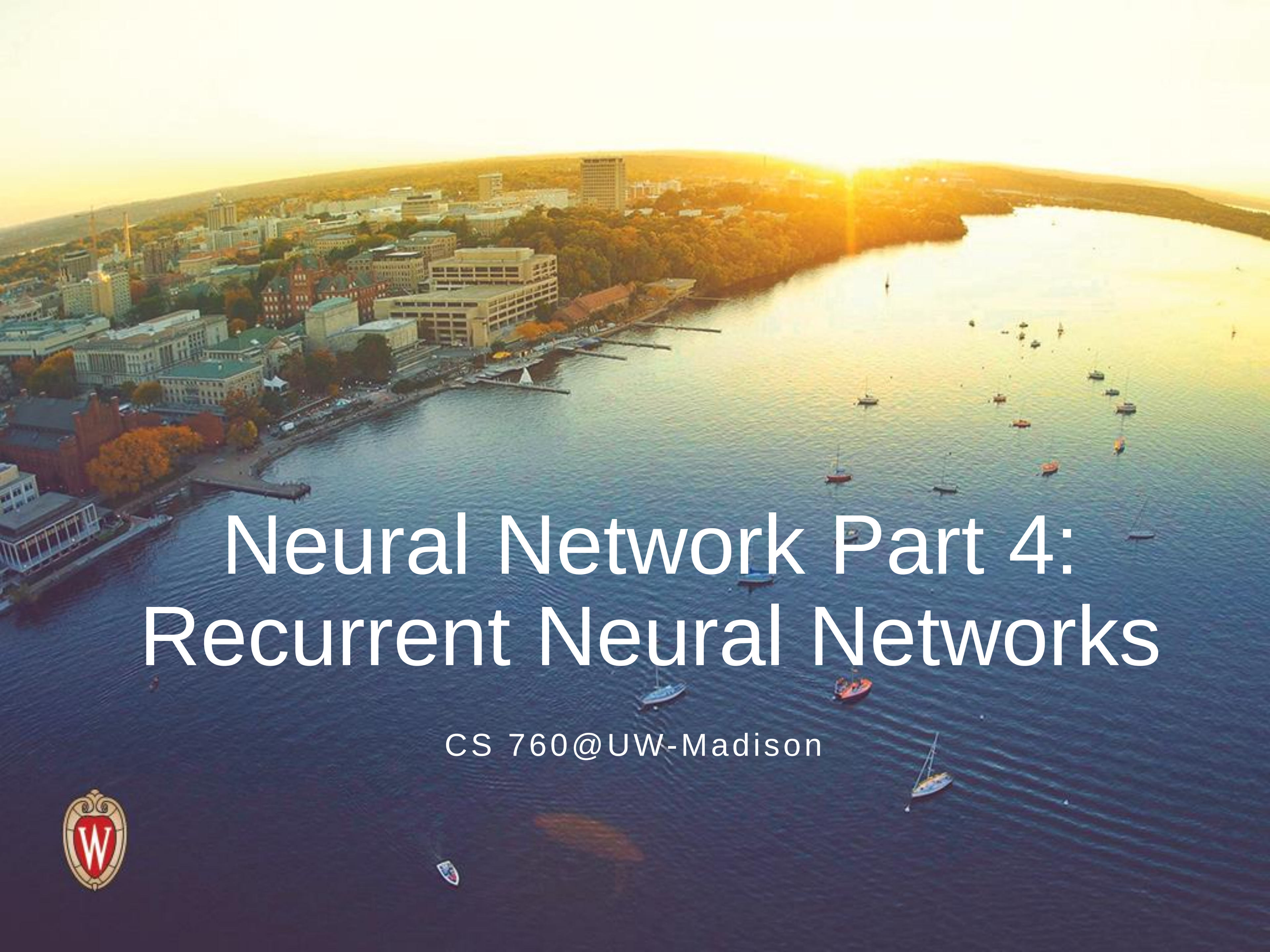


Neural Network Part 4: Recurrent Neural Networks

CS 760@UW-Madison

Goals for the lecture

you should understand the following concepts

- sequential data
- computational graph
- recurrent neural networks (RNN) and the advantage
- training recurrent neural networks
- bidirectional RNNs
- encoder-decoder RNNs

Introduction

Recurrent neural networks

- Dates back to (Rumelhart *et al.*, 1986)
- A family of neural networks for handling sequential data, which involves variable length inputs or outputs
- Especially, for natural language processing (NLP)

Sequential data

- Each data point: A sequence of vectors $x^{(t)}$, for $1 \leq t \leq \tau$
- Batch data: many sequences with different lengths τ
- Label: can be a scalar, a vector, or even a sequence

- Example
  - Sentiment analysis
  - Machine translation

Example: machine translation

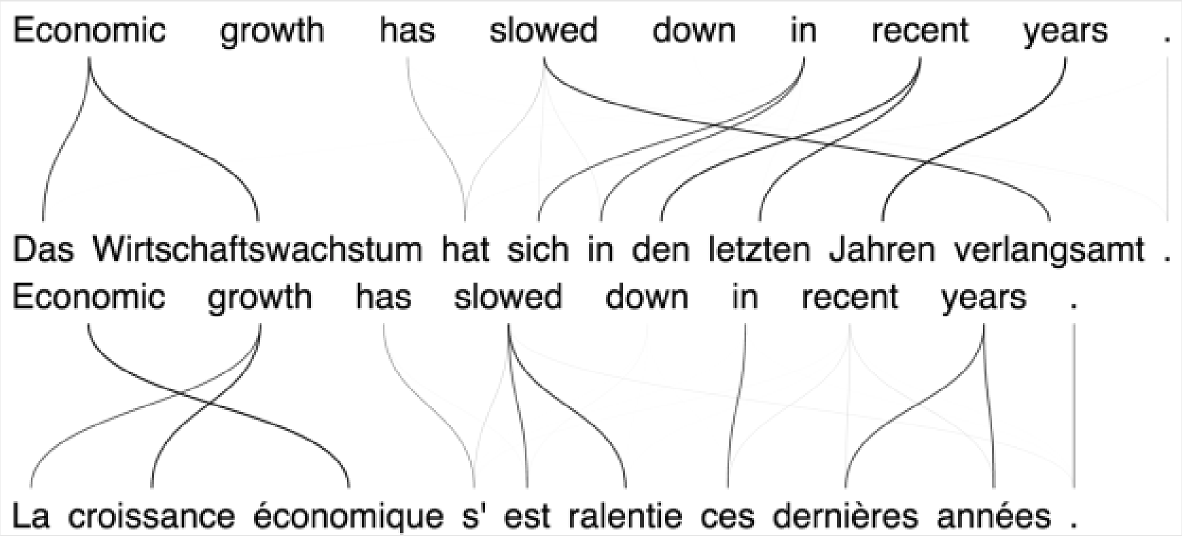


Figure from: devblogs.nvidia.com

More complicated sequential data

- Data point: two dimensional sequences like images
- Label: different type of sequences like text sentences

