Reinforcement Learning
Part 1

CS 760@UW-Madison

Goals for the lecture

you should understand the following concepts

- the reinforcement learning task
- Markov decision process
- value functions
- value iteration
- Q functions
- Q learning

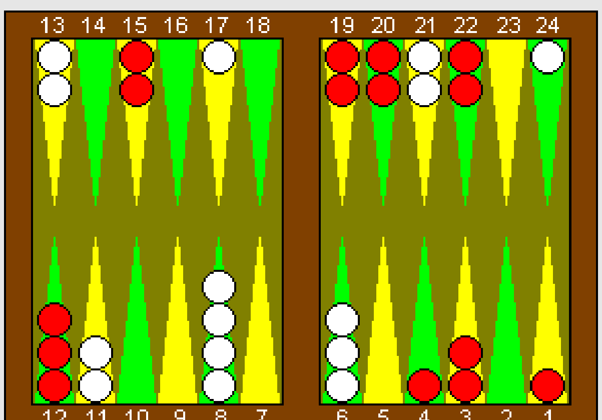
Reinforcement learning (RL)

Task of an agent embedded in an environment

repeat forever

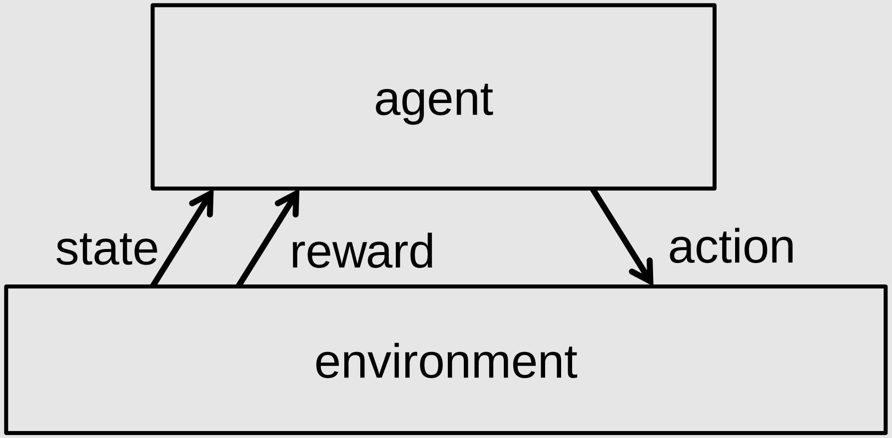
1) sense world
2) reason
3) choose an action to perform
4) get feedback (usually reward = 0)
5) learn

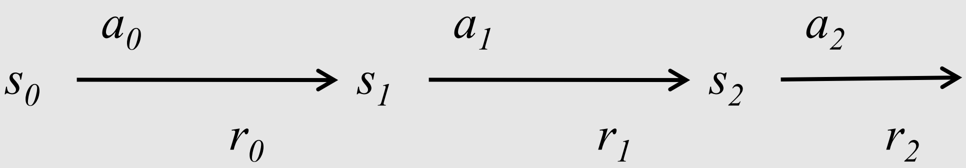
the environment may be the physical world or an artificial one



Reinforcement learning

- set of states S
- set of actions A
- at each time t, agent observes state $s_t \in S$ then chooses action $a_t \in A$
- then receives reward r_t and changes to state s_{t+1}





