

October 16, 2001

CS412, Spring 97

Prof. Ron

Exam #1

My Name is.....

My i.d. is.....

My seat during the exam time is: Row..... Seat.....

Grading table (not to filled by the student)

Question 1.	A	B	C	D	Tot:
Question 2.	A	B	C	D	Tot:
Question 3.	A	B	C	D	Tot:
Question 4.	A	B	C	D	Tot:

[1] (25=5+7+8+5)

- (a) Apply one iteration of the fixed point algorithm to the equation $x = \sin(x) + .5$, starting with $x_0 = .6$
- (b) Without iterating further, do you expect the fixed point iterations to converge? Explain.
- (c) Suggest an alternative method for solving the same equation, which in your opinion may be better. Reason your choice, and implement the first iteration of your method (with $x_0 = .6$).
- (d) Write the output of the `matlab` code
- ```
C=[7,0,-1;0,1,0]';
C(2,:)
```
-

[2] (25=10+5+5+5) You are given four data values  $(0, -1)$ ,  $(1, 0)$ ,  $(2, 1)$  and  $(4, 27)$ , and are asked to interpolate the data by a cubic polynomial (i.e., a polynomial in  $\Pi_3$ ).

(a) Construct a divided difference table suitable for that purpose.

(b) Using that table, find the polynomial interpolant. How would you check that you have the correct result?

(c) Your friend Jim tells you that there exists also a *quadratic* polynomial that interpolates these data. Is he right or wrong? Explain.

(d) Your friend Tina tells you that there exists a polynomial of *exact* degree 6 that interpolates these data. Is she right or wrong? Explain.

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[3] (30=8+10+7+5)

- (a) The error bound in cubic Hermite interpolation at  $k$  points is five times smaller than the error bound for spline interpolation at those points. Why would one then use spline interpolation?
- (b) You are asked to approximate the function  $f(x) = e^{-x} + x^3$  on the interval  $[0, 1]$  by cubic Hermite interpolation on equidistant partition. How many subintervals should you use in order to have an error  $\leq 10^{-6}$ ?
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(c) In (b), where do you expect the error to be larger: near  $x = 1$  or near  $x = 0$ ? Explain?

(d) Explain why polynomial interpolation is considered ‘global’ while cubic Hermite interpolation is considered ‘local’.

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[4] (30=10+5+7+8)

- (a) You have a calculator that performs only the four basic arithmetic operations. You wish to find  $x = 28^{1/3}$ , and know that  $x \approx 3$ . Suggest an efficient method for doing that, and iterate once with your method.

- (b) Here is a 5-line code that implements your method from (a). Complete the second and fourth line.

```
xold=3; xnew=4;
```

```
..... (xold-xnew)>1E-6
```

```
 xold=xnew;
```

```
 xnew=.....
```

```
end
```

- (c) Write the output of the following `matlab` code

```
x=[2,0; 2,1]; y=[2,1]'; x\ y
```

- (d) Write a `matlab` code (preferably without a loop) that evaluates the polynomial  $(x-2)(x-3)(x-4)\dots(x-51)$  at  $x = 2.4$ .