Dataflow Analysis
and
Dataflow-Analysis Frameworks

Roadmap

Last time:

– Data structures (and data) used to determine when it is safe (i.e., sound) to perform an optimizing transformation
  - Review dominators
  - SSA form
  - Dataflow analysis

This time:

– More dataflow analysis
  - Dataflow equations
  - Solving dataflow equations

– Dataflow-analysis frameworks

Dataflow-Analysis Example 1

Reaching definitions

Transfer function:
$\lambda S.\,(S - \{< p_i, x >\}) \cup \{< p2, x >\}$

Before p1: Ø
After p1: {<p1, x>}

p1: x = 1;

. . .

Before p2: {<p1, x>, …}
After p2: {<p2, x>, …}

p2: x = 2;

. . .

Before p3: {<p2, x>, …}
After p3: {<p2, x>, <p3, y>, …}

p3: y = x;

Data: sets of <program-point, variable> pairs

Note: for expository purposes, it is convenient to assume we have a statement-level CFG rather than a basic-block-level CFG.

Dataflow-Analysis Example 1

Reaching definitions

Meet operation: Union of sets (of <program-point, variable> pairs)

Before p1: Ø
After p1: {<p1, x>}

p1: x = 1;

. . .

Before p2: {<p1, x>, …}
After p2: {<p2, x>, …}

p2: x = 2;

. . .

p4: x = 7;

Before p4: Ø
After p4: {<p4, x>}

Before p3: {<p2, x>, <p4,x>, …}
After p3: {<p2, x>, <p3, y>, <p4,x>,…}

p3: y = x;

Dataflow-Analysis Example 1

Reaching definitions: Why is it useful?

Answers the question “Where could this variable have been defined?”

Before p1: Ø
After p1: {<p1, x>}

p1: x = 1;

. . .

Before p2: {<p1, x>, …}
After p2: {<p2, x>, …}

p2: x = 2;

p4: x = 7;

Before p4: Ø
After p4: {<p4, x>}

. . .

Before p3: {<p2, x>, <p4,x>, …}
After p3: {<p2, x>, <p3, y>, <p4,x>,…}

p3: y = x;

Dataflow-Analysis Example 2

Live Variables

Transfer function:
$\lambda S.(S - \{z\}) \cup \{x, y\}$

Before p1: ∅	
After p1: {x}	p1: x = 1;
	if (…) {
Before p2: {x}	
After p2: {x,y}	p2: y = 0;
Before p3: {x,y}	
After p3: ∅	p3: z = x + y;
	}
Before p4: ∅	
After p4: {x}	p4: x = 2;
Before p5: {x}	
After p5: {x}	p5: z = 3;
Before p6: {x}	
After p6: ∅	p6: cout << x;

Data: sets of variables

z is not live after p5, and thus p5 is a useless assignment (= dead code)

Dataflow-Analysis Direction

Forward analysis

– Start at the beginning of a function's CFG, work along the control edges (e.g., reaching definitions)

Backward analysis

– Start at the end of a function's CFG, work against the control edges (e.g., live variables)

Warning 2: There is another concept called "live variables."

- When variable x is "not live," a convenient shorthand is "Variable x is dead."
- When x is dead just after a statement s, that does not imply that s is dead code. (E.g., suppose s assigns to y.)
- When s is a useless assignment to x
  - Statement s is dead code (because dead = useless or unreachable)
  - x is not live just after s ("Variable x is dead just after s")
  - Because variable x is dead, s is a useless assignment, and thus statement s is dead code.

There … t for progra…

– Easy … to in state…
  - The region … redefined
  - In SSA form, from "x_i = …; … all uses of x_i, e.g., "… = f(…, x_i, …);"

– Easy to see when an assignment is *useless*
  - We have "x_i = …;" and there are *no uses* of x_i in any expression or assignment RHS
  - "'x_i = …;' is a useless assignment"
  - "'x_i = …;' is dead code"

In other words, some u… information is pre-computed, or at least easily reco…

Warning 1: Dead code = useless assignments + unreachable code

Dataflow-Analysis Example 3

Reachable uses

Before p1: {…}
After p1: {<p3, z>, <p2, z>, …}

p1: z = 1;

. . .