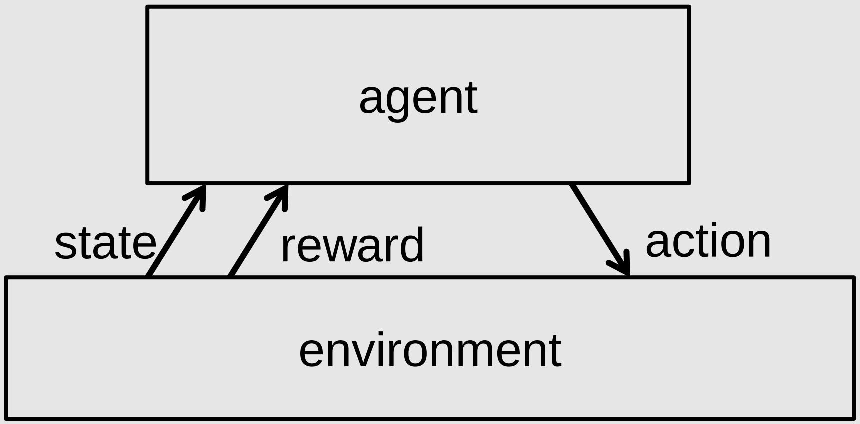
RL as Markov decision process (MDP)

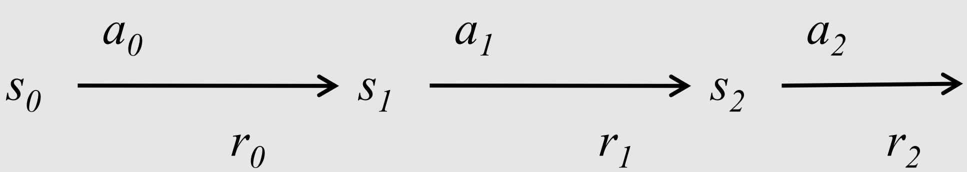
- Markov assumption

$$P(s_{t+1}|s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(s_{t+1}|s_t, a_t)$$

- also assume reward is Markovian

$$P(r_t|s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(r_t|s_t, a_t)$$





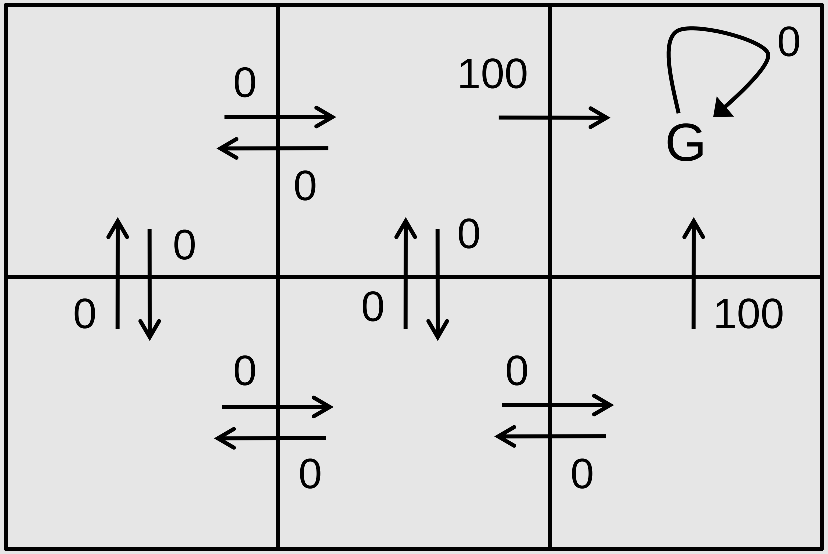
Goal: learn a policy $\pi : S \rightarrow A$ for choosing actions that maximizes

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \quad \text{where } 0 \le \gamma < 1$$

for every possible starting state s_0

Reinforcement learning task

- Suppose we want to learn a control policy $\pi : S \rightarrow A$ that maximizes $\sum_{t=0}^{\infty} \gamma^t E[r_t]$ from every state $s \in S$



each arrow represents an action a and the associated number represents deterministic reward $r(s, a)$

