CS154: Homework #3

Due: Wednesday, July 25, 2012 by 5PM

Problem 1. Let $\mathbf{L}$ be a regular language over some alphabet Σ. Prove that the language $\mathbf{T}$, defined as,

$$\mathbf{T} = \{w \in \Sigma^* \mid \text{for some string } x,\ wx \in \mathbf{L}\}$$

is also regular.

Note: Simply describing the DFA is *not* sufficient. You need to give a *formal* proof which has two sides, similar to that given in Lecture 1.

Problem 2. Let $\mathbf{L} \subset \{0,1\}^*$ be the language of all strings such that there are two 0's separated by a number of positions that is a **non-zero** multiple of 5. Prove that any DFA for this language must have **at least** 2^5 states.

Problem 3. Consider two regular languages $\mathbf{L}_1$ and $\mathbf{L}_2$ defined over the *same* alphabet Σ. Let the *shuffle* of $\mathbf{L}_1$ and $\mathbf{L}_2$ be the language consisting of strings

$$\{w \mid w = a_1 b_1 a_2 b_2 \dots a_k b_k, \text{ where } a_1 \dots a_k \in \mathbf{L}_1 \text{ and } b_1 \dots b_k \in \mathbf{L}_2 \text{ and each } a_i, b_i \in \Sigma\}$$

In other words, the shuffle consists of strings of equal length from $\mathbf{L}_1$ and $\mathbf{L}_2$ whose alphabets are **strictly alternated**. Prove that the shuffle of $\mathbf{L}_1$ and $\mathbf{L}_2$ is a regular language as well.

Hint: Let $\Sigma' = \Sigma \times \Sigma$ and consider the homomorphism $h : \Sigma' \to \Sigma$ defined as $h(a,b) = a$.

Problem 4. Consider the transition table of a DFA,

		0	*1*
$\rightarrow$	A	B	A
	B	A	C
	C	D	B
	D^*	D	A
	E	D	F
	F	G	E
	G	F	G
	H	G	D

a) Draw the table of distinguishabilities for this automaton.

b) Construct the minimum-state equivalent DFA.

Problem 5. Consider the CFG $\mathbf{G}$ defined by the productions

$$S \rightarrow aSbS \mid bSaS \mid \varepsilon$$

Prove that $\mathbf{L}(\mathbf{G})$ is the set of all strings with an equal numbers of a's and b's.