Propositional Logic

Reading: Chapter 7.1, 7.3 – 7.5

[Based on slides from Jerry Zhu, Louis Oliphant and Andrew Moore]

Logic

- If the rules of the world are presented formally, then a decision maker can use logical reasoning to make rational decisions
- Several types of logic:
  - Propositional Logic (Boolean logic)
  - First-Order Logic (aka first-order predicate calculus)
  - Non-Monotonic Logic
  - Markov Logic
- A logic includes:
  - **syntax**: what is a correctly-formed sentence?
  - **semantics**: what is the meaning of a sentence?
  - **Inference procedure** (reasoning, entailment): what sentence logically follows given knowledge?

Propositional Logic

- A **symbol** in PL is a symbolic *variable* whose value must be either True or False, and which stands for a natural language statement that could be either true or false
  - A = "Smith has chest pain"
  - B = "Smith is depressed"
  - C = "It is raining"

Propositional Logic Syntax

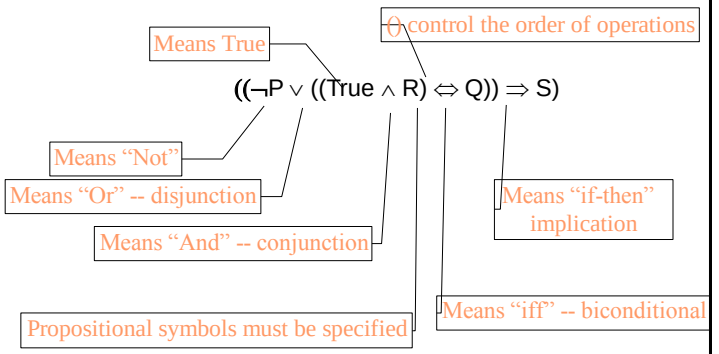
Sentence	→ *AtomicSentence* | *ComplexSentence*
AtomicSentence	→ True | False | *Symbol*
Symbol	→ P | Q | R | . . .
ComplexSentence	→ ¬ *Sentence*
|	(*Sentence* ∧ *Sentence*)
|	(*Sentence* ∨ *Sentence*)
|	(*Sentence* ⇒ *Sentence*)
|	(*Sentence* ⇔ *Sentence*)

BNF (Backus-Naur Form) grammar in propositional logic

((¬P ∨ ((True ∧ R) ⇔ Q)) ⇒ S well formed ("wff" or "sentence")

(¬(P ∨ Q) ∧ ⇒ S) not well formed

Propositional Logic Syntax



Propositional Logic Syntax

- Precedence (from highest to lowest):

  $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

- If the order is clear, you can leave off parentheses

$\neg P \vee True \wedge R \Leftrightarrow Q \Rightarrow S$ ok

$P \Rightarrow Q \Rightarrow S$ not ok

Semantics

- An **interpretation** is a complete True / False assignment to all propositional symbols
  - Example symbols: P means "It is hot", Q means "It is humid", R means "It is raining"
  - There are 8 interpretations (TTT, ..., FFF)
- The semantics (meaning) of a sentence is the set of interpretations in which the sentence evaluates to True
- Example: the semantics of the sentence $P \vee Q$ is the set of 6 interpretations:
  - P=True, Q=True, R=True or False
  - P=True, Q=False, R=True or False
  - P=False, Q=True, R=True or False
- A **model** of a set of sentences is an interpretation in which all the sentences are true

Evaluating a Sentence under an Interpretation

- Calculated using the definitions of all the connectives, recursively

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

- Pay attention to ⇒
  - "5 is even implies 6 is odd" is True!
  - If P is False, regardless of Q, P⇒Q is True
  - No causality needed: "5 is odd implies the Sun is a star" is True

Understanding "⇒"

- This is an operator. Although we call it "implies" or "implication," do not try to understand its semantic form from the name. We could have called it "foo" instead and still defined its semantics the same way.
- A ⇒ B "means" A is *sufficient* but not *necessary* to make B true
- Example:
  - Let A be "has a cold" and B be "drink water"
  - A ⇒ B can be interpreted as "should drink water" when "has a cold."
  - However, you can drink water even when you do not have a cold. Thus A ⇒ B is still true when A is ***not*** true.