Before p2: {<p3, z>, <p2, z>, …}
After p2: {<p3, x>, <p3, z>, …}

p2: x = z;

Before p3: {<p3, x>, <p3, z>}
After p3: ∅

p3: y = x+z;

Transfer function:
$\lambda S.(S - \{< p_i, x >\}) \cup \{< p2, z >\}$

Data: sets of <program-point, variable> pairs

Dataflow-Analysis Example 3

Reachable uses

Meet operation: Union of sets (of <program-point, variable> pairs)

Before p1: {<p3, z>, <p2, z>, <p4, z>, …}
After p1: {<p3, z>, <p2, z>, <p4, z>, …}

p1: if (…)

. . .

Before p2: {<p3, z>, <p2, z>, …}
After p2: {<p3, x>, <p3, z>, …}

p2: x = z;

. . .

Before p3: {<p3, x>, <p3, z>}
After p3: Ø

p3: y = x+z;

p4: x = 3*z;

Before p4: {<p3, z>, <p4, z>}
After p4: {<p3, x>, <p3, z>}

Dataflow-Analysis Example 3

Reachable uses: Why is it useful?

Answers the question “What could this variable definition reach?”

After p0: {<p3, z>, <p2, z>, <p4, z>, …}

p0: z = 5;

Before p1: {<p3, z>, <p2, z>, <p4, z>, …}

After p1: {<p3, z>, <p2, z>, <p4, z>, …}

p1: if (…)

. . .

p4: x = 3*z;

Before p4: {<p3, z>, <p4, z>}

After p4: {<p3, x>, <p3, z>}

Before p2: {<p3, z>, <p2, z>, …}

After p2: {<p3, x>, <p3, z>, …}

p2: x = z;

. . .

Before p3: {<p3, x>, <p3, z>}

After p3: ∅

p3: y = x+z;

Dataflow-Analysis Example 3

Reachable uses: Why is it useful?

Answers the question “What could this variable definition reach?”

p0: z = 5;

After p0: {<p3, z>, <p2, z>, <p4, z>, …}

Before p1: {<p3, z>, <p2, z>, <p4, z>, …}

p1: if (…)

After p1: {<p3, z>, <p2, z>, <p4, z>, …}

. . .

Before p4: {<p3, z>, <p4, z>}

p4: x = 3*z;

After p4: {<p3, x>, <p3, z>}

Before p2: {<p3, z>, <p2, z>, …}

p2: x = z;

After p2: {<p3, x>, <p3, z>, …}

. . .

Before p3: {<p3, x>, <p3, z>}

p3: y = x+z;

After p3: ∅

Reaching definitions versus reachable uses: really just an indexing question. At which end of the edges do you want to collect the information?

