A comparative study of four fluid-solid coupling methods for applications in ground vehicle mobility

Arman Pazouki*, **Hammad Mazhar***, **Mridul Aanjaneya**#, **Paramsothy Jayakumar**†, **Eftychios Sifakis**#, **Dan Negrut***

* Department of Mechanical Engineering
University of Wisconsin-Madison
1513 University Ave., Madison, WI, 53706, USA
[pazouki, hmazhar, negrut]@wisc.edu

Department of Computer Sciences
University of Wisconsin-Madison
1210 W Dayton St., Madison, WI, 53706, USA
[aanjneya, sifakis]@cs.wisc.edu

†US Army TARDEC
Warren, MI, 48397, USA
paramsothy.jayakumar.civ@mail.mil

Abstract

Motivated by the desire to investigate vehicle fording scenarios, we analyze four frameworks for the simulation of the fluid-solid interaction problem. While all of these approaches rely on a general multibody dynamics simulation framework that supports impact, contact, and constraint, they differ in (*i*) the fluid representation; (*ii*) the simulation methodology; and (*iii*) the fluid-solid interfacing mechanism.

The first approach relies on an explicit-implicit, Lagrangian-Lagrangian (LL), solution to the coupled Navier-Stokes and Newton-Euler equations of motion. The fluid momentum and continuity equations,

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\mathbf{v} + \mathbf{f} \tag{1}$$

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{v}, \tag{2}$$

are solved using a weakly compressible Smoothed Particle Hydrodynamics (SPH) method [1]. In Eqs. (1) and (2), ρ, $\mathbf{v}$, p, $\mathbf{f}$, and μ are density, velocity, pressure, volumetric force, and dynamic viscosity, respectively. The incompressibility is satisfied to an appropriate degree by using a state equation that relates the pressure and density. The two way coupling of the solid and fluid phases is enforced using Lagrangian markers on the solid surfaces as well as within a layer inside the solid object. These so-called Boundary Condition Enforcing (BCE) markers have been employed and validated for a range of particle suspension problems as documented in [2].

The second approach relies on a coupled semi-implicit Lagrangian-Lagrangian method called "constraint fluids" [3] where a many-body density constraint is formulated using each SPH marker and its surrounding neighbors. More specifically, Eq. (2) is replaced with a constant density constraint written as $\rho/\rho_0 - 1 = 0$, where ρ_0 is the target density. At each step a quadratic optimization problem is solved where the unknowns represent impulses on the SPH particles that enforce incompressibility through the density constraint. The major benefit of this approach is that coupling between fluid and rigid is straightforward as both the solid and fluid phases can be modeled in the same system of equations.

Unlike the first two approaches, the third method implements an Eulerian framework for the discretization of the inviscid incompressible Navier-Stokes equations on a marker-and-cell (MAC) grid [4]. Herein, Eq. (1) is written in the form of a partial differential equation where $d\mathbf{v}/dt$ is replaced with $\partial\mathbf{v}/\partial t + \mathbf{v}\cdot\nabla\mathbf{v}$. In a projection method [5] implemented to enforce the divergence free constraint, i.e. $\nabla\cdot\mathbf{v} = 0$ instead of Eq. (2), an explicit advection step is first carried out using a second-order accurate MacCormack-style method [6]. The pressure projection equation is subsequently solved using the second-order cut-cell method [7]. Before updating the incompressible velocity through advection, they are extrapolated across the interface using the fast extension method [8] in order to define valid ghost node values. We use the level set method [9] to track the free surface of the incompressible fluid. The level set function ϕ is advected using the particle level set method [10] and the semi-Lagrangian advection scheme [11].

The last and least accurate method approximates fluid as a set of frictionless rigid spheres which interact with each other through contact and, if desired, friction and cohesion. This method is based on a differential variational inequality approach where friction and contact are handled at the velocity level using a semi-implicit time stepping scheme [12]. The fluid–solid coupling is trivial as both phases are

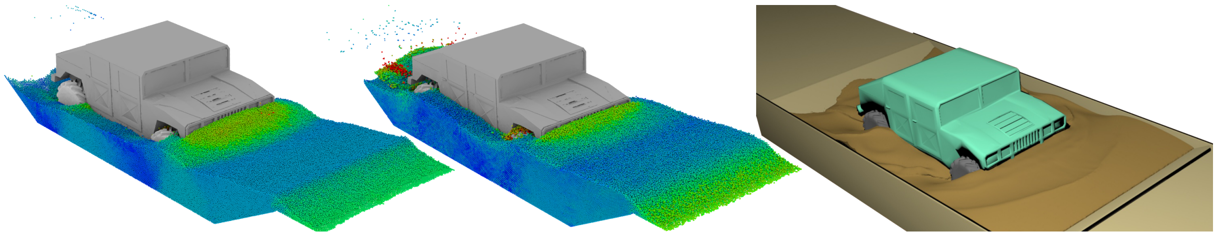

Figure 1: A kinematically driven vehicle moving inside an incompressible fluid simulated using 3 methods. Left: 1 million frictionless rigid bodies, Center: 1 million SPH markers using constraint fluid, Right: A $256 \times 256 \times 512$ Cartesian MAC grid.

characterized by a set of constrained Newton-Euler equations of motion. In this framework, the numerical approach calls for the solution of a large cone complementarity problem subsequently cast as a quadratic optimization problem with conic constraints. The numerical solution of the latter is found using a Nesterov-type algorithm [13], which yields at each time step the set of frictional contact forces for all pairs of bodies experiencing mutual contact.

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