Example

$\neg P \vee Q \wedge R \Rightarrow Q$

Example

$\neg P \vee Q \wedge R \Rightarrow Q$

P	Q	R	~P	Q^R	~PvQ^R	~PvQ^R->Q
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	0	0	0	1
1	1	0	0	0	0	1
1	1	1	0	1	1	1

Satisfiable: a sentence that is **true** under ***some*** interpretations

Deciding satisfiability of a sentence is NP-complete

Example

$(P \wedge R \Rightarrow Q) \wedge P \wedge R \wedge \neg Q$

Example

$(P \wedge R \Rightarrow Q) \wedge P \wedge R \wedge \neg Q$

P	Q	R	~Q	R^~Q	P^R^~Q	P^R	P^R->Q	final
0	0	0	1	0	0	0	1	0
0	0	1	1	1	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	0	0	0	0	1	0
1	0	0	1	0	0	0	1	0
1	0	1	1	1	1	1	0	0
1	1	0	0	0	0	0	1	0
1	1	1	0	0	0	1	1	0

Unsatisfiable: a sentence that is **false** under ***all*** interpretations

Also called **inconsistent** or a **contradiction**

Example

$(P \Rightarrow Q) \vee P \wedge \neg Q$

Example

$(P \Rightarrow Q) \vee P \wedge \neg Q$

P	Q	R	~Q	P->Q	P^~Q	(P->Q)vP^~Q
0	0	0	1	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	0	1
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	1	0	1	0	1

Valid: a sentence that is **true** under ***all*** interpretations

Also called a **tautology**

Knowledge Base (KB)

- A knowledge base, KB, is a *set of sentences*. Example KB:
  - TomGivingLecture $\Leftrightarrow$ (TodayIsTuesday $\vee$ TodayIsThursday)
  - $\neg$ TomGivingLecture
- It is equivalent to a single long sentence: the ***conjunction*** of all sentences
  - (TomGivingLecture $\Leftrightarrow$ (TodayIsTuesday $\vee$ TodayIsThursday)) $\wedge \neg$ TomGivingLecture
- **A model of a KB is an interpretation in which *all* sentences in KB are *true***

Entailment

- **Entailment** is the relation of a sentence β ***logically following*** from other sentences α (e.g., KB)

$$\alpha \models \beta$$

- $\alpha \models \beta$ if and only if, in every interpretation in which α is true, β is also true; whenever α is true, so is β; all models of α and also models of β
- Deduction theorem: **$\alpha \models \beta$ if and only if $\alpha \Rightarrow \beta$ is valid** (always true)
- Proof by contradiction (refutation, *reductio ad absurdum*): **$\alpha \models \beta$ if and only if $\alpha \wedge \neg\beta$ is unsatisfiable**
- There are 2^n interpretations to check, if KB has *n* symbols

Entailment

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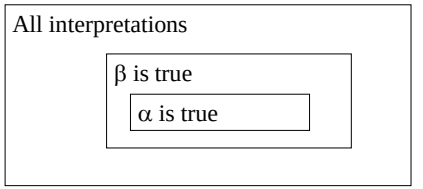


Deductive Inference

- Let's say you write an algorithm which, according to you, proves whether a sentence β is entailed by α
- The thing your algorithm does is called ***deductive inference***
- We don't trust your inference algorithm (yet), so we write things your algorithm finds as

$$\alpha \vdash \beta$$

- It reads "β is **derived from** α by your algorithm"
- What properties should your algorithm have?
  - **Soundness**: the inference algorithm only derives entailed sentences. That is, **if $\alpha \vdash \beta$ then $\alpha \models \beta$**
  - **Completeness**: all entailment can be inferred. That is, **if $\alpha \models \beta$ then $\alpha \vdash \beta$**

Soundness and Completeness

- **Soundness** says that any wff that follows deductively from a set of axioms, KB, is valid (i.e., true in all models)
- **Completeness** says that all valid sentences (i.e., true in all models of KB), can be proved from KB and hence are theorems

Method 1: Inference by Enumeration

Also called **Model Checking** or **Truth Table Enumeration**

LET: KB = A ∨ C, B ∨ ¬C α = A ∨ B

QUERY: KB ⊨ α ?

A	B	C
false	false	false
false	false	true
false	true	false
false	true	true
true	false	false
true	false	true
true	true	false
true	true	true

NOTE: The computer doesn't know the meaning of the proposition symbols

So, all logically distinct cases must be checked to prove that a sentence *can* be derived from KB

Inference by Enumeration

LET: KB = A ∨ C, B ∨ ¬C α = A ∨ B

QUERY: KB ⊨ α ?