Value Function

Value function for a policy

- given a policy $\pi : S \rightarrow A$ define

$$V^{\pi}(s) = \sum_{t=0}^{\infty} \gamma^t E[r_t]$$

assuming action sequence chosen according to π starting at state s

- we want the optimal policy π^* where

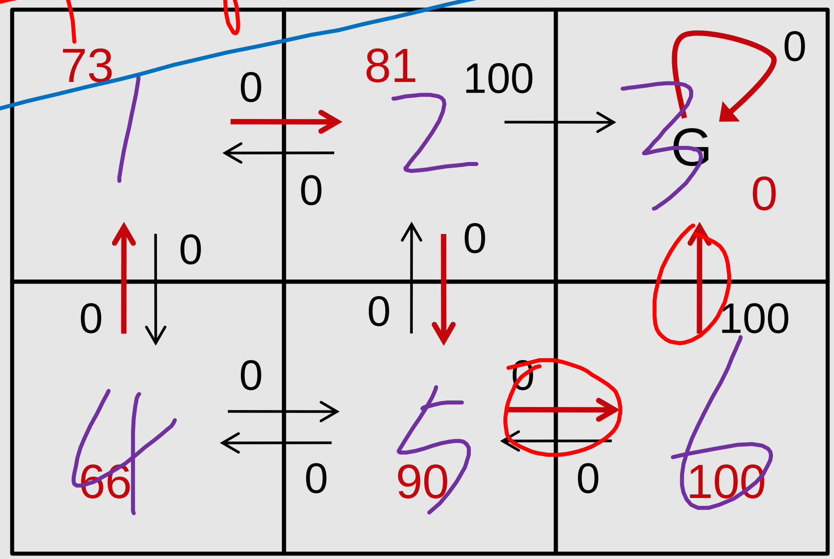
$$p^* = \arg\max_p V^p(s) \quad \text{for all } s$$

we'll denote the value function for this optimal policy as $V^*(s)$

Value function for a policy π

- Suppose π is shown by red arrows, $\gamma = 0.9$

$100 + \gamma \cdot 0 + \gamma^2 \times 0 + \ldots$

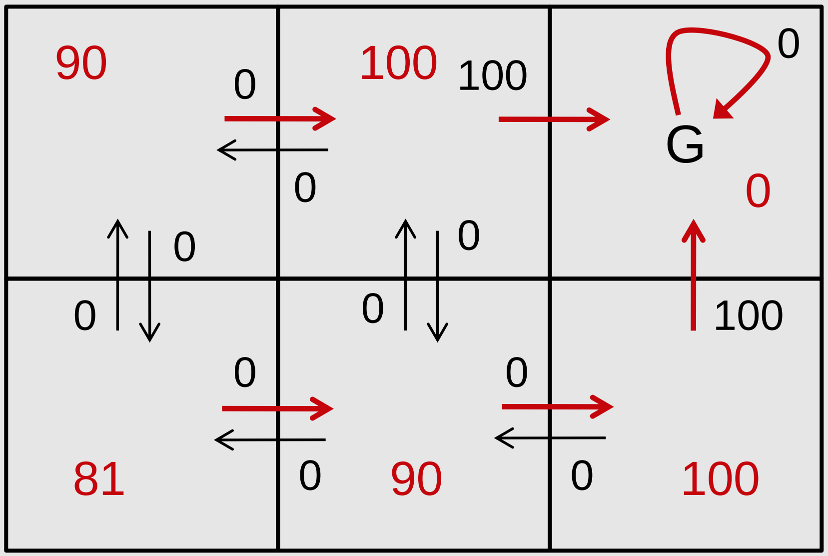


$V^\pi(s)$ values are shown in red

$90 = 0 + \gamma \times 100 + \gamma^2 \times 0 + \gamma^3 \times 0 + \ldots$

Value function for an optimal policy π^*

- Suppose π^* is shown by red arrows, $\gamma = 0.9$



$V^*(s)$ values are shown in red

Using a value function

define a new function, closely related to V^*

$$Q(s,a) \leftarrow E\left[r(s,a)\right] + \gamma E_{s'|s,a}\left[V^*(s')\right]$$

Key property (Bellman equation):

$$\pi^*(s) \leftarrow \arg\max_a Q(s,a) \qquad V^*(s) \leftarrow \max_a Q(s,a)$$

If we know $V^*(s)$, $r(s_t, a)$, and $P(s_t \mid s_{t-1}, a_{t-1})$ we can compute $\pi^*(s)$