Obtaining a Dataflow-Analysis Solution

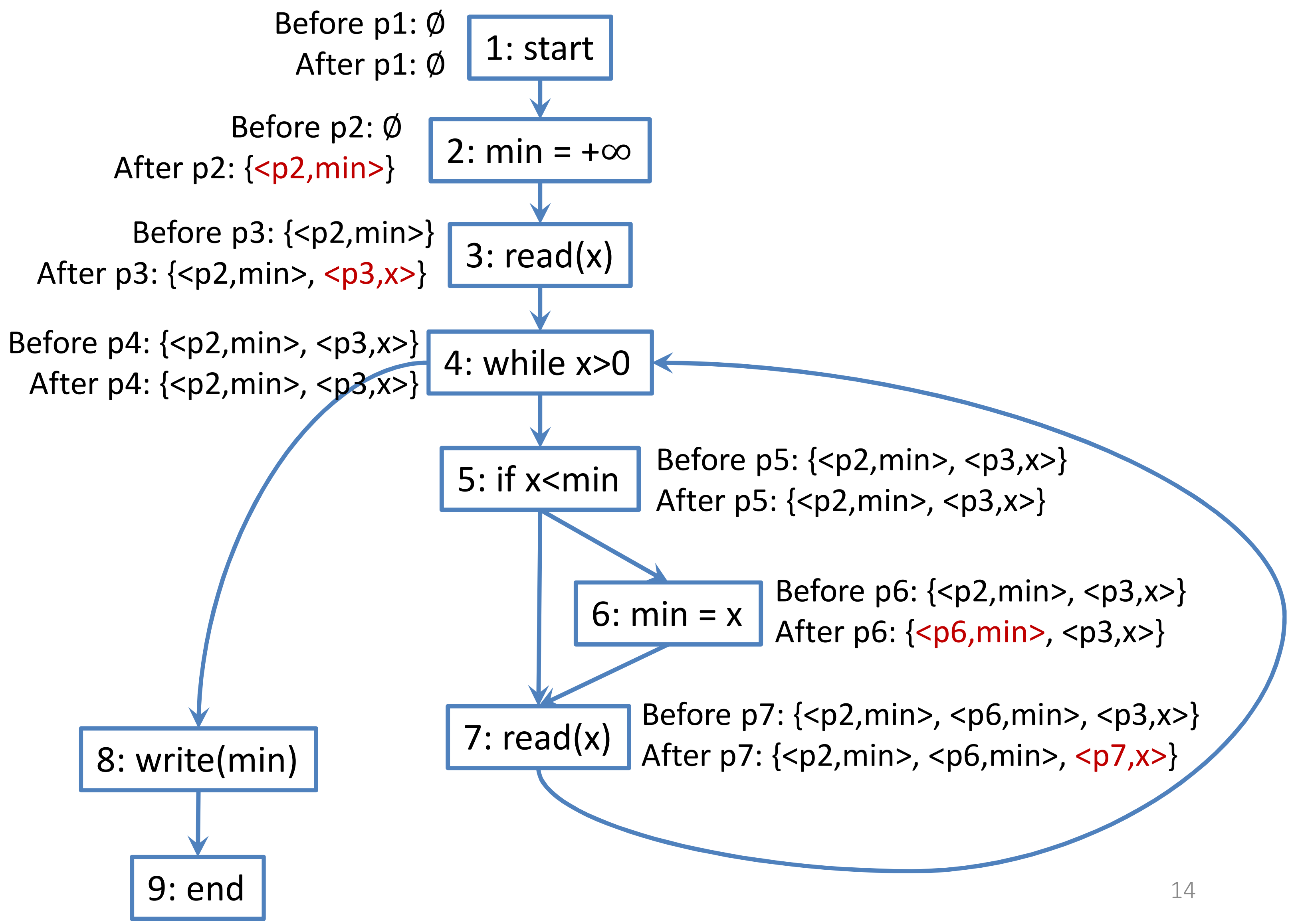
Successive approximation:

- Assign to each node in the CFG a (dataflow-problem-specific) default value
  - Typically either Ø or the universe of the sets you are working with, e.g., {all variables in the procedure}
- Assign a special value to the entry node
- Propagate values until quiescence, as follows:

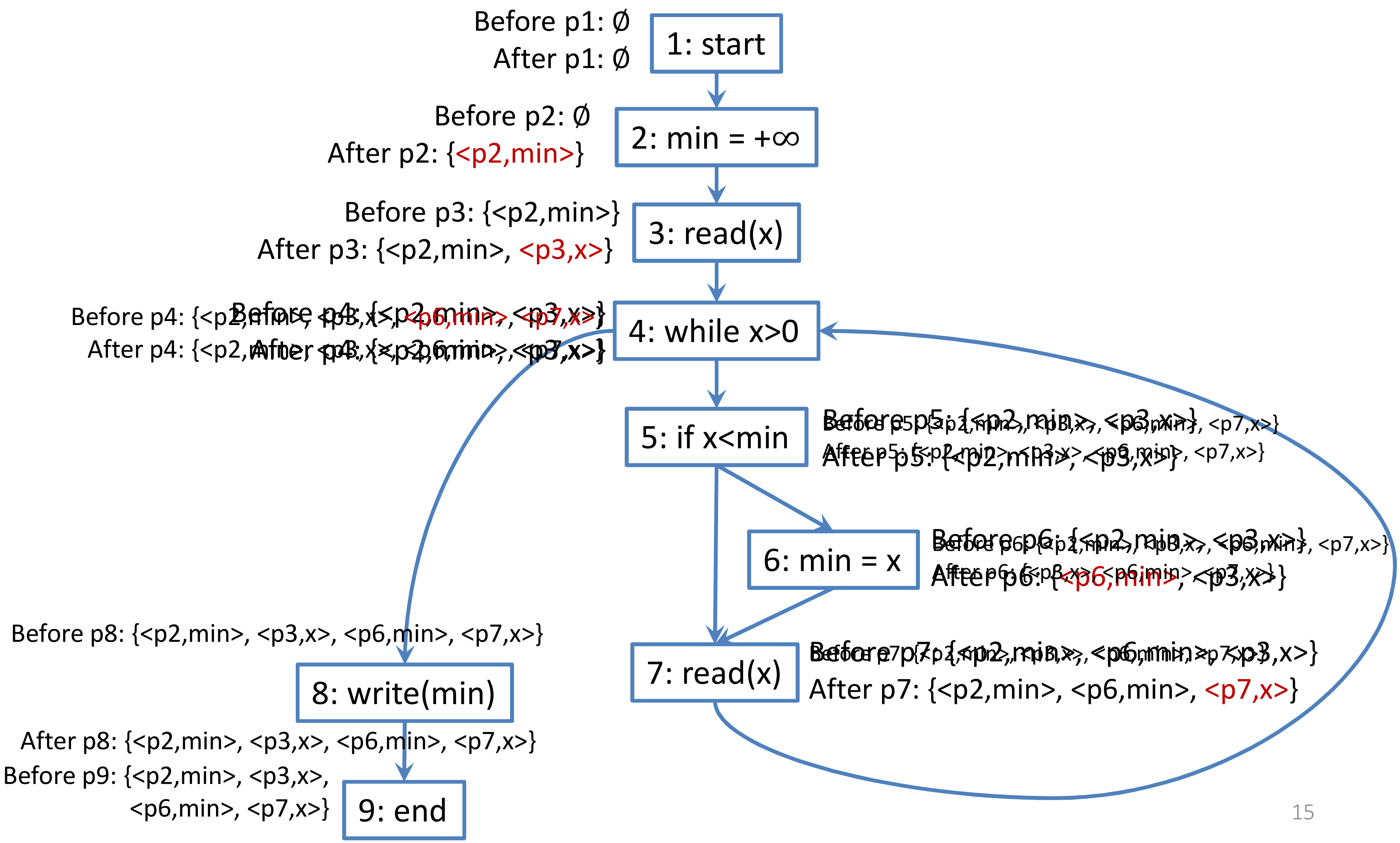
  Repeatedly
  - Pick a node
  - Find input values from predecessors
  - Apply transfer function

  Until no change is possible

Example: Reaching Definitions



Example: Reaching Definitions



Obtaining a Dataflow-Analysis Solution by Successive Approximation

for all nodes n, RdBefore[n] := ∅ and RdAfter[n] := ∅

workset := { start}

while (workset ≠ ∅) {

    select and remove a node n from workset

    oldValueAfter := RdAfter[n]

    RdBefore[n] := $\cup_{<p,n> \in Edges}$RdAfter[p]

    RdAfter[n] := F_n(RdBefore[n])

    if oldValueAfter ≠ RdAfter[n] then

        for all $<n, w> \in Edges$, insert w into workset

}

Successive Approximation!? Does That Always Work?

To find a solution $x^* = F(x^*)$, perform $x_{k+1} = F(x_k)$

Let's try: $x^2 = 2$, using $x = \frac{2}{x}$

Iterate on $x_{k+1} = \frac{2}{x_k}$

Pick any $x_0 \neq 0$,

$x_1 = \frac{2}{x_0}$, $x_2 = x_0$, $x_3 = \frac{2}{x_0}$, $x_4 = x_0$, failure ☹

Successive Approximation!? Does That Always Work?