A	B	C	A∨C	B∨ ¬C	KB
false	false	false	false	true	false
false	false	true	true	false	false
false	true	false	false	true	false
false	true	true	true	true	true
true	false	false	true	true	true
true	false	true	true	false	false
true	true	false	true	true	true
true	true	true	true	true	true

Rows where all of sentences in KB are true are the models of KB

Inference by Enumeration

LET: KB = A ∨ C, B ∨ ¬C α = A ∨ B

QUERY: KB ⊨ α ? ***YES!***

A	B	C	A∨C	B∨ ¬C	KB	A∨B	KB⇒α
false	false	false	false	true	false	false	true
false	false	true	true	false	false	false	true
false	true	false	false	true	false	true	true
false	true	true	true	true	true	true	true
true	false	false	true	true	true	true	true
true	false	true	true	false	false	true	true
true	true	false	true	true	true	true	true
true	true	true	true	true	true	true	true

α is entailed by KB if all models of KB are models of α, i.e., all rows where KB is true, α is also true

In other words: KB ⇒ α is valid

Inference by Enumeration

- Using inference by enumeration to build a complete truth table in order to determine if a sentence is entailed by KB is a **complete** inference algorithm for Propositional Logic
- But very slow: takes exponential time

Method 2: Natural Deduction using Sound Inference Rules

Goal: Define a more efficient algorithm than enumeration that uses a set of inference rules to incrementally deduce new sentences that are true given the initial set of sentences in KB plus uses all logical equivalences

Logical Equivalences

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$
$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$
$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$
$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$
$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan}$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan}$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

You can use these equivalences to derive or modify sentences

Sound Inference Rules

- ***Modus Ponens*** (Latin: mode that affirms)

$$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

- **And-Elimination**

$$\frac{\alpha \wedge \beta}{\alpha}$$

Note: Prove that an inference rule is sound by using a truth table

P	Q	P	P⇒Q	P ∧ (P⇒Q)	Q	(P ∧ (P⇒Q)) ⇒ Q
T	T	T	T	T	T	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	F	T	F	F	T

Some Sound Inference Rules

- **Implication-Elimination, IE (Modus Ponens, MP)** $\dfrac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$
- **And-Elimination, AE** $\dfrac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$
- **And-Introduction, AI** $\dfrac{\alpha_1, \alpha_2, \ldots, \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$
- **Or-Introduction, OI** $\dfrac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$
- **Double-Negation Elimination, DNE** $\dfrac{\neg\neg\alpha}{\alpha}$

Inference Rules

- Each inference rule formalizes the idea that "A infers B" (A |- B) in terms of "logically follows" (A |= B)
- Doesn't say anything about **deducibility** – just says for each interpretation that makes A true, that interpretation also makes B true

Question

What's the difference between

≡ (logical equivalence)

|= (entailment)

|- (derived from)

Natural Deduction = Constructing a Proof

- A **Proof** is a sequence of inference steps that leads from α (i.e., KB) to β
- This is a search problem!

KB:

1. $(P \wedge Q) \Rightarrow R$
2. $(S \wedge T) \Rightarrow Q$
3. S
4. T
5. P

β:

R

Proof by Natural Deduction

1. S Premise (in KB)
2. T Premise
3. $S \wedge T$ Conjunction(1, 2) (And-Introduction)
4. $(S \wedge T) \Rightarrow Q$ Premise
5. Q Modus Ponens(3, 4)
6. P Premise
7. $P \wedge Q$ Conjunction(5, 6)
8. $(P \wedge Q) \Rightarrow R$ Premise
9. R Modus Ponens(7, 8) Last line is query sentence β

Monotonicity Property

- Note that natural deduction relies on the **monotonicity property** of Propositional Logic:
- Deriving a new sentence and adding it to KB does NOT affect what can be entailed from the *original* KB
- Hence **we can incrementally add new true sentences that are derived in *any order***
- Once something is proved true, it will remain true

Method 3: Resolution

- Your algorithm can use all the logical equivalences, *Modus Ponens,* And-Elimination to derive new sentences
- **Resolution rule**: a ***single*** inference rule
  - **Sound**: only derives entailed sentences
  - **Complete**: can derive any entailed sentence
    - **Resolution is only refutation complete**: if KB |= β, then KB $\wedge \neg \beta$ |- *False*.
  - But the sentences need to be preprocessed into a special form
  - But all sentences can be converted into this form

