UNIVERSITY OF WISCONSIN - MADISON

Computer Sciences Department

CS513, Spring 99

Prof. Ron

Midterm Exam

My name is: .....

Answer every question below. Write your answer into your blue book. Use a new page for each question. You are *not* allowed to use books or notes. A calculator of any type is permitted, provided that it is not preprogrammed with code relevant to cs513. Be brief and to the point with your answers. "Yes/no" answers carry "no" credit, unless reasoning is provided. *Turn in your exam sheet together with your blue book.*

(1) (30 points, 5+15+10) You are given the following 2×2 matrix A

$$A = \begin{pmatrix} 2 & -1 \\ 0 & \sqrt{3} \end{pmatrix}$$

(a) Find the spectrum of this matrix.
(b) Find the 2-, ∞- and 1-norm of this matrix.
(c) Find the 2-condition number of A (if possible, without computing the inverse of A). State any result from class that you use here.

(2) (25 points, 10+15)
(a) Let v be the column vector defined by

$$v' := (\,3 \quad 2 \quad 4 \quad 4\,).$$

Find a Householder matrix H such that

$$(Hv)' = (\,3 \quad a \quad 0 \quad 0\,),$$

with a some number (that you might choose to suit your needs.)
(b) QR-factor the matrix

$$\begin{pmatrix} 5 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 0 \\ 0 & 4 & 0 \end{pmatrix}.$$

(3) (20 points, 10+10)
(a) Describe briefly an efficient algorithm for computing the determinant of a square $n \times n$ matrix. What is the complexity of your algorithm?
(b) Use (a) in order to find the determinant of the matrix

$$A = \begin{pmatrix} +4 & -2 & +2 \\ -2 & +2 & -2 \\ +2 & -2 & +3 \end{pmatrix}$$

(4) (20 points, 5+10+5)

(a) A is a square matrix. Define the notion: 'A is positive definite'.

(b) State two conditions, each of which is equivalent, for a symmetric matrix A, to the positive definiteness of that matrix.

(c) Check whether the matrix in the previous problem (3(b)) is positive definite.

(5) (25 points =5+20 points) Let V be the linear span of the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

You are asked to find a vector v in V which is 'as close as possible' to the vector

$$u = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

(a) Define rigorously the notion of 'as close as possible'. (If there are several possible definitions, choose the one that will allow you to solve (b) below.)

(b) Find that 'closest' vector v.