- Example: image captioning

Image captioning

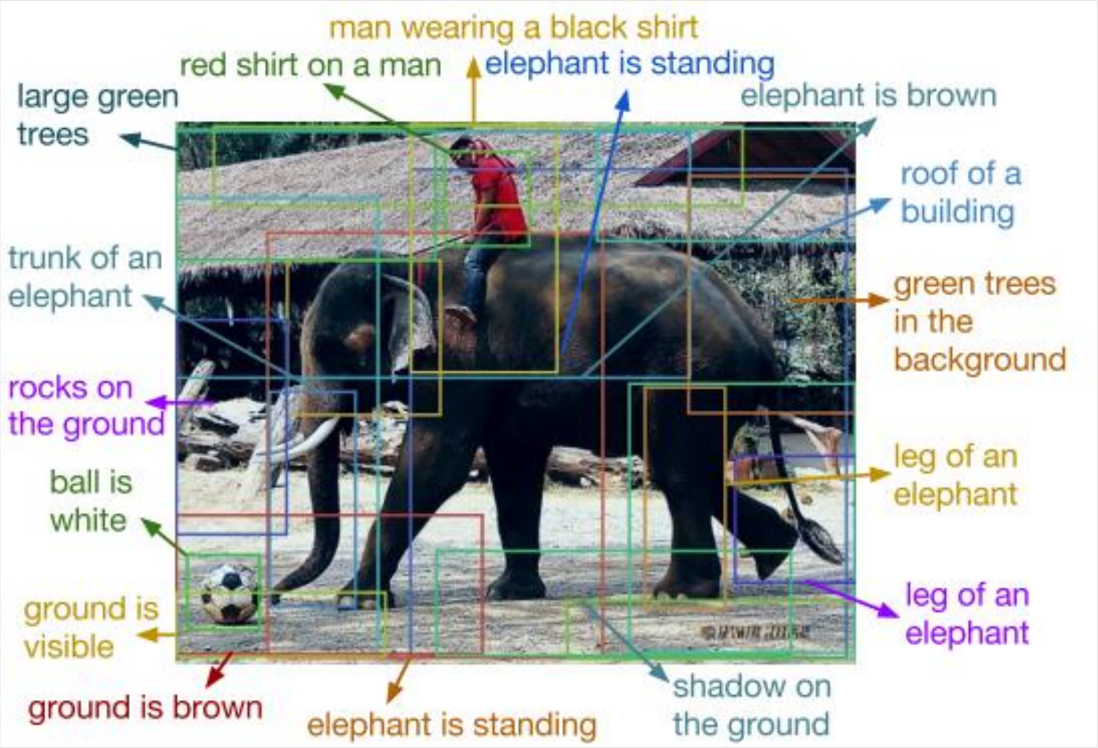
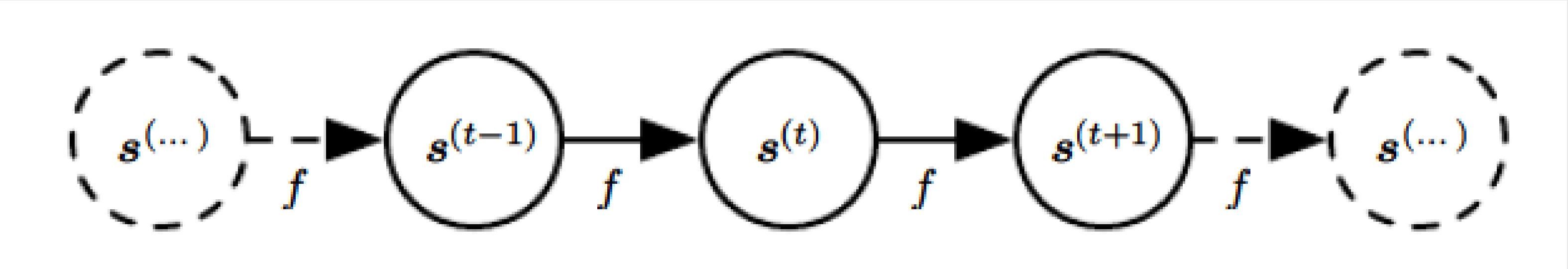


Figure from the paper “DenseCap: Fully Convolutional Localization Networks for Dense Captioning”, by Justin Johnson, Andrej Karpathy, Li Fei-Fei

Computational graphs

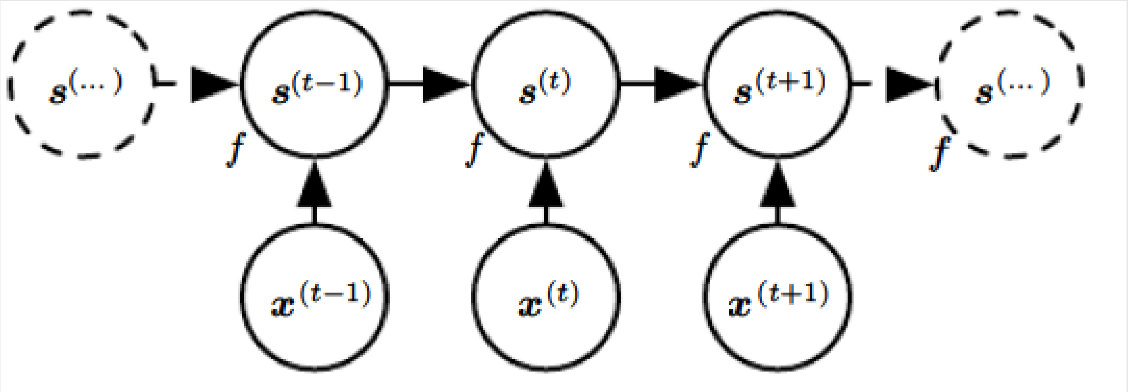
A typical dynamic system



$$s^{(t+1)} = f(s^{(t)}; \theta)$$

Figure from *Deep Learning*,
Goodfellow, Bengio and Courville

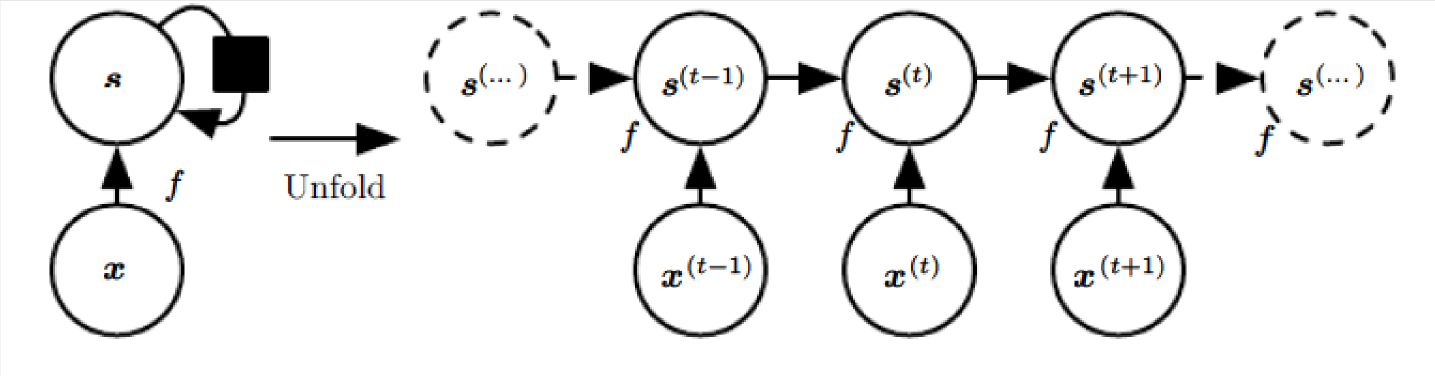
A system driven by external data



$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta)$$

Figure from *Deep Learning*,
Goodfellow, Bengio and Courville

Compact view

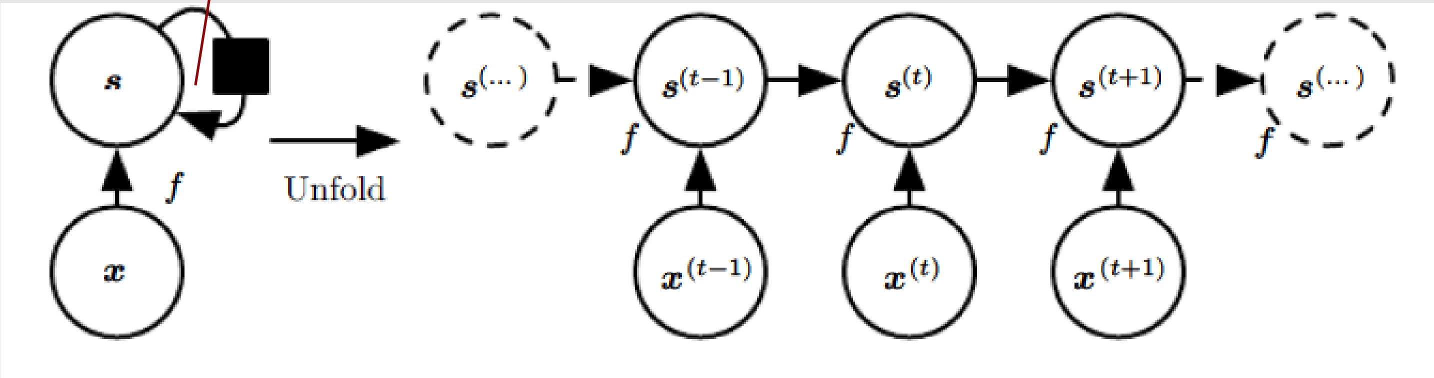


$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta)$$

Figure from *Deep Learning*,
Goodfellow, Bengio and Courville

Compact view

square: one step time delay



$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta)$$

Key: the same f and θ for all time steps

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks (RNN)

