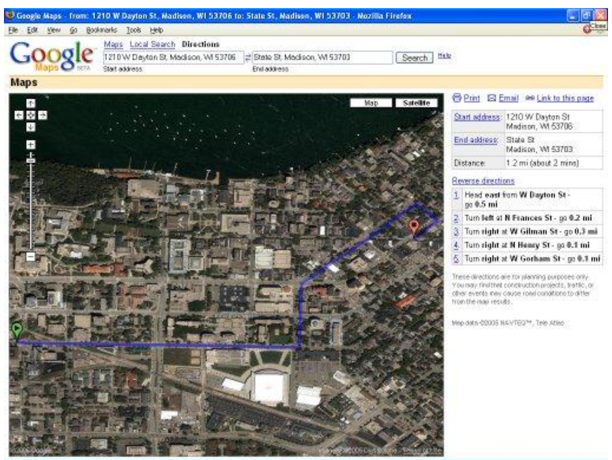
Uninformed Search

Chapter 3.1 – 3.4

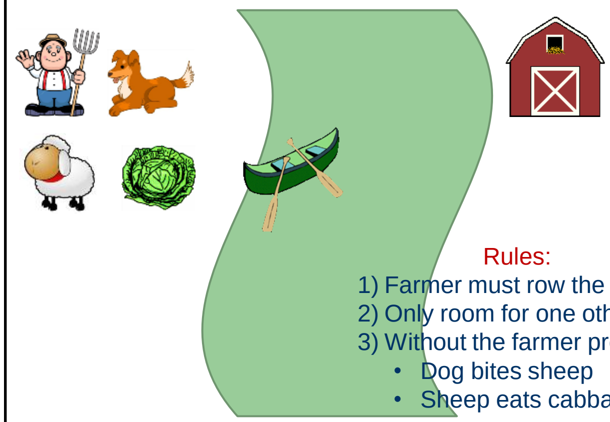
Many AI Tasks can be Formulated as Search Problems

- **Puzzles**
- **Games**
- **Navigation**
- **Assignment**
- **Layout**
- **Scheduling**
- **Routing**

Search Example: Route Finding



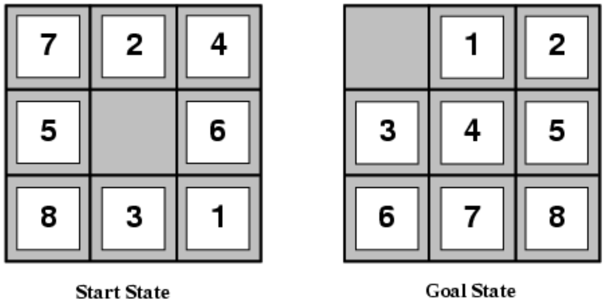
Search Example: River Crossing Problem



Rules:

1) Farmer must row the boat
2) Only room for one other
3) Without the farmer present:
   - Dog bites sheep
   - Sheep eats cabbage

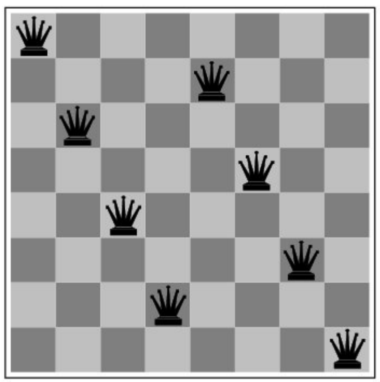
Search Example: 8-Puzzle



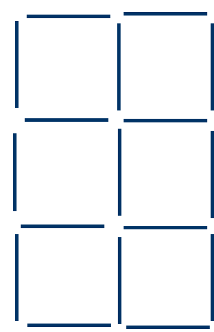
Search Example: Water Jugs Problem

Given 4-liter and 3-liter pitchers, how do you get exactly 2 liters into the 4-liter pitcher?