Value iteration for learning $V^*(s)$

initialize $V(s)$ arbitrarily

loop until policy good enough

{

    loop for $s \in S$

    {

        loop for $a \in A$

        {

$$Q(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in S} P(s'|s,a) V(s')$$

        }

$$V(s) \leftarrow \max_a Q(s,a)$$

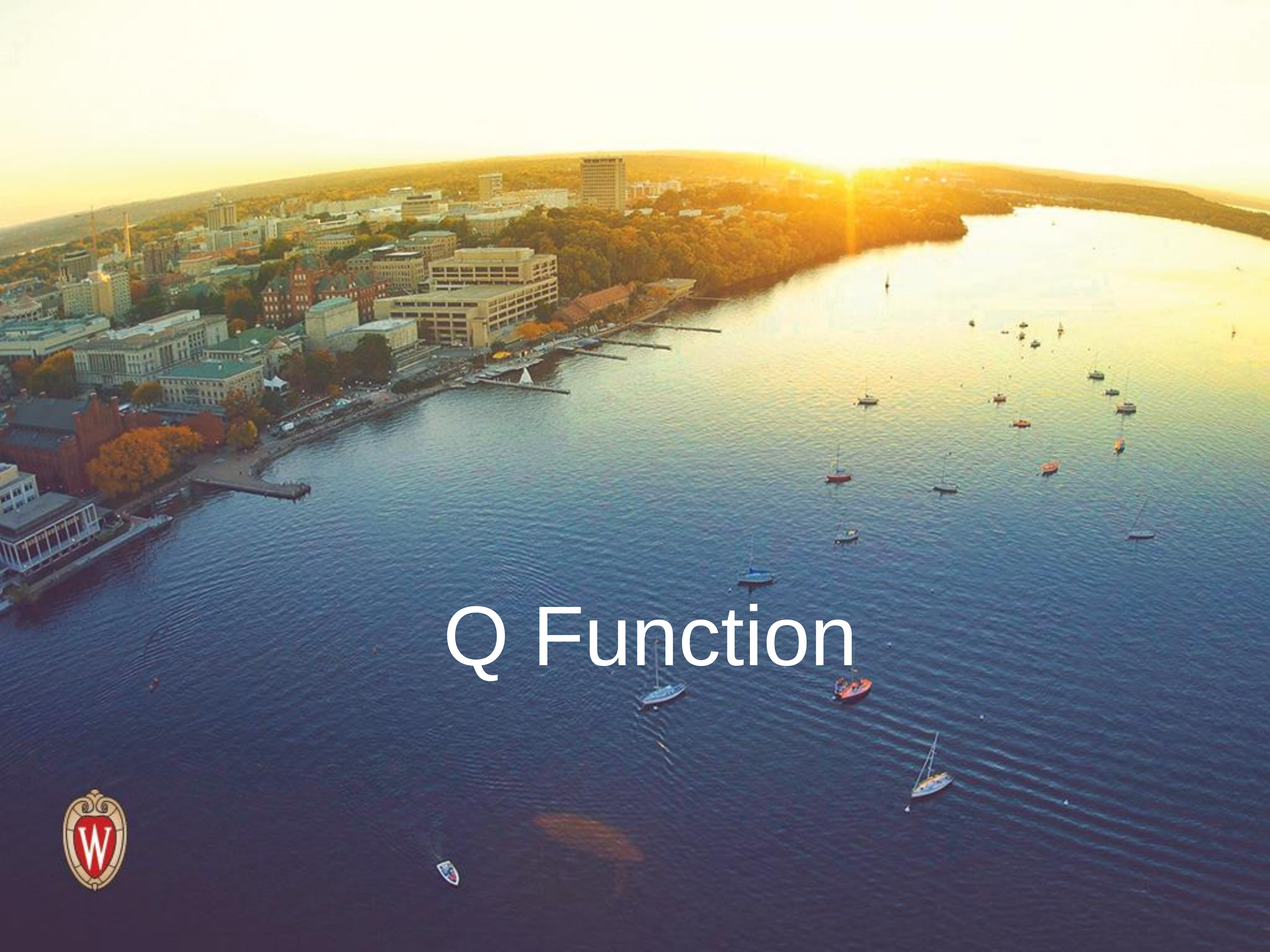
    }

}

Value iteration for learning $V^*(s)$

- $V(s)$ converges to $V^*(s)$
- works even if we randomly traverse environment instead of looping through each state and action methodically
  - but we must visit each state infinitely often
- implication: we can do online learning as an agent roams around its environment

