Signal Denoising

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In this project, you will experiment with denoising of signals using wavelet systems, wavelet packets and cosine packets. For wavelet and cosine packets, we will use the wavelet software `WaveLab`.

We consider noisy data of the form:

$$y_i = s_i + z_i, i = 1, ..., n$$

where $s = (s_i)$ is the original signal and $z = (z_i)$ is i.i.d. Gaussian white noise with variance 1. The goal is to try to recover s from y.

We define the signal-to-noise ratio (SNR) for noisy data to be $\|s\|_{\ell_2}/\|z\|_{\ell_2}$ and we measure the performance of the denoising method by the normalized ℓ_2 error

$$\frac{\|s - s_{rec}\|_{\ell_2}}{\|s\|_{\ell_2}}$$

where s_{rec} is the recovered signal for s.

Level I: Wavelet denoising of signals

In this part, you will write your own MATLAB programs to implement wavelet transform denoising.
Generate at least six test signals for denoising. Some good examples to start with are: $\sin, B_1, B_2$ and B_3. The variety of signals should be large enough to draw a good conclusion. You can find various signals in `The IDR FrameNet Portal` . (If you try hard and still have difficulties in finding signals, contact your mentor.) You can use the MATLAB **randn** function to generate the noise. Consider various systems including: `HAAR, DAUB4, DAUB8, BIOR 7/9, BIOR 6/10` (You may add the other wavelet systems as well). Run the following experiments for each of the possible pair of a signal and a system. Compare the performance of the various systems and decide which wavelet systems perform best for each of your chosen signals.

(a) Experiment with various denoising methods:

- (i) Hard thresholding with threshold level $t = \sqrt{2 * \log(n)}$
- (ii) Soft thresholding with threshold level $t = \sqrt{2 * \log(n)}$
- (iii) Find experimently the threshold level (for both hard and soft thresholdings) t to minimize the normalized ℓ_2 error
- (iv) Find experimently the threshold level (for both hard and soft thresholdings) t to give optimal "perceptual" performance (explain your criteria).

(b) Experiment with various signal-to-noise ratios including SNR = 1, 2 and 4.

Level II: Wavelet & Cosine Packets

`Do the assignment for Level I and the following.`

This course has focused on the Fast Wavelet Transform, which iterates over the low-frequency output in a filter bank. An alternative approach is to expand the entire tree by iterating over the high frequency components as well.

Wavelet packet analysis expands a signal into a full binary tree, and stores all coefficients and at all levels. The result is a redundant representation with $n \log(n)$ coefficients. Wavelet packet methods for denoising attempt to select a best orthogonal basis for thresh-holding from the redundant wavelet packet tree.

Cosine packet methods attempt to select a best orthogonal basis for thresh-holding from a full binary expansion in terms of local trigonometric basis elements.

Use the `WaveLab` function **WPDeNoise** and **CPDeNoise** to denoise your signals from Level I.

Your mentor will help you get acquainted with both the material and the software.

For **WPDeNoise**, use the following systems: `HAAR, Symmlet 4, Symmlet 8`, and `Coiflet 4`.

For **CPDeNoise**, take bell = `'Sine'`.

How do the results compare to your results from Wavelet Transform denoising in Level I?

Level III: Cycle Spinning

`Do the assignment for Level I, II and the following.`

Denoising in a wavelet basis often exhibits artifacts related to the lack of translation invariance of the wavelet basis. Cycle spinning attempts to suppress those artifacts by averaging out the translation dependence. Refer to the paper: "Translation-Invariant De-Noising," Coifman & Donoho, 1995 Available from: `http://www-stat.stanford.edu/∼donoho/Reports/`.

Write your own MATLAB code for the denoising experiments with 16 spin-cycles. Repeat the same denoising experiments as in Level I and Level II. Compare the result with the previous results.

Turn in a report on your experiments, along with your MATLAB code that you used for your experiments, and a discussion of your observations and conclusions.