Recurrent neural networks

- Use the same computational function and parameters across different time steps of the sequence
- Each time step: takes the input entry and the previous hidden state to compute the output entry
- Loss: typically computed at every time step

Recurrent neural networks

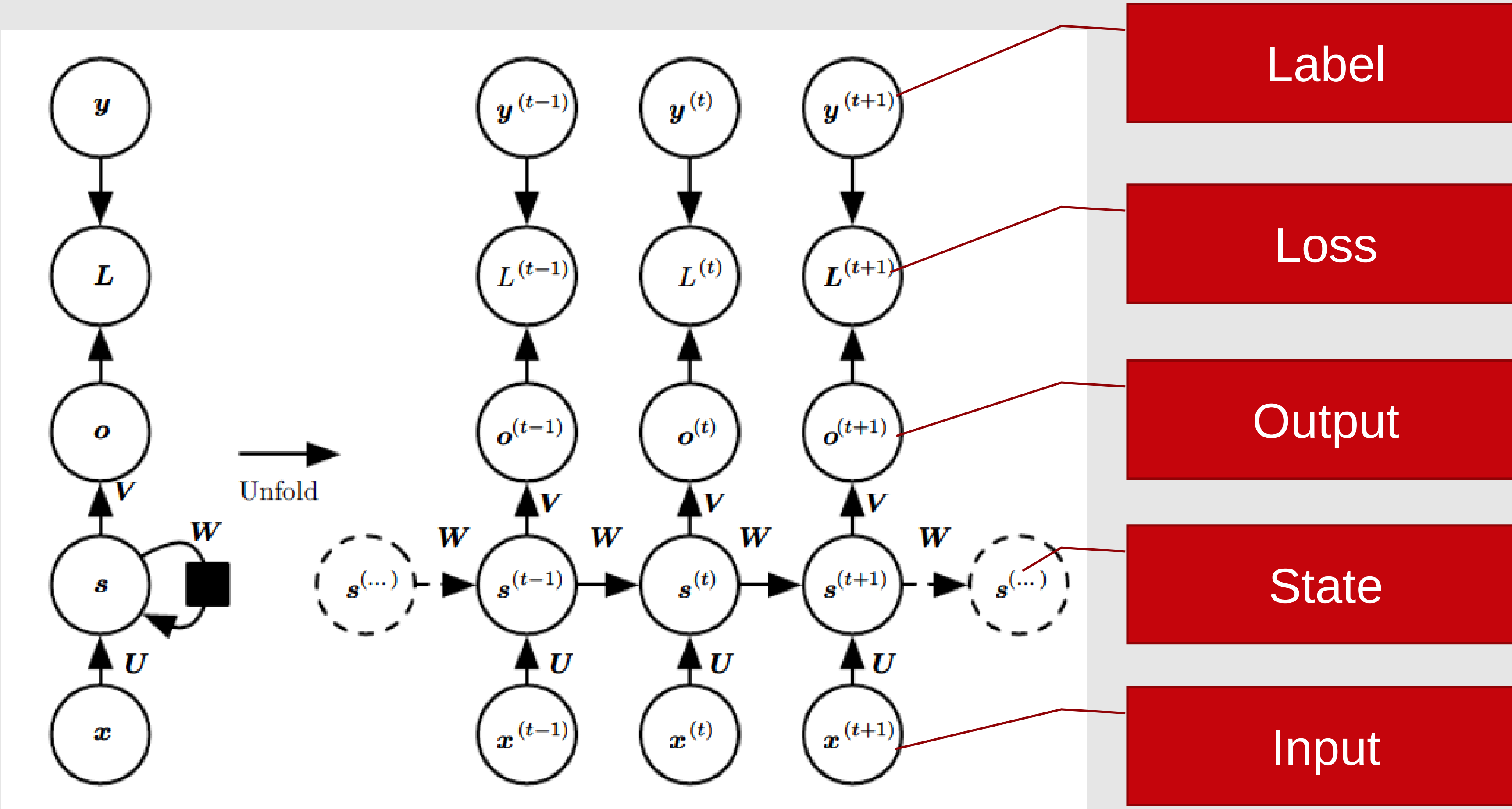
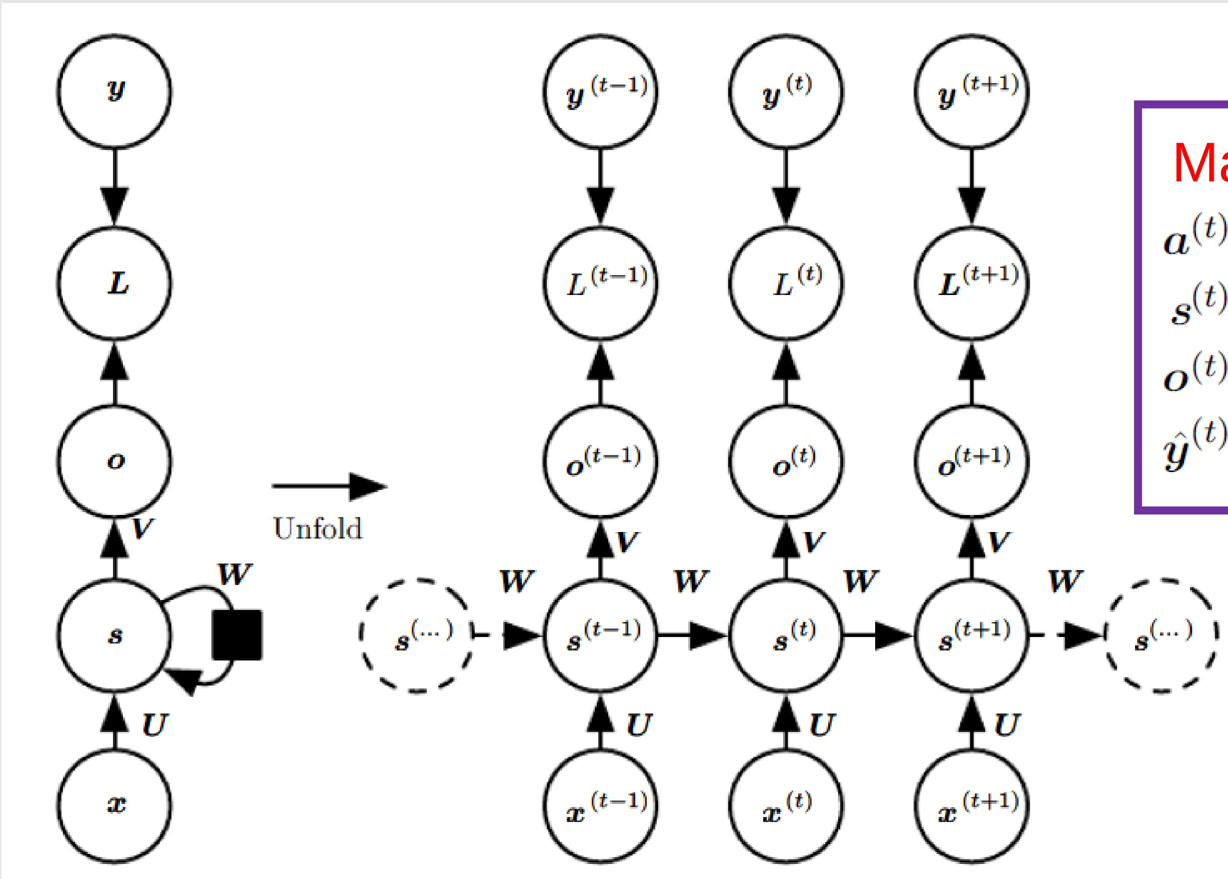


Figure from *Deep Learning*, by Goodfellow, Bengio and Courville

Recurrent neural networks



Math formula:

$$\begin{aligned}
\boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{s}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)} \\
\boldsymbol{s}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}) \\
\boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{s}^{(t)} \\
\hat{\boldsymbol{y}}^{(t)} &= \mathrm{softmax}(\boldsymbol{o}^{(t)})
\end{aligned}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Advantage