Resolution

- Take any two clauses where one contains some symbol, and the other contains its complement (negative)

$$P \vee Q \vee R \qquad \neg Q \vee S \vee T$$

- Merge (resolve) them, throw away the symbol and its complement

$$P \vee R \vee S \vee T$$

- If two clauses resolve and there's no symbol left, you have derived the ***empty clause*** (**False**), so KB |= β
- If no new clauses can be added, KB does not entail β

Resolution Rule of Inference

- Show KB |= α by proving that KB $\wedge \neg\alpha$ is unsatsifiable, i.e., deducing False from KB $\wedge \neg\alpha$
- **Resolution Rule** of Inference

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- Examples

$$\frac{A \vee B, \neg B}{A} \qquad \frac{A \vee B \vee \neg C \vee D, \neg A \vee \neg E \vee F}{B \vee \neg C \vee D \vee \neg E \vee F}$$

called "unit resolution"

Resolution Refutation Algorithm

1. Add negation of query to KB
2. Pick 2 sentences that haven't been used before and can be used with the Resolution Rule of inference
3. If none, halt and answer that the query is *NOT* entailed by KB
4. Compute resolvent and add it to KB
5. If False in KB
   - Then halt and answer that the query *IS* entailed by KB
   - Else Goto 2

Conjunctive Normal Form (CNF)

$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$

- Replace all $\Leftrightarrow$ using iff/biconditional elimination
  - $\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$
- Replace all $\Rightarrow$ using implication elimination
  - $\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$
- Move all negations inward using
  - double-negation elimination
    - $\neg(\neg\alpha) \equiv \alpha$
  - de Morgan's rule
    - $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$
    - $\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$
- Apply distributivity of $\vee$ over $\wedge$
  - $\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$ + 1 more

Convert Sentence into CNF

$A \Leftrightarrow (B \vee C)$	starting sentence
$(A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$	iff/biconditional elimination
$(\neg A \vee B \vee C) \wedge (\neg(B \vee C) \vee A)$	implication elimination
$(\neg A \vee B \vee C) \wedge ((\neg B \wedge \neg C) \vee A)$	move negations inward
$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$	distribute $\vee$ over $\wedge$

called a "clause"

Resolution Steps

- Given KB and β (query)
- Add $\neg\beta$ to KB, and convert all sentences to CNF
- Show this leads to False (aka "empty clause"). Proof by contradiction
- Example KB:
  - $A \Leftrightarrow (B \vee C)$
  - $\neg A$
- Example query: $\neg B$

Resolution Preprocessing

- Add $\neg \beta$ to KB, and convert to CNF:

  a1: $\neg A \vee B \vee C$
  a2: $\neg B \vee A$
  a3: $\neg C \vee A$
  b: $\neg A$
  c: B

- Want to reach goal: False (empty clause)

Resolution Example

a1: $\neg A \vee B \vee C$
a2: $\neg B \vee A$
a3: $\neg C \vee A$
b: $\neg A$
c: B

Resolution Example

a1: $\neg A \vee B \vee C$
a2: $\neg B \vee A$
a3: $\neg C \vee A$
b: $\neg A$
c: B

Step 1: resolve a2, c: A

Step 2: resolve above and b: *empty*

Example

- Given:
  - $P \vee Q$
  - $P \Rightarrow R$
  - $Q \Rightarrow R$

- Prove: R

Example

- Given:
  - $(P \Rightarrow Q) \Rightarrow Q$
  - $(P \Rightarrow P) \Rightarrow R$
  - $(R \Rightarrow S) \Rightarrow \neg(S \Rightarrow Q)$
- Prove: R

Example

- Given:
  - P
  - $\neg P$
- Prove: R

Efficiency of the Resolution Algorithm

- Run time can be exponential in the worst case
  - Often much faster
- Factoring: if a new clause contains duplicates of the same symbol, delete the duplicates

  $P \vee R \vee P \vee T \equiv P \vee R \vee T$
- If a clause contains a symbol and its complement, the clause is a tautology and is useless; it can be thrown away

  a1: $(\neg A \vee B \vee C)$

  a2: $(\neg B \vee A)$

  Resolvent of a1 and a2 is: $B \vee C \vee \neg B$

  Which is valid, so throw it away

Method 4: Chaining with Horn Clauses

- Resolution is more powerful than we need for many practical situations
- A weaker form: **Horn clauses**
  - Disjunction of literals with **at most one positive**

    $\neg R \vee P \vee Q$ no

    $\neg R \vee \neg P \vee Q$ yes
  - KB = conjunction of Horn clauses
  - What's the big deal?