Search Example: 8-Queens



Remove 5 Sticks Problem



Remove exactly 5 of the 17 sticks so the resulting figure forms exactly 3 squares

Basic Search Task Assumptions (usually, though not games)

- **Fully observable**
- **Deterministic**
- **Static**
- **Discrete**
- **Single agent**

What Knowledge does the Agent Need?

- The information needs to be
  - sufficient to describe all relevant aspects for reaching the goal
  - adequate to describe the world state/situation
- **Fully observable** assumption, also known as the ***closed world assumption***, means
  - *All necessary information about a problem domain is accessible so that each state is a complete description of the world; there is no missing information at any point in time*

How should the Environment be Represented?

- **Knowledge representation problem:**
  - What information from the sensors is relevant?
  - How to represent domain knowledge?
- ***Determining what to represent is difficult and is usually left to the system designer to specify***
- Problem ***State*** = representation of all necessary information about the environment
- ***State Space*** (aka **Problem Space**) = all possible valid configurations of the environment

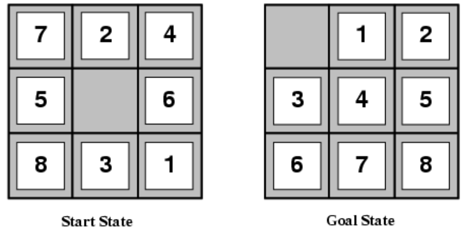
What Goal does the Agent want to Achieve?

- **How do you describe the goal?**
  - as a task to be accomplished
  - as a state to be reached
  - as a set of properties to be satisfied
- **How do you know when the goal is reached?**
  - with a **goal test** that defines what it means to have achieved/satisfied the goal
  - or, with a set of **goal states**
- ***Determining the goal is usually left to the system designer or user to specify***

What Actions does the Agent Need?

- **Discrete and Deterministic task assumptions imply**
- **Given:**
  - an action (aka operator or move)
  - a description of the current state of the world
- **Action completely specifies:**
  - if that action *can* be applied (i.e., legal)
  - what the exact state of the world will be after the action is performed in the current state (no "history" information needed to compute the successor state)

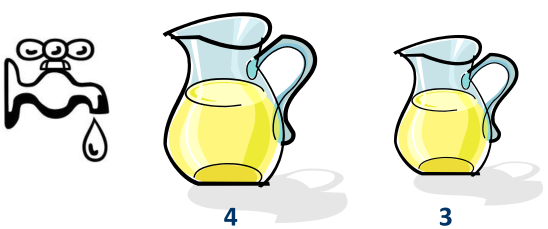
Search Example: 8-Puzzle



- **States = configurations**
- **Actions = up to 4 kinds of moves: up, down, left, right**

Water Jugs Problem

Given 4-liter and 3-liter pitchers, how do you get exactly 2 liters into the 4-liter pitcher?



State: (*x, y*) for # liters in 4-liter and 3-liter pitchers, respectively
Actions: empty, fill, pour water between pitchers
Initial state: (0, 0)
Goal state: (2, *)

Actions / Successor Functions

1. $(x, y \mid x < 4) \rightarrow (4, y)$ — Fill *4*
2. $(x, y \mid y < 3) \rightarrow (x, 3)$ — Fill *3*
3. $(x, y \mid x > 0) \rightarrow (0, y)$ — Empty *4*
4. $(x, y \mid y > 0) \rightarrow (x, 0)$ — Empty *3*
5. $(x, y \mid x+y \geq 4 \text{ and } y > 0) \longrightarrow (4, y - (4 - x))$ — Pour from *3* to *4* until *4* is full
6. $(x, y \mid x+y \geq 3 \text{ and } x > 0) \longrightarrow (x - (3 - y), 3)$ — Pour from *4* to *3* until *3* is full
7. $(x, y \mid x+y \leq 4 \text{ and } y > 0) \longrightarrow (x+y, 0)$ — Pour all water from *3* to *4*

Formalizing Search in a State Space

- **A state space is a *graph*: (*V*, *E*)**
  - *V* is a set of nodes (vertices)
  - *E* is a set of arcs (edges)
    each arc is *directed* from one node to another node
- **Each node is a data structure that contains:**
  - a **state** description
  - other information such as:
    - link to parent node
    - name of action that generated this node (from its parent)
    - other bookkeeping data

