CS515 Spring 08

Prof. Ron

Assignment #8

Due 08 May 2008

Question # 1.

Find the refinement mask of the B-spline B_5 with support $[0, 5]$ and integer simple knots. Find explicitly the two subdivision rules (for updating old knots and for inserting new ones).

Now, use the subdivision rules that you found in order to check experimentally convexity preservation of the spline representation:

(a) Choose a few coefficients (at least six) so that the control polygon is convex.

(b) Use subdivision to compute the associated spline curve, and verify its convexity.

Question # 2.

In this problem, you get to experiment with the effect of knot placement on the error in the approximation.

Use the `splinetool` to construct an approximation to the titanium heat data by a cubic spline with five interior knots whose maximum absolute error is as small as possible (try to get close to .04 max error).

For this, use the `matlab` command `splinetool`, then click on the `Titanium Heat Data`, and choose as an approximation method `Least Squares Approximation`. Then, under `Data, breaks/knots, weights` choose `knots`. You are ready to add, move or delete knots.

To add a knot, choose `add knot` from the `edit` menu. Use the crosshairs and the mouse to mark the location of the knot. Left-click the mouse to drop the knot at the marked location.

To delete a knot, click on it, and then choose `delete knot` from the `edit` menu.

To move a knot, use the `edit` window below the list of `Data, break/knots...` frame. Click in the $\pm$ in order to move the highlighted knot. You control the shift size by typing it in the `edit` window between the $+$ and the $-$.

When you found a satisfactory placement for the five knots, print out the graph of the approximation (open the `File` menu), and its error, as well as the location of the five knots. Hand in all these printouts.

Question # 3.

Prove the following claim: let $\underline{t}$ be a knot sequence, and let x be some point. Let

$$f = \sum_j a_j B_{jk,\underline{t}}$$

be a spline curve of order k with respect to the knot sequence $\underline{t}$. Let $\widehat{\underline{t}}$ be the knot sequence obtained from $\underline{t}$ by inserting x $k-1$ times into the sequence, and let

$$f = \sum_j \widehat{a}_j B_{jk,\widehat{\underline{t}}}$$

be the new representation. Then, $f(x)$ coincides with one of the new coefficients (which one?) What is the significance of this result?