

CS412: Homework #2

Due: Tuesday, February 24th, 2015 (by 11:59PM)

Sum of all problems: 120%, Maximum possible score: 100%.

For the following problems, even if you do not answer a given question, you are still allowed to use its result in order to answer a subsequent question.

1. [10%] Express the following polynomial in the correct form for evaluation by Horner's method:

$$p(t) = 5t^3 - 3t^2 + 7t - 2$$

2. [30%] How many multiplications are required to evaluate a polynomial $p(t)$ of degree $n - 1$ at a given point t
 - (a) Represented in the monomial basis?
 - (b) Represented in the Lagrange basis?
 - (c) Represented in the Newton basis?
3. (a) [10%] Determine the polynomial interpolant to the data

t	1	2	3	4
y	11	29	65	125

using the monomial basis.

- (b) [10%] Determine the Lagrange polynomial interpolant to the same data and show that the resulting polynomial is equivalent to that obtained in part (a).
 - (c) [30%] Compute the Newton polynomial interpolant to the same data using each of the three methods discussed in class (triangular matrix, incremental interpolation, and divided differences) and show that each produces the same result as the previous two methods.
4. (a) [15%] For a given set of data points t_1, \dots, t_n , define the function $\pi(t)$ by

$$\pi(t) = (t - t_1)(t - t_2) \dots (t - t_n)$$

Show that

$$\pi'(t_j) = (t_j - t_1) \dots (t_j - t_{j-1})(t_j - t_{j+1}) \dots (t_j - t_n)$$

- (b) [15%] Use the result of part (a) to show that the j th Lagrange basis function can be expressed as

$$l_j(t) = \frac{\pi(t)}{(t - t_j)\pi'(t_j)}$$