- assumes we have a model of the world: i.e. know $P(s_t \mid s_{t-1}, a_{t-1})$
- What if we don't?



Q Function

Q learning

Review:

$$Q(s,a) \leftarrow E[r(s,a)] + \gamma E_{s'|s,a}[V^*(s')]$$

$$\pi^*(s) \leftarrow \arg\max_a Q(s,a) \qquad V^*(s) \leftarrow \max_a Q(s,a)$$

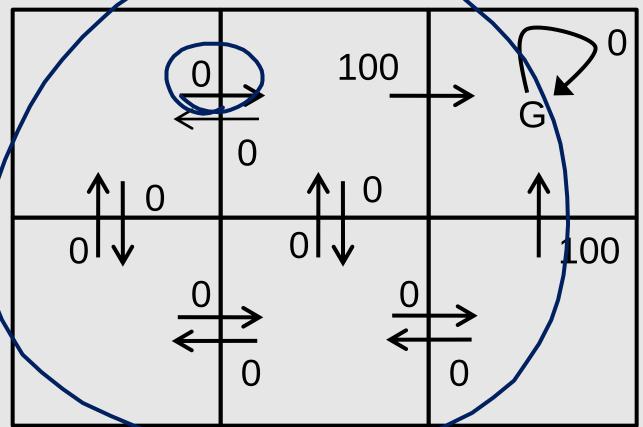
if agent knows $Q(s, a)$, it can choose optimal action without knowing $P(s' \mid s, a)$

and it can learn $Q(s, a)$ without knowing $P(s' \mid s, a)$

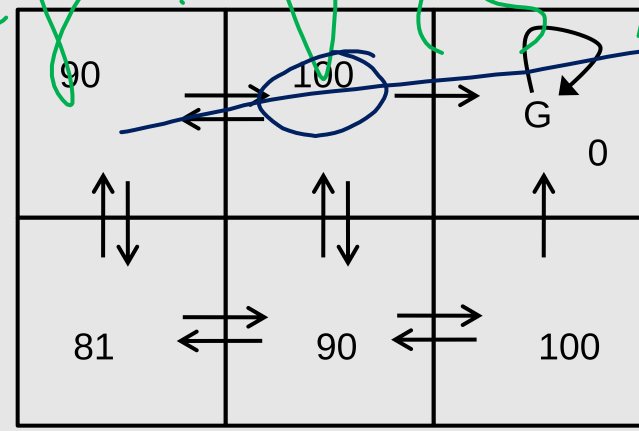
$$Q(s,a) \leftarrow r(s,a) + \gamma V^*(s') = r(s,a) + \gamma \max_{a'} Q(s',a')$$

Q values

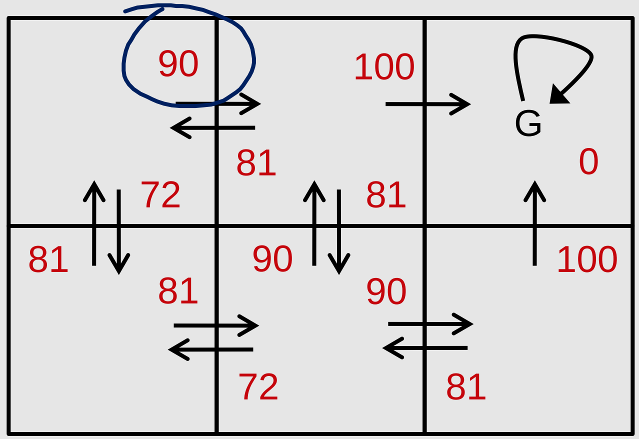
Q(s, a) = direct + γ · V*(s')



