CS515 Spring 08
Prof. Ron

Assignment #6

Due 22 April 2008

Question # 1.
Let $F = \Pi_3 = \operatorname{ran} W$, with $W = [()^0, ()^1, ()^2, ()^3]$ (hence $(Wa)(x) = a(1) + a(2)x + a(3)x^2 + a(4)x^3$); also let

$$\Lambda' : f \mapsto (f(0), f'(0), f(1), f'(1)).$$

(i) Prove that Λ' is 1-1 on Π_3. (Hint: how many zeros, counting multiplicities, must a polynomial f have if $\Lambda' f = 0$?)
(ii) Construct the Gram matrix $\Lambda' W$.
(iii) Construct the basis V for Π_3 for which $\Lambda' V = \text{id}$.
(iv) Construct the unique cubic polynomial that matches the function $x \mapsto \sin(\pi(x - 1/2))$ in value and slope at the two points, 0 and 1.

Question # 2.
The `MATLAB Spline Toolbox` command `bs = spmak(t,1);` returns a description of the B-spline with knot sequence `t(1:end)` whose values at the entries of the vector (or matrix) `x` can be obtained by the command `fnval(bs,x)`. (Note that `MATLAB` will determine the order k of the B-spline from the length of the vector `t`.)

Generate (and hand in, along with a listing of the `MATLAB` script that generated it) a plot that shows, for $\mathbf{t} := (t_1, \ldots, t_4) = (0, 2, 3, 4.7)$ and on the interval $[-1 \,..\, 6]$, (i) $B_{1,3,\mathbf{t}}$; (ii) $\omega_{1,3} B_{1,2,\mathbf{t}}$; (iii) $(1 - \omega_{2,3}) B_{2,2,\mathbf{t}}$, with, to recall, $\omega_{i,k}(t) = (t - t_i)/(t_{i+k-1} - t_i)$.

Comment on the significance of this plot.

Comment on notation: the B-spline $B_{j,k,\mathbf{t}}$ is the B-spline $B_{j,k}$ with respect to the knot sequence $\mathbf{t} = (t_0, \ldots, t_N)$.

Question # 3.
Use the recurrence relation to construct

$$B(\cdot|0,1,1), \quad B(\cdot|0,0,1);$$

$$B(\cdot|0,1,1,1), \quad B(\cdot|0,0,1,1), \quad B(\cdot|0,0,0,1);$$

$$B(\cdot|0,1,1,1,1), \quad B(\cdot|0,0,1,1,1), \quad B(\cdot|0,0,0,1,1), \quad B(\cdot|0,0,0,0,1).$$

Can you guess a formula for

$$B(\cdot|\underbrace{0,\ldots,0}_{i \text{ terms}},\underbrace{1,\ldots,1}_{j \text{ terms}})$$

Question # 4.
Use the de Boor - Fix dual functionals λ_{jk} in order to prove the following property of splines: if f is a linear combination of B-splines, and it vanishes outside an interval $[t_{j+1}, t_{j+k}]$ for some j (and where k is the order of the B-splines), then f is zero everywhere.

Note that this last result is tight: a spline f that vanishes outside an interval of the form $[t_j, t_{j+k}]$ needs not to be 0 everywhere (why?)