- Hidden state: a lossy summary of the past
- Shared functions and parameters: greatly reduce the capacity and good for generalization in learning
- Explicitly use the prior knowledge that the sequential data can be processed by in the same way at different time step (e.g., NLP)

Advantage

- Hidden state: a lossy summary of the past
- Shared functions and parameters: greatly reduce the capacity and good for generalization in learning
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- Yet still powerful (actually universal): any function computable by a Turing machine can be computed by such a recurrent network of a finite size (see, e.g., Siegelmann and Sontag (1995))

Training RNN

- Principle: unfold the computational graph, and use backpropagation
- Called back-propagation through time (BPTT) algorithm
- Can then apply any general-purpose gradient-based techniques

Training RNN

- Principle: unfold the computational graph, and use backpropagation
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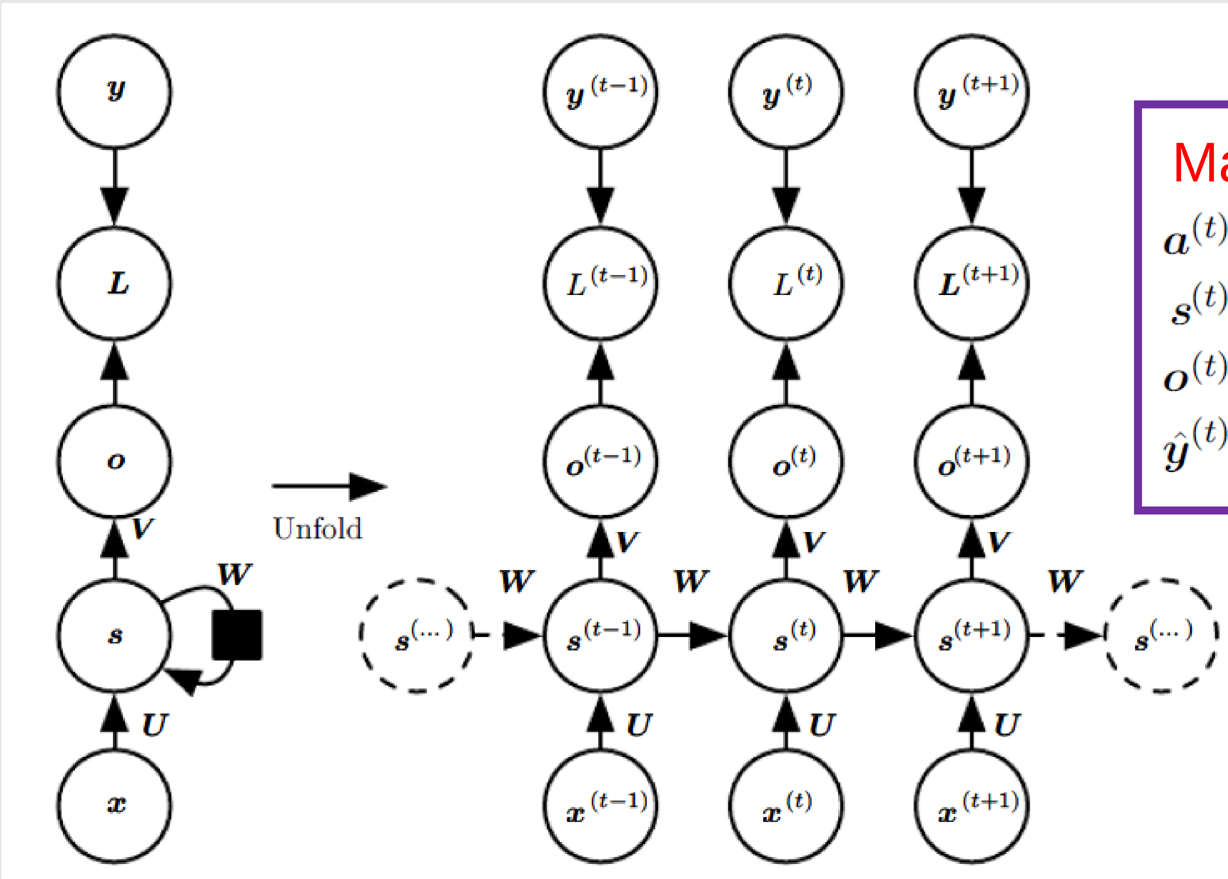
- Can then apply any general-purpose gradient-based techniques

- Conceptually: first compute the gradients of the internal nodes, then compute the gradients of the parameters

Recurrent neural networks



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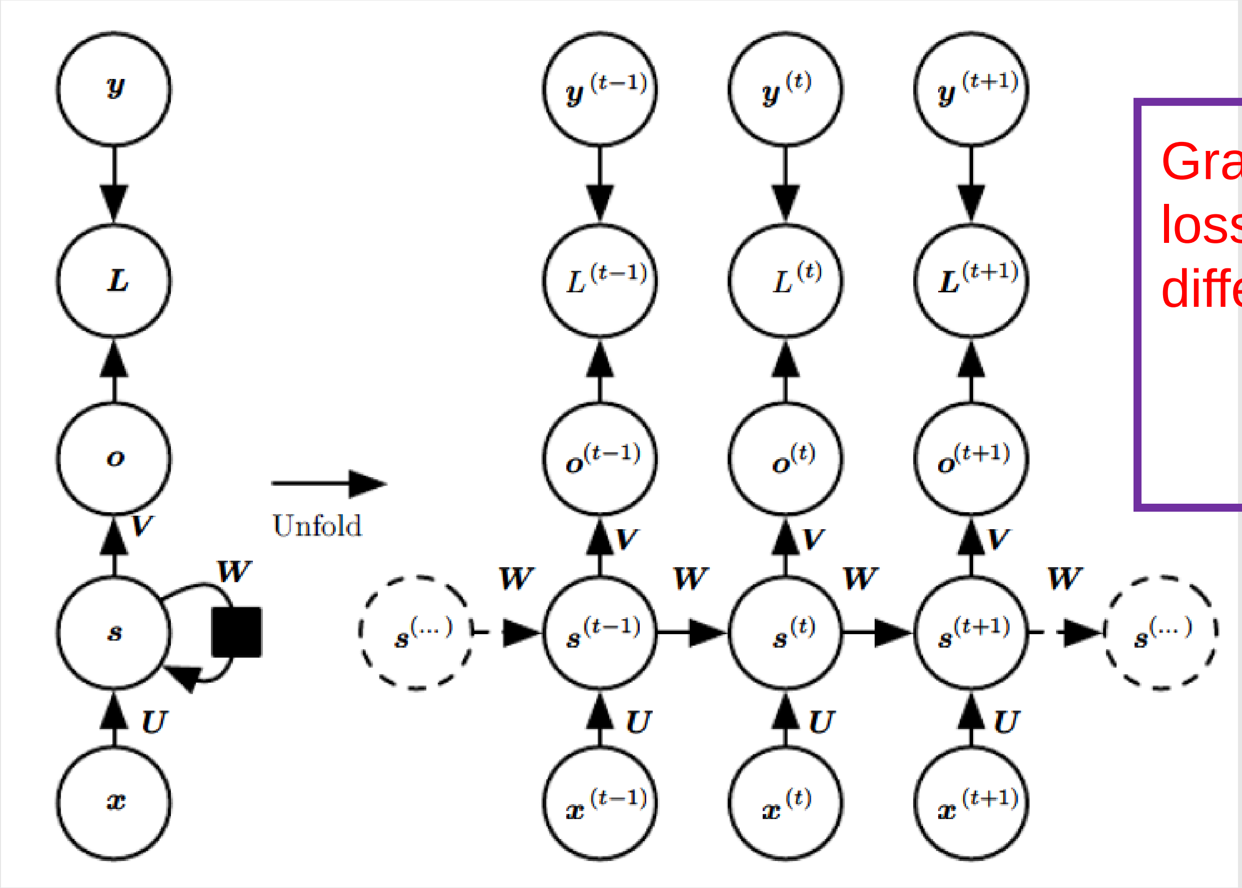
\end{aligned}
$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks

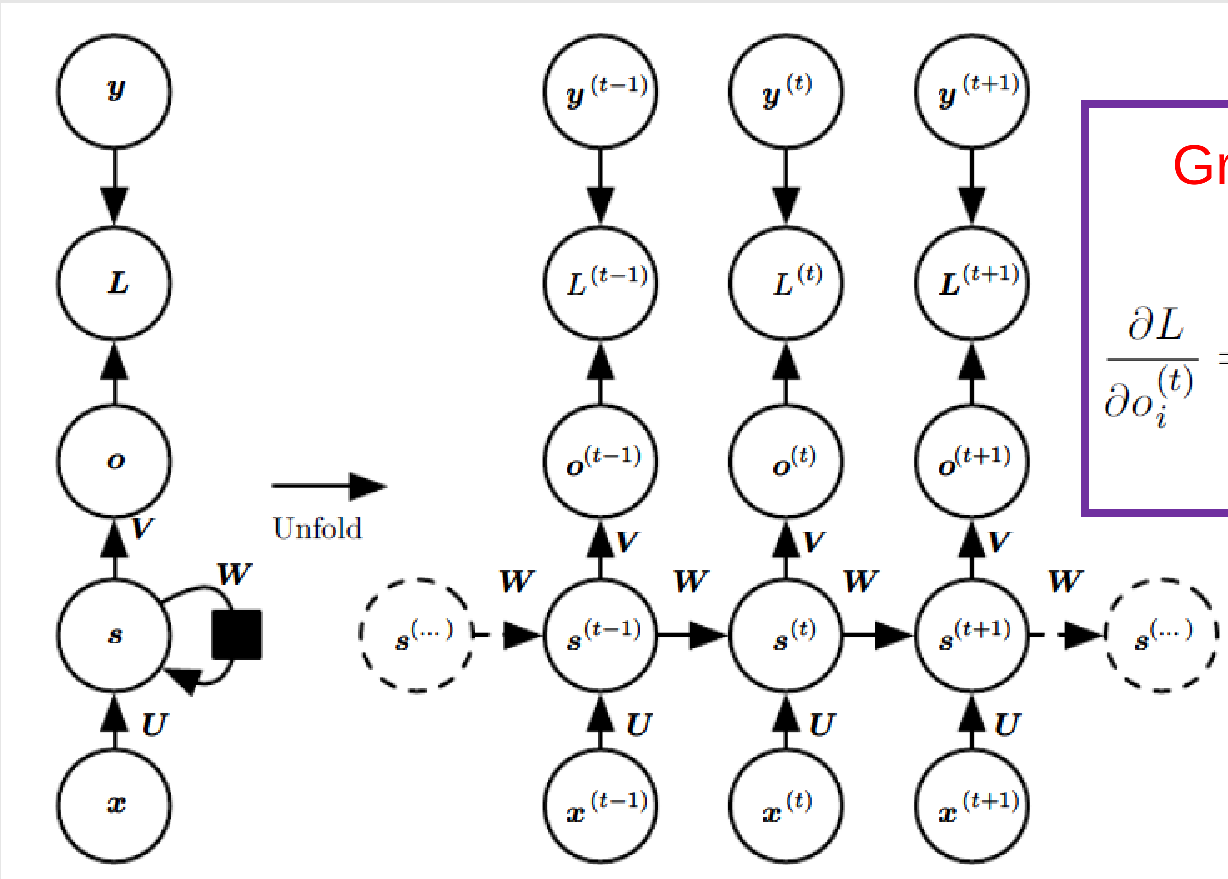


Gradient at $L^{(t)}$: (total loss is sum of those at different time steps)

$$\frac{\partial L}{\partial L^{(t)}} = 1.$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks

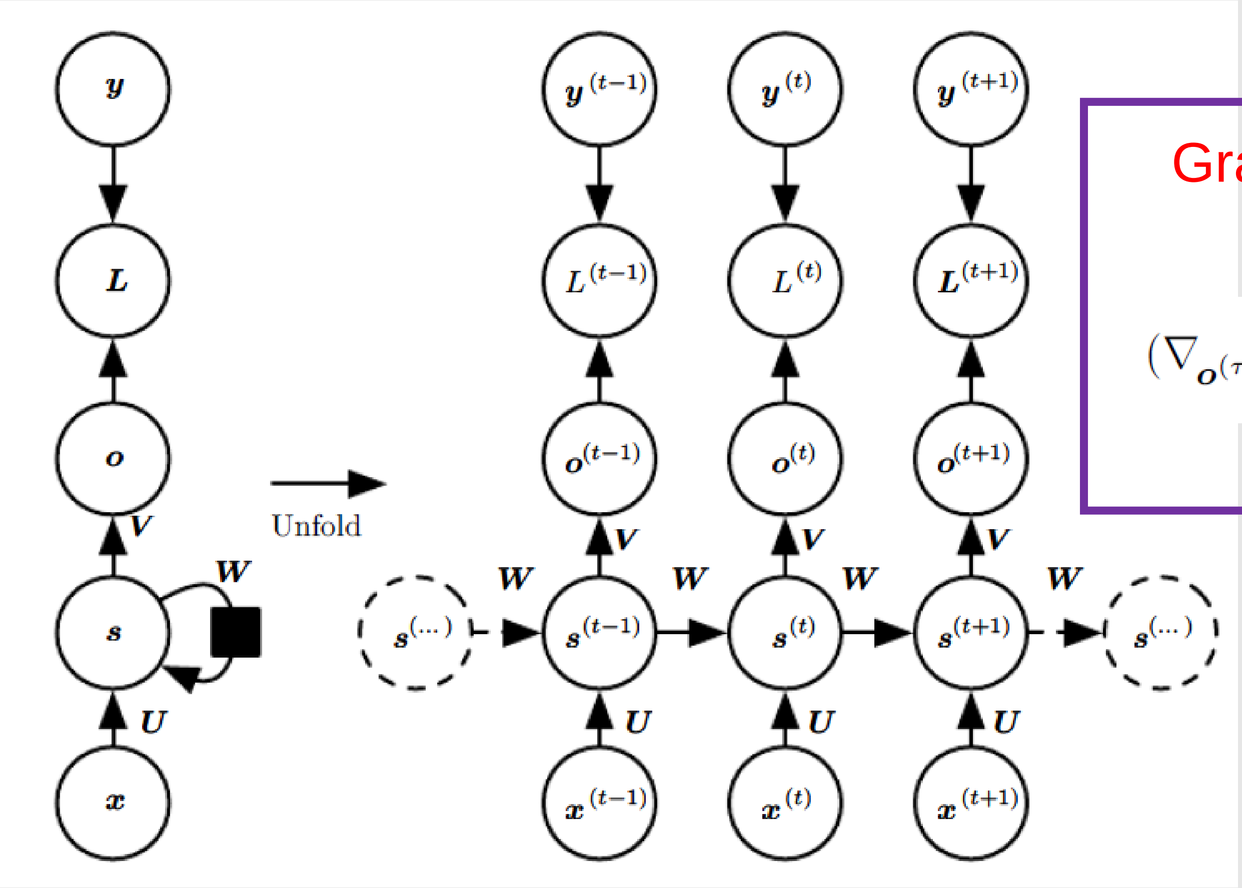


Gradient at $o^{(t)}$:

$$\frac{\partial L}{\partial o_i^{(t)}} = \frac{\partial L}{\partial L^{(t)}} \frac{\partial L^{(t)}}{\partial o_i^{(t)}} = \hat{y}_i^{(t)} - \mathbf{1}_{i,y^{(t)}}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks

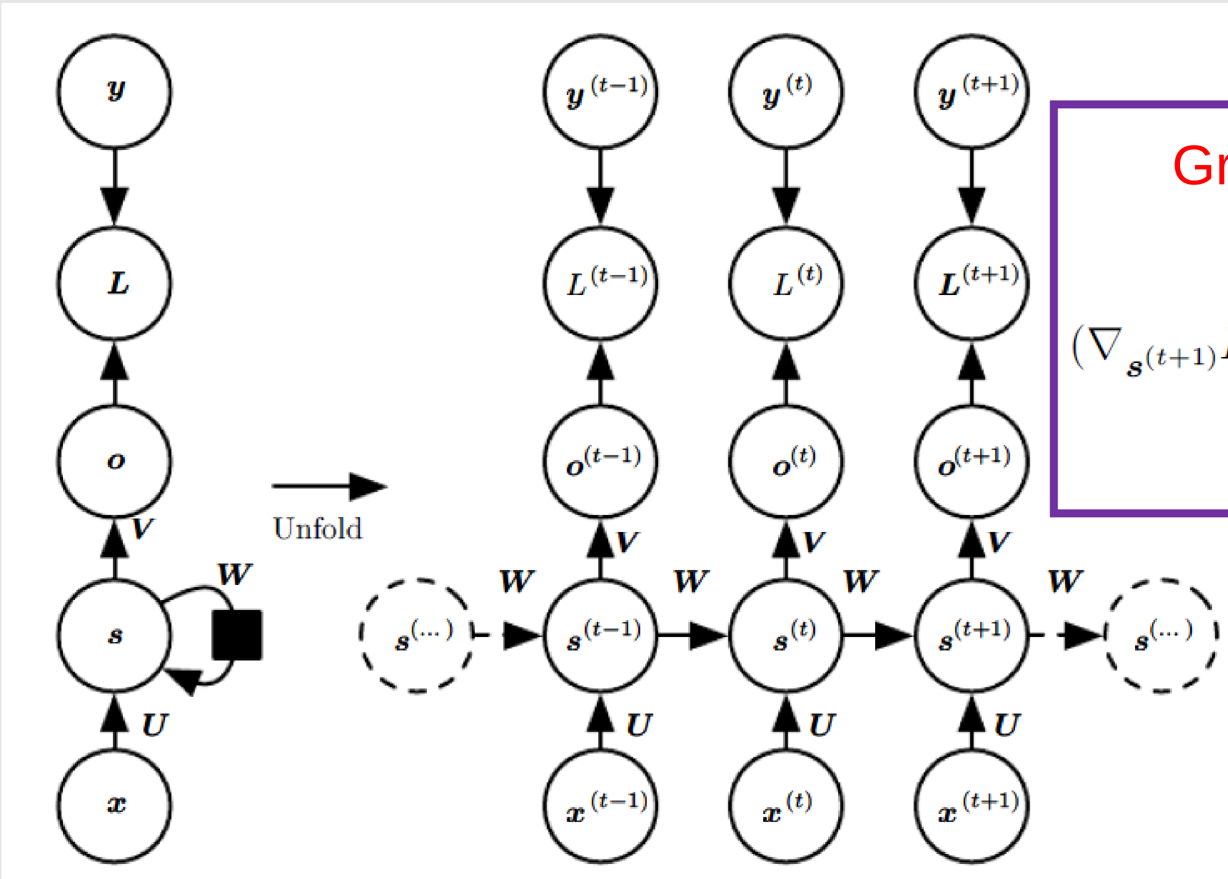


Gradient at $s^{(\tau)}$:

$$(\nabla_{o^{(\tau)}} L) \frac{\partial o^{(\tau)}}{\partial s^{(\tau)}} = (\nabla_{o^{(\tau)}} L) V$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks

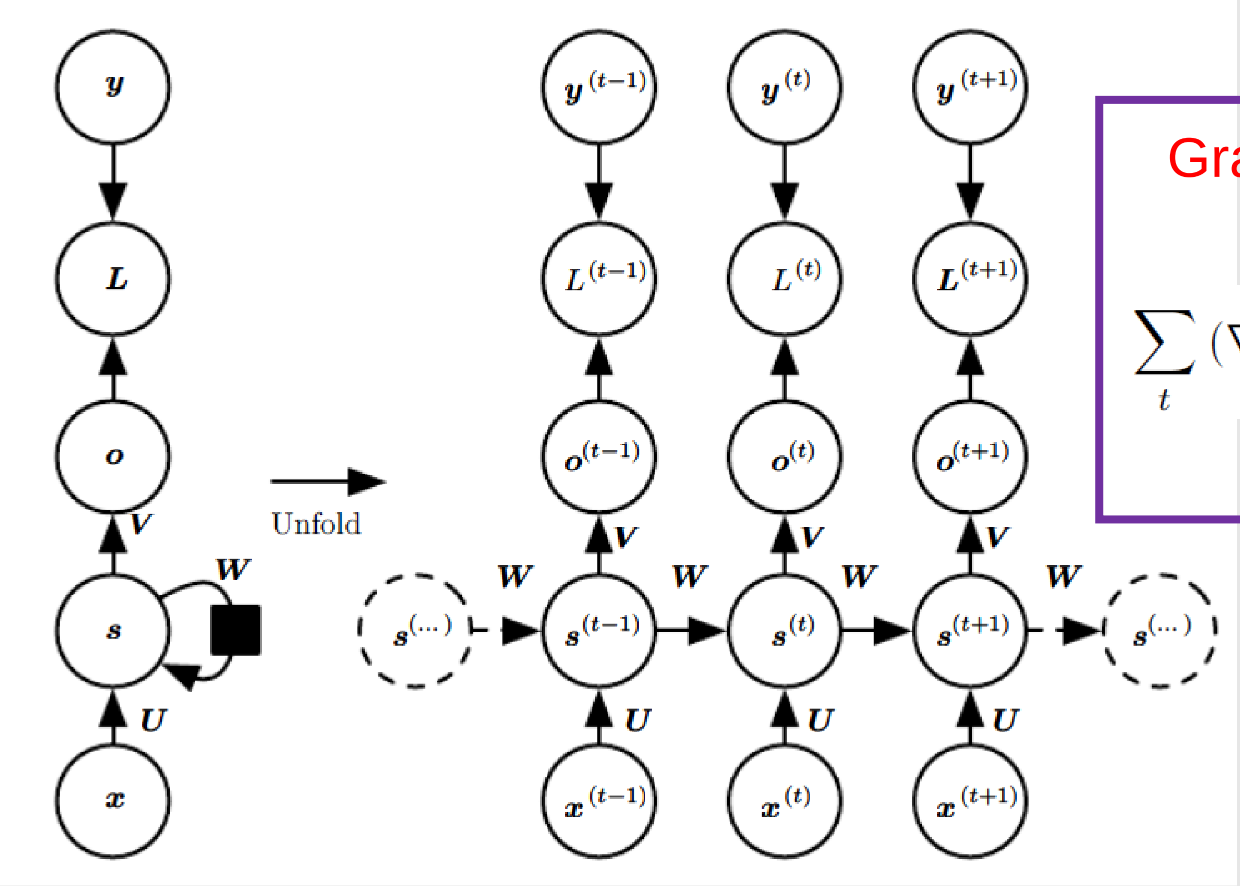


Gradient at $s^{(t)}$:

$$(\nabla_{\boldsymbol{s}^{(t+1)}} L) \frac{\partial \boldsymbol{s}^{(t+1)}}{\partial \boldsymbol{s}^{(t)}} + (\nabla_{\boldsymbol{o}^{(t)}} L) \frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{s}^{(t)}}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Recurrent neural networks



Gradient at parameter V:

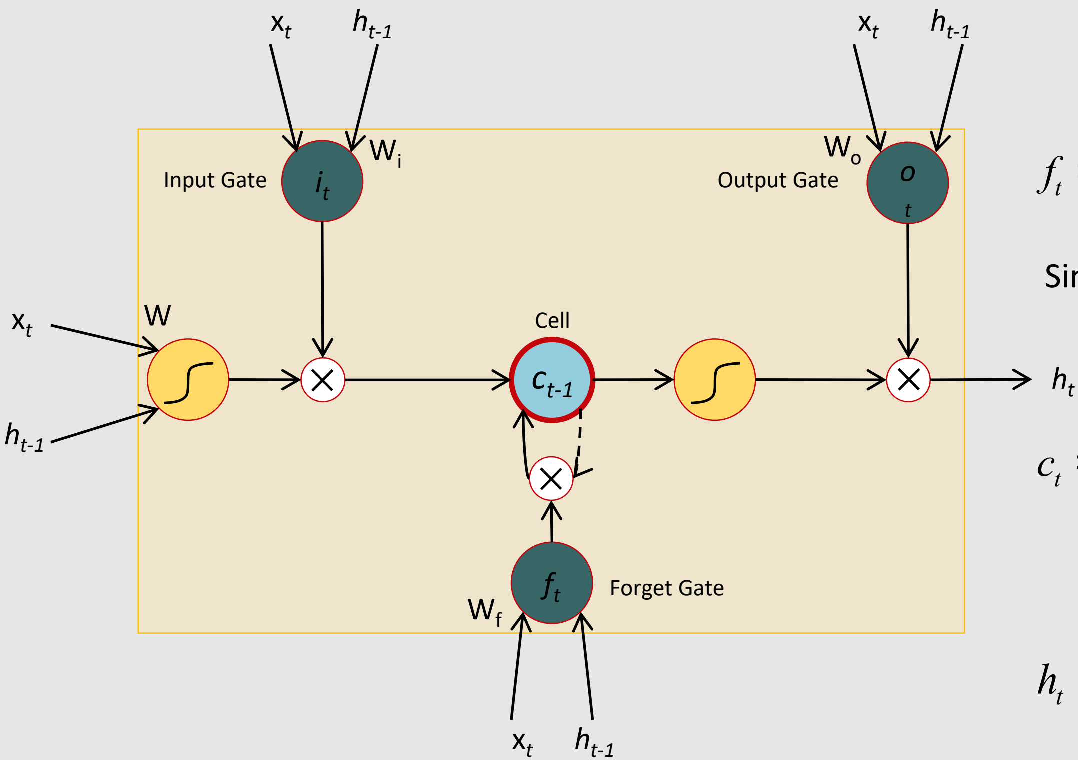
$$\sum_t \left(\nabla_{\boldsymbol{o}^{(t)}} L\right) \frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{V}} = \sum_t \left(\nabla_{\boldsymbol{o}^{(t)}} L\right) \boldsymbol{s}^{(t)\top}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