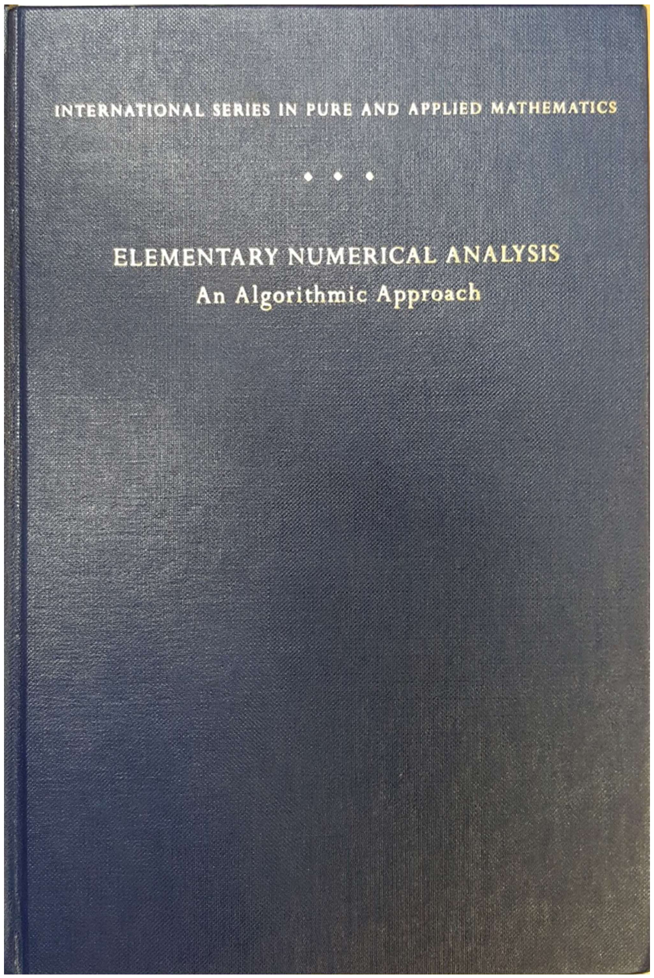
To find a solution $x^* = F(x^*)$, perform $x_{k+1} = F(x_k)$

