CS515 Spring 05
Prof. Ron

Assignment #6a

Due 04 April 2005

Question # 1.
Write a short Matlab code that performs one iteration of the subdivision operator

$$x \longleftarrow 2h * (x_{\uparrow}).$$

Use this code iteratively in order to draw graphs of a few refinable functions, as follows:

(a) Draw the graphs of the B-splines of order 3, 4, and 5. Remember that the mask of the B-spline of order k is

$$\left(\frac{1+e^{-i\omega}}{2}\right)^k.$$

(Hint: in lieu of finding explicitly the filter h associated with the mask, you can write the filter as the convolution product of k simple filters.)

(b) Draw the graph of the Deslauries-Dubuc refinable function. The mask of this function is

$$\cos^4(\omega/2)(\cos^2(\omega/2) + 3\sin^2(\omega/2)) = \cos^4(\omega/2)(2 - \cos\omega).$$

Naturally, you will need to find first the lowpass filter h. You may use here the hint in (a).

(c) Draw the graph of Daub4. The mask of Daub4 is the squareroot of the DD mask from (b).

Turn in the graphs, the code you wrote, and all the manual calculations you did.

Question # 2.
In order to understand the decomposition and reconstruction algorithms of wavelets, you will perform in this question, at complete generality, one cycle of decomposition and reconstruction using a simple wavelet system. The system is the tight frame that is based on the centered hat function (and is dubbed RS2 in Framenet).

(a) Given a signal $x = (x(k))_k$, perform one step of decomposition using that system. You need to write explicitly the derived signals. For example,

$$\nu_{-1,0}(k) = \frac{x(k-1) + 2x(k) + x(k+1)}{4}.$$

(b) Now check that the reconstruction step recovers x from the three subsignals $\nu_{-1,i}$, $i = 0, 1, 2$.

Question # 3.
This problem introduces you to the features of the two-dimensional discrete wavelet analysis of the Matlab Wavelet Toolbox.

At the Matlab prompt, type `helpwin`. In the help navigator window, go to `Wavelet Toolbox` > `Getting Started` > `Using Wavelets` > `Two-Dimensional Discrete Wavelet Analysis` > `Two-Dimensional Analysis`. There you will see

- How to load an image,
- How to perform single-level and multilevel image decompositions and reconstructions,
- How to do global thresholding,

and so on. Now do the following:

(1) Load the image *wbarb* into Matlab.
(2) Apply a 2-D DWT using Haar wavelet to the image.

(4) Set all detail coefficients to zero except the largest 50% of them, and reconstruct the image using the modified coefficients.
(5) Add Gaussian white noise with standard deviation 15 to the original image.
(6) Apply a 2-D DWT using Haar wavelet to the noisy image.
(7) Set all detail coefficients zero except those whose magnitude is larger than a 45 and reconstruct the image using the modified coefficients.
(8) Experiment with other thresholds and choose the best.

Turn in your code and the results from this experiment.