The problem of exploding/vanishing gradient

- What happens to the magnitude of the gradients as we backpropagate through many layers?
  - If the weights are small, the gradients shrink exponentially.
  - If the weights are big the gradients grow exponentially.
- Typical feed-forward neural nets can cope with these exponential effects because they only have a few hidden layers.
- In an RNN trained on long sequences (*e.g.* 100 time steps) the gradients can easily explode or vanish.
  - We can avoid this by initializing the weights very carefully.
- Even with good initial weights, its very hard to detect that the current target output depends on an input from many time-steps ago.
  - So RNNs have difficulty dealing with long-range dependencies.

The Popular LSTM Cell



$$f_t = s\left(W_f\begin{pmatrix}x_t\\h_{t-1}\end{pmatrix} + b_f\right)$$

Similarly for i_t, o_t

$$c_t = f_t \otimes c_{t-1} +$$

$$i_t \otimes \tanh W\begin{pmatrix}x_t\\h_{t-1}\end{pmatrix}$$

$h_t = o_t$ Ä $\tanh c_t$

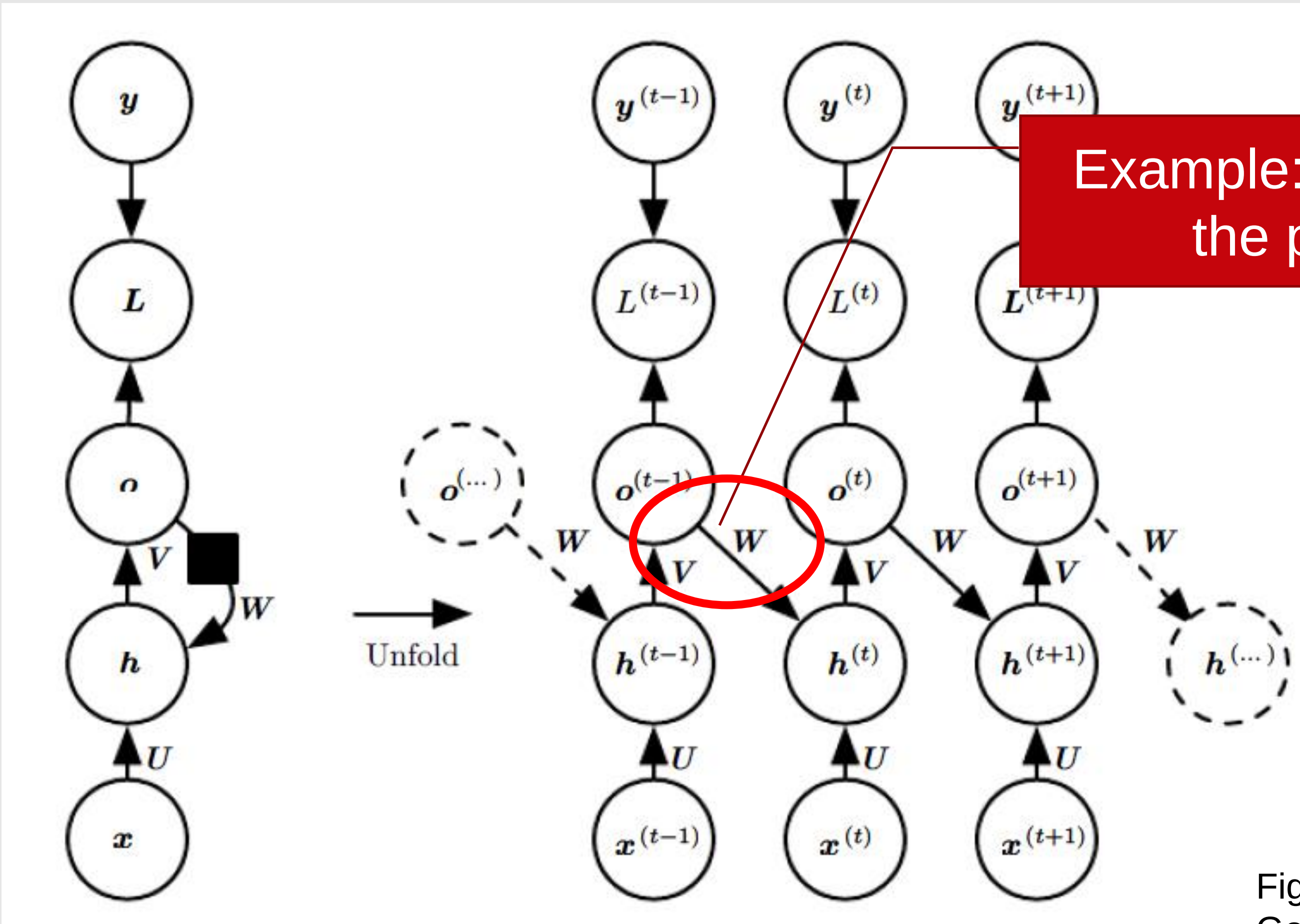
* Dashed line indicates time-lag

Some Other Variants of RNN

RNN

- Use the same computational function and parameters across different time steps of the sequence
- Each time step: takes the input entry and the previous hidden state to compute the output entry
- Loss: typically computed every time step

- Many variants
  - Information about the past can be in many other forms
  - Only output at the end of the sequence