$r(s, a)$ (immediate reward) values



$V^*(s)$ values



$Q(s, a)$ values

0 + 0.9 × 100 = 90

Q learning update rule

for each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

observe current state s

do forever

  select an action a and execute it

  receive immediate reward r

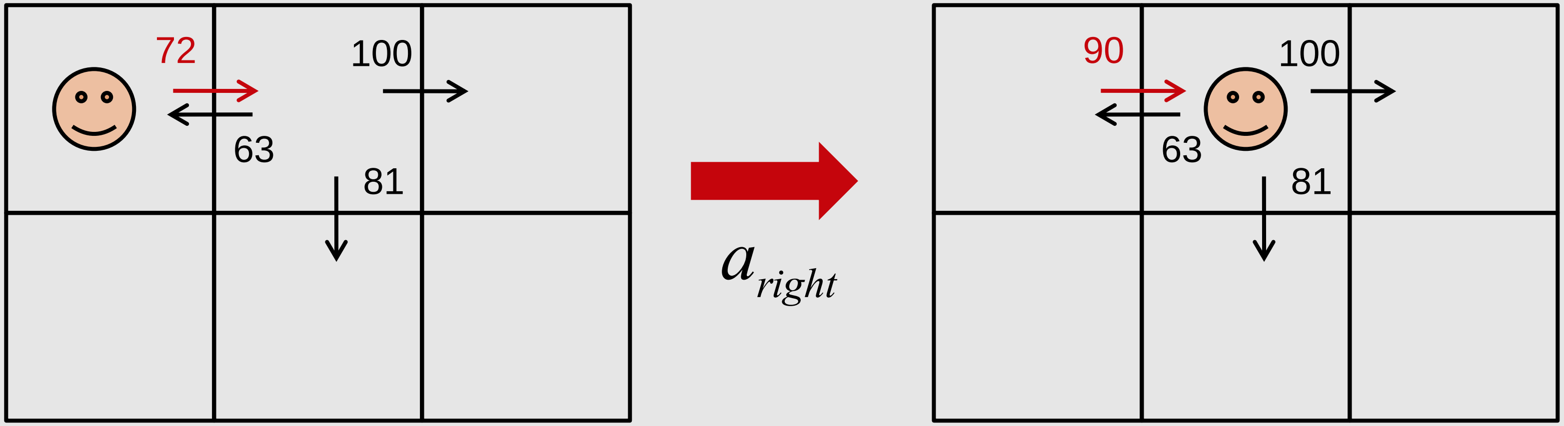
  observe the new state s'

  update table entry

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

  $s \leftarrow s'$

Updating Q



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$

$$\leftarrow 0 + 0.9 \max\{63, 81, 100\}$$

$$\leftarrow 90$$

Q learning: incremental update

for each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

observe current state s

do forever

select an action a and execute it

receive immediate reward r

observe the new state s'

update table entry

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n \left[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a') \right]$$

$s \leftarrow s'$

where α_n is a parameter dependent on the number of visits to the given (s, a) pair

$$a_n = \frac{1}{1 + \text{visits}_n(s,a)}$$

Convergence of Q learning

- Q learning will converge to the correct Q function
  - in the deterministic case
  - in the nondeterministic case (using the update rule just presented)

- in practice it is likely to take many, many iterations

THANK YOU

Some of the slides in these lectures have been adapted/borrowed from materials developed by Mark Craven, David Page, Jude Shavlik, Tom Mitchell, Nina Balcan, Elad Hazan, Tom Dietterich, and Pedro Domingos.