Problem 1. For any DFA, we extend the transition function $\delta$ by breaking the input string $w = xa$ during the inductive step, where $x$ is any string followed by a single symbol $a$. However, we informally think of $\delta$ as describing what happens along a path with a certain string of symbols, and if so, then it should not matter how we break the input string. Show that in fact,

$$\delta(q, xy) = \delta(\delta(q, x), y)$$

for any state $q$ and strings $x$ and $y$.

Problem 2. Give DFA’s accepting the following languages over the alphabet $\{0, 1\}$.

a) The set of all strings such that each block of four consecutive symbols contains at least two 0’s.

b) The set of strings such that the number of 0’s is divisible by 3, and the number of 1’s is divisible by 3.

Problem 3. Consider the following $\varepsilon$-NFA.

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>${q, r}$</td>
<td>$\emptyset$</td>
<td>${q}$</td>
<td>${r}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\emptyset$</td>
<td>${p}$</td>
<td>${r}$</td>
<td>${p, q}$</td>
</tr>
<tr>
<td>$r^{*}$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Recall that starred states denote accept/final states.

a) Compute the $\varepsilon$-closure of each state.

b) Give all the strings of length three or less accepted by the automaton.

c) Convert the automaton to a DFA.

Problem 4. Write regular expressions for the following languages. In all parts the alphabet is $\{0, 1\}$.

a) $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$.

b) $\{w \mid w \text{ has at most one pair of consecutive } 1\text{'s}\}$.

c) $\{w \mid \text{the number of } 0\text{'s in } w \text{ is divisible by } 3\}$.

d) $\{w \mid \text{every pair of adjacent } 0\text{'s in } w \text{ appears before any pair of adjacent } 1\text{'s}\}$.

e) $\{w \mid \text{every odd position of } w \text{ is a } 1\}$.

Problem 5. Prove that the language $L = \{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$ is not regular.