$x^2 = 2$, so $x = \frac{2}{x}$

Add x to both sides: $x + x = x + \frac{2}{x}$ That is, $2x = x + \frac{2}{x}$

Iterate on $x_{k+1} = \frac{1}{2}\left(x_k + \frac{2}{x_k}\right)$

$x_0 = 1.00000$
$x_1 = 1.50000$
$x_2 = 1.41666$
$x_3 = 1.41421$
$x_4 = 1.41421$



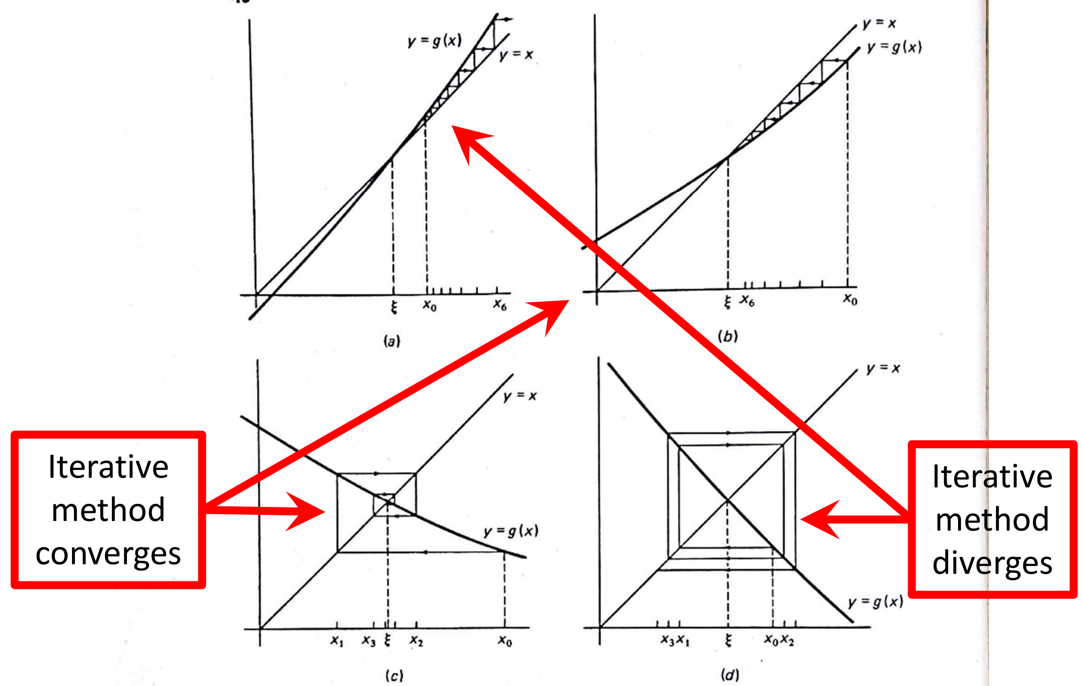


Fig. 2.3 Fixed-point iteration.

Theorem 2.1 Let $g(x)$ be an iteration function satisfying Assumptions 2.1 and 2.3. Then $g(x)$ has exactly one fixed point ξ in I, and starting with any point x_0 in I, the sequence $x_1, x_2, \ldots$ generated by fixed-point iteration of Algorithm 2.6 converges to ξ.

To prove this theorem, recall that we have already proved the existence of a fixed point ξ for $g(x)$ in I. Now let x_0 be any point in I. Then, as we remarked earlier, fixed-point iteration generates a sequence $x_1, x_2, \ldots$ of points all lying in I, by Assumption 2.1. Denote the error in the nth iterate by

$$e_n = \xi - x_n \qquad n = 0, 1, 2, \ldots$$

Then since $\xi = g(\xi)$ and $x_n = g(x_{n-1})$, we have

$$e_n = \xi - x_n = g(\xi) - g(x_{n-1}) = g'(\eta_n)e_{n-1} \tag{2.19}$$

Successive Approximation!? Does That Always Work?

To find a solution $x^* = F(x^*)$, perform $x_{k+1} = F(x_k)$

- Fact: For reaching definitions and live variables, successive approximation always works
- Why?
  - (An approximation to) an answer is two sets per program point
  - The sets at each program point are finite and of *a priori* bounded size
  - Each sets always increases in size (⊆)
  - Approximations to answers get bigger and bigger, but cannot grow without bound
    - Therefore the algorithm must terminate
    - When the algorithm terminates, the sets solve the equations

Equations?
What equations?

Equations? What Equations?

Two equations for each node n:

RdBefore[n] = $\cup_{<p,n> \in Edges}$ RdAfter[p]

RdAfter[n] = F_n(RdBefore[n])

Successive approximation:

RdBefore$_{k+1}$[n] = $\cup_{<p,n> \in Edges}$ RdAfter$_k$[p]

RdAfter$_{k+1}$[n] = F_n(RdBefore$_k$[n])

In iterative algorithm:

RdBefore[n] := $\cup_{<p,n> \in Edges}$ RdAfter[p]

RdAfter[n] := F_n(RdBefore[n])

Equations? What Equations?

<u>Equations</u>:

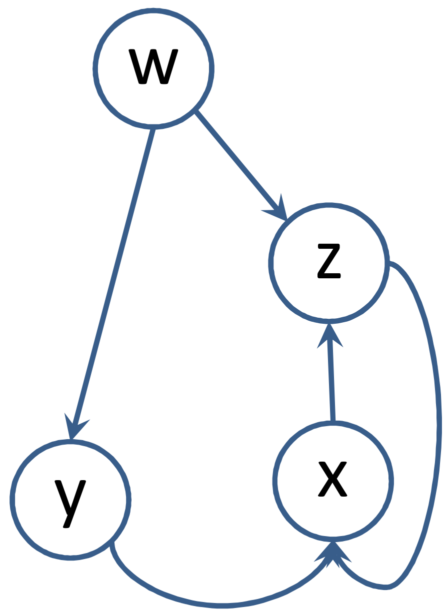
x = 3y + 4z

y = 2w + 2

z = 7w – x

w = 17

Equation dependence graph:



Dataflow equations

Control-flow graph

DATAFLOW-ANALYSIS FRAMEWORKS

What is a Dataflow Framework?

