CS412, Fall 06
Prof. Ron

Final Exam

The Formulæ page of cs412's Final Exam, Fall 06

Numerical integration

The numbers j, k are certain integers that differ from one formula to another. Note that the error for the composite rules are not given.

(I) The error in the basic **rectangle rule** is

$$\frac{(b-a)^j}{2} f^{(k)}(c)$$

(II) The error in the basic **trapezoid rule** is

$$\frac{(b-a)^j}{-12} f^{(k)}(c)$$

(III) The error in the basic **midpoint rule** is

$$\frac{(b-a)^j}{24} f^{(k)}(c)$$

(IV) The error in the basic **Simpson's rule** is

$$\frac{(b-a)^j}{2880} f^{(k)}(c)$$

IVP's

Euler's method is the rectangle rule, beginning at the last partition point t_j.

Modified Euler is a midpoint rule, beginning at t_{j-1}.

Simpson's method (for IVP) is the Simpson rule, beginning at t_{j-1}.

RK2 is a predictor-corrector method, with Euler being the predictor, and trapezoid being the corrector.

The **RK4** method is as follows:

$$Y_{j+1} := Y_j + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4),$$

where

$$F_1 := hY_j' = hf(t_j, Y_j), \quad F_2 = hf(t_j+h/2, Y_j+hY_j'/2), \quad F_3 := hf(t_j+h/2, Y_j+\frac{F_2}{2}), \quad F_4 = hf(t_{j+1}, Y_j+F_3).$$

The **AB method** is

$$Y_{j+1} := Y_j + \frac{h}{24}(55Y_j' - 59Y_{j-1}' + 37Y_{j-2}' - 9Y_{j-3}'),$$

while the AM method is

$$Y_{j+1} := Y_j + \frac{h}{24}(9Y_{j+1}' + 19Y_j' - 5Y_{j-1}' + Y_{j-2}'),$$

and Milne's method is

$$Y_{j+1} := Y_{j-3} + \frac{4h}{3}(2Y_j' - Y_{j-1}' + 2Y_{j-2}').$$

Errors in IVPs

You need to know the *order* of the local truncation errors of the various methods. You do not need to know the further details of these error formulæ.