CS515 Spring 08
Prof. Ron

Assignment #4b

Prepared by Narfi Stefansson

Due February 28, 2008

1. Let ϕ be a refinable function whose mask is a trigonometric polynomial, and $\psi_2, \ldots, \psi_m$ be finite linear combinations of $\phi_{1,k}, k \in \mathbb{Z}$. By our assumption on ϕ and $\psi_2, \ldots, \psi_m$ there exist trigonometric polynomials $H_1, H_2, \ldots, H_m$ such that

$$\begin{aligned} \hat{\phi}(2\omega) &= H_1(\omega)\hat{\phi}(\omega) \\ \hat{\psi}_2(2\omega) &= H_2(\omega)\hat{\phi}(\omega) \\ &\vdots \\ \hat{\psi}_m(2\omega) &= H_m(\omega)\hat{\phi}(\omega) \end{aligned}$$

Furthermore, we assume that all the exponents in $H_1, \ldots, H_m$ are non-negative. Then we can write

$$H_j(\omega) = \sum_{k=0}^{n-1} \texttt{h(j,k+1)} e^{ik\omega}$$

for $j = 1, \ldots, m$.

Thus, we have related the masks $H_1, H_2, \ldots, H_m$ to a matrix $\texttt{h} = \begin{bmatrix} \texttt{h(1,1)} & \cdots & \texttt{h(1,n)} \\ \vdots & & \vdots \\ \texttt{h(m,1)} & \cdots & \texttt{h(m,n)} \end{bmatrix}$

Your assignment is to write a `Matlab` function called `checkIfTight` that accepts one input argument, a matrix `h`, which we interpret to contain the coefficients of masks $H_1, H_2, \ldots, H_m$ in the manner described above. The function should return true if the masks $H_2, \ldots, H_m$ satisfy the *unitary extension principle*, false otherwise.

If `false`, your function should print out a string describing which of two required conditions failed to hold. Print out '`masks are not orthogonal`' or '`the sum of squares of absolute values of the masks is not equal to 1`', or both. Your function should not print out anything else.

Turn in a printout of the function, as well as the results you get when you run the script `TestMasks.m` which is available from the course homepage. Hint: it is much better to do everything on the time domain using convolutions.

2. Use the identity

$$(\cos^2(\omega/2) + \sin^2(\omega/2))^4 = 1$$

is order to construct a tight frame based on the centered B-spline $\phi = E^{-2}B_4$. You may assume that you already know that this ϕ is refinable with mask $H_0(\omega) = \cos^4(\omega/2)$. There will be four mother wavelets in your system.

Specifically, you will need:

To find the four masks $H_1, \ldots, H_4$ of the mother wavelets.

To check that the conditions of the Unitary Extension Principle are valid.

To find the Fourier transform of each of four mother wavelets in your system.

To write explicitly *one* of the mother wavelets as a linear combination of translates of ϕ. (This means that you will need to write one of masks H_l as a linear combination of exponentials. You are allowed to choose any of the four, so choose the one that looks the simplest to this end).

Hint: if you search FrameNet, you might find that system somewhere there. Of course, you cannot simply copy the details from there, but at least you should be able to double-check yourself.