September 13, 2011

CS412, Fall 11
Prof. Ron

Assignment #1

Due September 27, 2011, 2:15 p.m.
(after that time assignments should be put into the mbox of NISHA KIRAN)

(1) Pertinent notions to this question are the derivative of a function, the loss of significance in finite precision environment, and the speed/order of an algorithm.

In our web site (look under `mfiles`) you will find a file named ck1.m . That file contains a small `Matlab` program. Copy the file to your account (into a special directory where you intend to run your matlab experiments). Then read carefully the file, determine what it does, and complete the command line marked there. Then run it. Then change the definition of D in the file to be
`D=(atan(3+h(i))-atan(3-h(i)))/(2*h(i))`
and rerun. Turn in the diaries of these runs, together with comments that explain briefly what the program does. What conclusions can be drawn based on these results? Two conclusions, of totally different nature, can be drawn. (You may also wish to test both variants with a function other than `atan`), or to change the value 3 to some other value).

(2) This question addresses the notion of loss of significance. You are encouraged to revisit the Taylor series expansion that you have learned in calculus, as you will need to apply it here.

Explain why in each of the following expressions a loss of significance may occur and rearrange each of them to avoid it.
(a) $\frac{x^4-y^4}{x^3-y^3}$, near $x = y = 3$.
(b) $\sin x - \cos x$ near $x = \pi/4$.
(c) $e^{2y} - e^{-y}$, near $y = 0$.

(3) Write a `Matlab` program that, for a given fixed-point equation

$$x = g(x),$$

runs the fixed-point algorithm

$$x_{new} := g(x_{old})$$

until either two consecutive approximations to the root are close enough (tolerance of 10^{-10} would suffice), or divergence is determined.

Now, use this routine to examine the convergence of fixed-point algorithms: you are given the equation

$$\tan(x^3) = x,$$

and are asked to find the root of this equation which is closest to 1. To do that, convert the equation in *five* different ways to the form

$$x = g(x)$$

and run your routine on it. Make your five choices for g so that at least once you have linear convergence, at least once you have quadratic convergence, and at least once you do not converge to the required root. Draw connections between the theory as discussed in class, and the numerical observations you are able to make (e.g., could you tell in advance, by studying g first, what kind of convergence, if at all, is expected?)

Submit the edited-commented diaries of your runs, together with your explanations of the observed convergence rates, and the connections you found between the experiment and the theory.