    $\neg R \vee \neg P \vee Q$

    $\equiv \neg(R \wedge P) \vee Q$

    $\equiv$?

Horn Clauses

$\neg R \vee \neg P \vee Q$
$\equiv \neg(R \wedge P) \vee Q$
$\equiv (R \wedge P) \Rightarrow Q$ Every rule in KB is in this form
P (special case, no negative literals): fact
$\neg R \vee \neg P$ (special case, no positive literal): goal clause

- The big deal:
  - KB easy for humans to read
  - Natural forward chaining and backward chaining algorithms; proof easy for humans to read
  - Can decide entailment with Horn clauses in time **linear** with KB size
- But …
  - Can only ask atomic queries

Horn Clauses

- Only 1 rule of inference needed
- **Generalized Modus Ponens**:

$$\frac{P,\ Q,\ (P \wedge Q) \Rightarrow R}{R}$$

Forward Chaining

- "Apply" any rule whose premises are satisfied in the KB
- Add its conclusion to the KB until query is derived

KB:

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

query: Q

- Forward chaining with Horn clause KB is **complete**

Forward Chaining

1. **P ⇒ Q**
2. **L ∧ M ⇒ P**
3. **B ∧ L ⇒ M**
4. **A ∧ P ⇒ L**
5. **A ∧ B ⇒ L**
6. **A**
7. **B**
8. **L** GMP(5,6,7)
9. **M** GMP(3,7,8)
10. **P** GMP(2,8,9)
11. **Q** GMP(1,10)

Backward Chaining

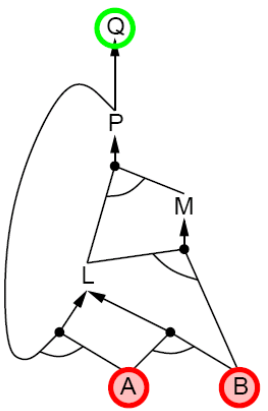
- Forward chaining problem: can generate a lot of irrelevant conclusions
  - Search forward, start state = KB, goal test = state contains query
- Backward chaining
  - **Work backwards from goal to premises**
  - Find all implications of the form

    $(\ldots) \Rightarrow$ query
  - Prove all the premises of one of these implications
  - Avoid loops: check if new subgoal is already on the goal stack
  - Avoid repeated work: check if new subgoal
    1. Has already been proved true, or
    2. Has already failed

Backward Chaining

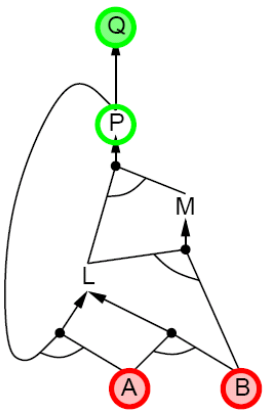
1.	$P \Rightarrow Q$	
2.	$L \wedge M \Rightarrow P$	
3.	$B \wedge L \Rightarrow M$	
4.	$A \wedge P \Rightarrow L$	
5.	$A \wedge B \Rightarrow L$	
6.	A	
7.	B	
8.	Q	Goal
9.	P	Subgoal(1,8)
10.	$L \wedge M$	Subgoal(2,9)
11.	L	Subgoal(10)
12.	$A \wedge B$	Subgoal(5,11)
13.	A	True(6)
14.	B	True(7)
15.	L	True(5,13,14)
16.	M	True(14,15)
17.	P	True(15,16)
18.	Q	True(1,17)

Backward Chaining



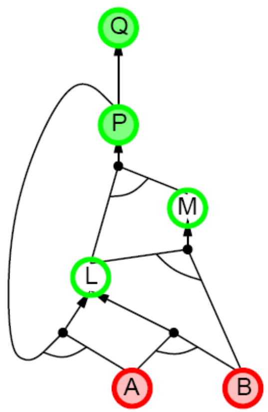
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining



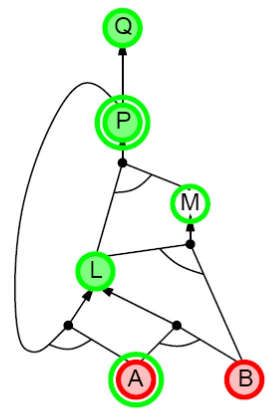
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining



