goal
- Backward chaining is goal-driven
 - Appropriate for problem solving
- Some form of bi-directional search is even better

What you should know

- A lot of terms
- Use truth tables
- Proofs
- Conjunctive Normal Form
- Proofs with resolution
- Horn clauses
- Forward chaining algorithm
- Backward chaining algorithm