Example: use the output at the previous step

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

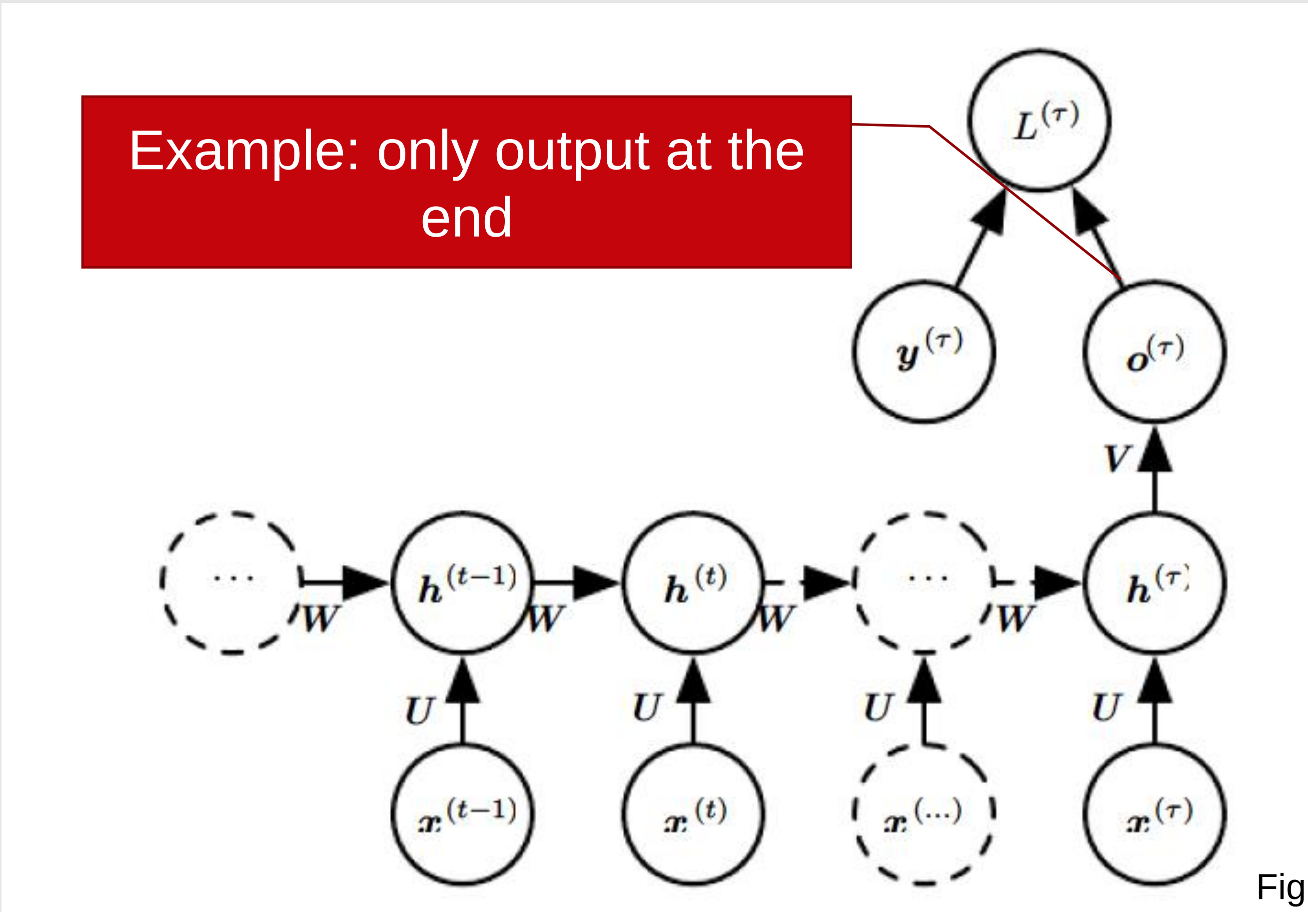


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Bidirectional RNNs

- Many applications: output at time t may depend on the whole input sequence
- Example in speech recognition: correct interpretation of the current sound may depend on the next few phonemes, potentially even the next few words
- Bidirectional RNNs are introduced to address this

BiRNNs

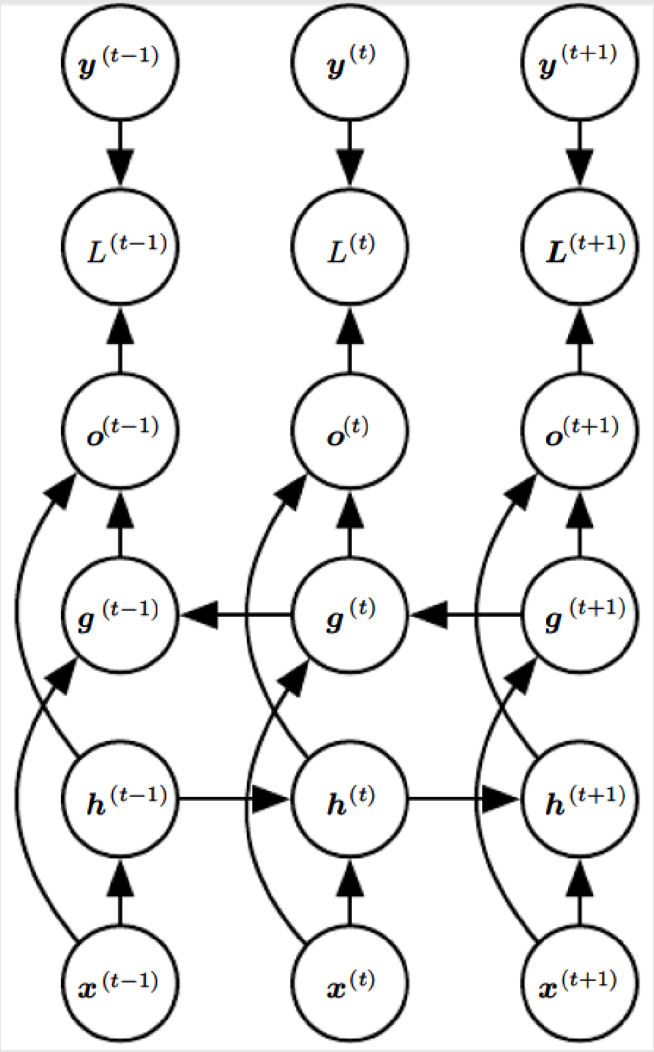


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Encoder-decoder RNNs

- RNNs: can map sequence to one vector; or to sequence of same length
- What about mapping sequence to sequence of different length?
- Example: speech recognition, machine translation, question answering, etc

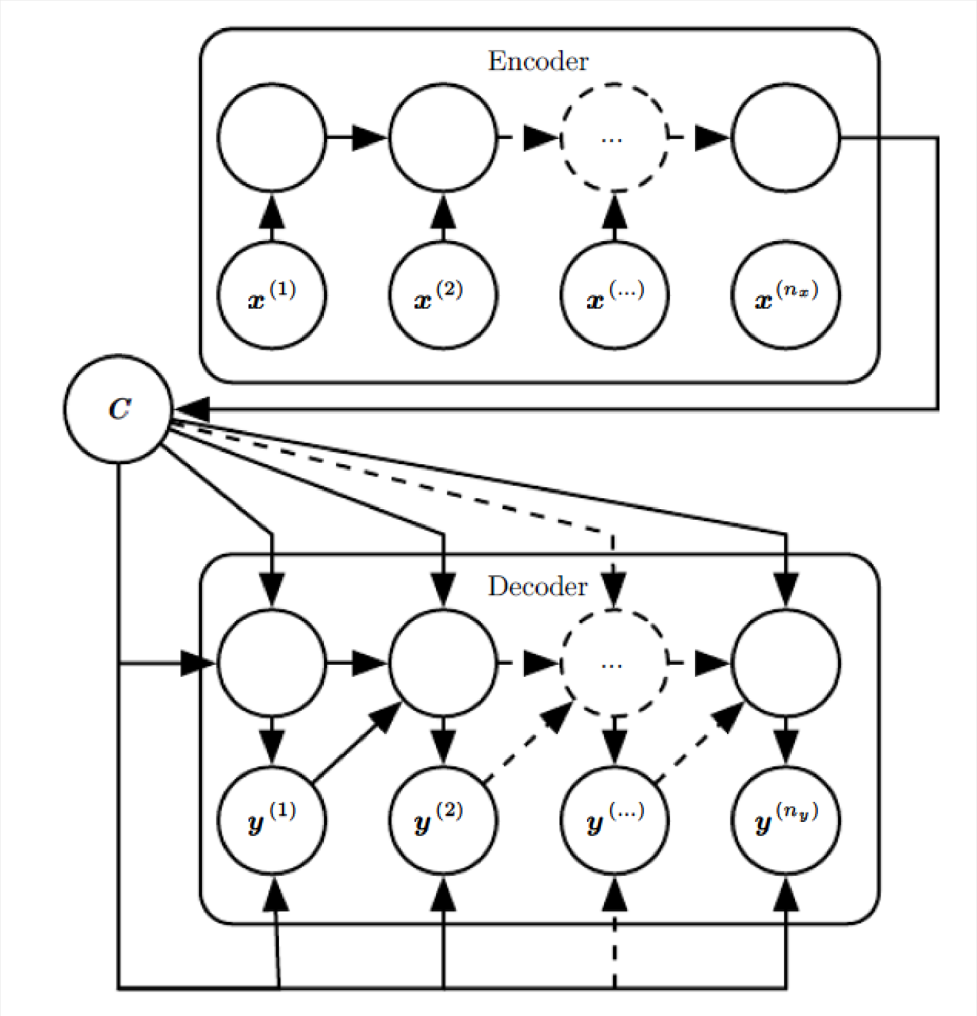


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

THANK YOU

Some of the slides in these lectures have been adapted/borrowed from materials developed by Mark Craven, David Page, Jude Shavlik, Tom Mitchell, Nina Balcan, Matt Gormley, Elad Hazan, Tom Dietterich, Pedro Domingos, and Geoffrey Hinton.