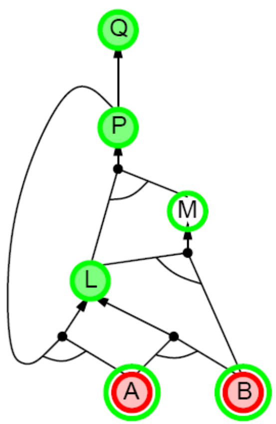
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining



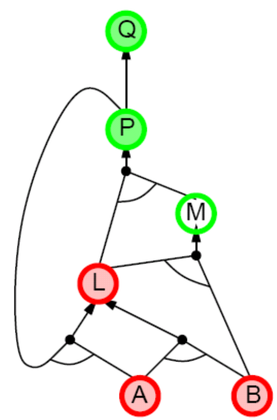
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining



$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining



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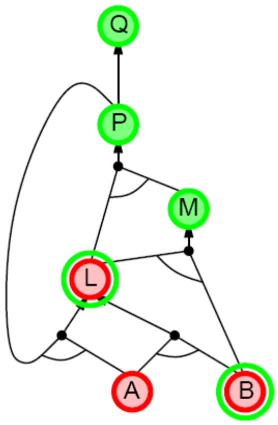

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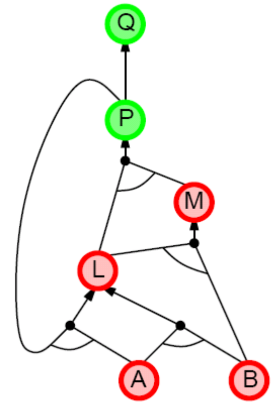

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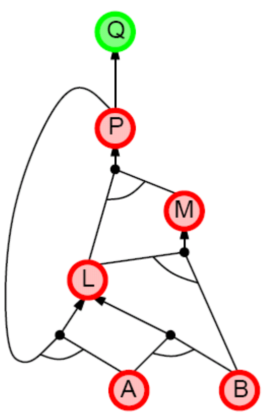

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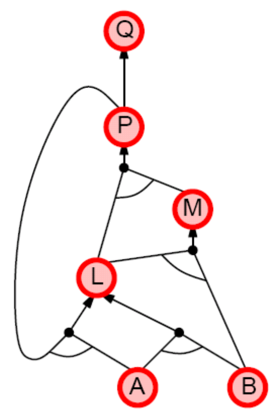

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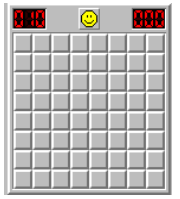
$A \wedge B \Rightarrow L$
A
B

Forward vs. Backward Chaining

- Forward chaining is data-driven
  - May perform lots of work irrelevant to the goal
- Backward chaining is goal-driven
  - Appropriate for problem solving
  - Time complexity can be much less than linear in size of KB
- Some form of bi-directional search may be even better

Problems with Propositional Logic

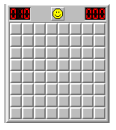
- Consider the game "minesweeper" on a 10x10 field with only one land mine



- How do you express the knowledge, with Propositional Logic, that the squares adjacent to the land mine will display the number 1?

Problems with Propositional Logic

- Consider the game "minesweeper" on a 10x10 field with only one land mine



- How do you express the knowledge, with Propositional Logic, that the squares adjacent to the land mine will display the number 1?
- Intuitively with a rule like

  $\text{Landmine}(x,y) \Rightarrow \text{Number1}(\text{Neighbors}(x,y))$

  but Propositional Logic **cannot** do this

Problems with Propositional Logic

- Propositional Logic has to say, e.g. for cell (3,4):
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_2\_3}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_2\_4}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_2\_5}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_3\_3}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_3\_5}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_4\_3}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_4\_4}$
  - $\text{Landmine\_3\_4} \Rightarrow \text{Number1\_4\_5}$
  - And similarly for *each* of Landmine_1_1, Landmine_1_2, Landmine_1_3, …, Landmine_10_10
- Difficult to express large domains concisely
- Don't have objects and relations
- First-Order Logic is a powerful upgrade

What You Should Know

- A lot of terms
- Use truth tables (inference by enumeration)
- Natural deduction proofs
- Conjuctive Normal Form (CNF)
- Resolution Refutation algorithm and proofs
- Horn clauses
- Forward chaining algorithm
- Backward chaining algorithm