Many analyses can be formulated in terms of how data is transformed over the control flow graph

- Propagate information from:
  - After (before) some node, to
  - Before (after) some other node
- Put information together when control flow merges (or diverges)

A framework captures these uniformities

- In object-oriented-program terms: like an abstract class AC
- To use the framework
  - You define certain data and methods (required by AC)
  - AC supplies other methods (already implemented, so you don't have to worry about implementing them yourself)

Dataflow Framework: What You Supply

The type of data (a.k.a. dataflow facts)

– A collection of values with an order, such as $\subseteq$
– (Sometimes called a "meet semi-lattice")
– Default value and value to use at entry (or exit)

Transfer functions

– Specify how data is propagated across a node

A meet operation ($\sqcap$)

– The operation for combining values that come across multiple edges

Direction (forward or backward)

Dataflow Framework Instantiated for Reaching-Definitions Analysis

The type of data (a.k.a. dataflow facts):

Sets of <program-point, variable> pairs

Transfer functions:

For "p: $id = exp;$" and "p: read id"

$\lambda S.(S - \{< p_i, id >\}) \cup \{< p, id >\}$

For "if exp …" and "write exp

$\lambda S.S$

The meet operation (for combining values that come across multiple edges):

Set union (∪)

Direction:

Forward

Dataflow Framework Instantiated for Live-Variable Analysis

The type of data (a.k.a. dataflow facts):

- Sets of variables

Transfer functions:

- For "$id = exp;$"
  - $\lambda S.(S - \{id\}) \cup \{x \in exp\}$
- For "if exp", and "write exp"
  - $\lambda S.S \cup \{x \in exp\}$
- For "read id"
  - $\lambda S.(S - \{id\})$

The meet operation (for combining values that come across multiple edges):

- Set union (∪)

Direction:

- Backward

Obtaining a Dataflow-Analysis Solution by Successive Approximation

```
for all nodes n, ValBefore[n] := T and ValAfter[n] := T
workset := {start}
while (workset ≠ Ø) {
    select and remove a node n from workset
    oldValueAfter := ValAfter[n]
    ValBefore[n] := ⊓_{<p,n>∈Edges} ValAfter[p]
    ValAfter[n] := F_n(ValBefore[n])
    if oldValueAfter ≠ ValAfter[n] then
        for all <n,w> ∈ Edges, insert w into workset
}
```

Obtaining a Dataflow-Analysis Solution by Successive Approximation

```
for all nodes n, ValAfter[n] := T and ValBefore[n] := T
workset := { end}
while (workset ≠ Ø) {
    select and remove a node n from workset
    oldValueBefore := ValBefore[n]
    ValAfter[n] := ⊓_{<n,p>∈Edges} ValBefore[p]
    ValBefore[n] := F_n(ValAfter[n])
    if oldValueBefore ≠ ValBefore[n] then
        for all <w, n> ∈ Edges, insert w into workset
}
```

Dataflow-Analysis Example 3

Available-expressions analysis

- Whether an expression that has been previously computed may be reused
- Forward dataflow problem: from expression to points of re-use
- Meet semi-lattice:

True

|

False

- Meet operation:
  - AND of all predecessors
- At the beginning of each block, everything is True

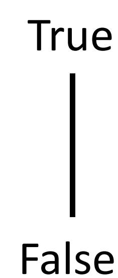
  * This causes some problems for loops

Dataflow-Analysis Example 4

Very-Busy-Expression analysis

- An expression is very busy at a point *p* if it is guaranteed that it will be computed at some time in the future

- Backwards dataflow problem: from computation to use
- Meet Lattice:



- Meet operation: AND

The end: or is it?

Covered a broad range of topics

– Some formal concepts

– Some practical concepts

What we skipped

– Linking and loading

– Interpreters

– Register allocation

– Performance analysis / Proofs