Formalizing Search in a State Space

- **Each arc corresponds to one of the finite number of actions:**
  - when the action is applied to the state associated with the arc's source node
  - then the resulting state is the state associated with the arc's destination node
- **Each arc has a fixed, positive cost:**
  - corresponds to the cost of the action

Formalizing Search in a State Space

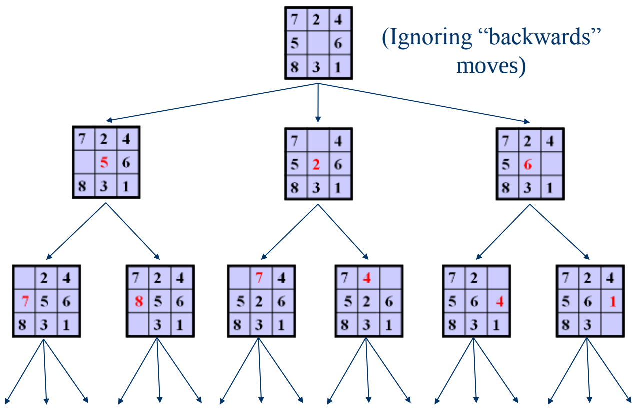
- **Each node has a finite set of successor nodes:**
  - corresponds to all of the legal actions
    that can be applied at the source node's state
- **Expanding a node means:**
  - generate *all* of the successor nodes
  - add them and their associated arcs to the state-space search tree

Formalizing Search in a State Space

- **One or more nodes are designated as start nodes**
- **A goal test is applied to a node's state to determine if it is a goal node**
- **A solution is a sequence of actions associated with a path in the state space from a start to a goal node:**
  - just the goal state (e.g., cryptarithmetic)
  - a path from start to goal state (e.g., 8-puzzle)
- **The cost of a solution is the sum of the arc costs on the solution path**

8-Puzzle State Space

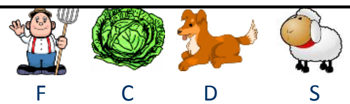
(Ignoring "backwards" moves)



Sizes of State Spaces

Problem	Nodes	Brute-Force Search Time (10 million nodes/second)
• Tic-Tac-Toe	3^9	
• 8 Puzzle	10^5	.01 seconds
• 2^3 Rubik's Cube	10^6	.2 seconds
• 15 Puzzle	10^{13}	6 days
• 3^3 Rubik's Cube	10^{19}	68,000 years
• 24 Puzzle	10^{25}	12 billion years
• Checkers	10^{40}	
• Chess	10^{120}	

Formalizing Search



A search problem has five components:

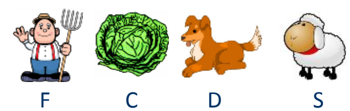
S, I, G, actions, cost

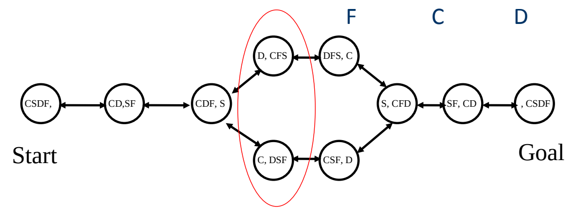
1. **State space *S*: all valid configurations**
2. **Initial states *I* ⊆ *S*: a set of start states** *I* = {(FCDS,)} ⊆ *S*
3. **Goal states *G* ⊆ *S*: a set of goal states** *G* = {(,FCDS)} ⊆ *S*
4. **An action function *successors*(*s*) ⊆ *S* : states reachable in one step (one arc) from *s***

   successors((FCDS,)) = {(CD,FS)}
   successors((CDF,S)) = {(CD,FS), (D,FCS), (C,FSD)}
5. **A cost function cost(*s, s′*): The cost of moving from *s* to *s′***

- **The goal of search is to find a solution path from a state in *I* to a state in *G***

State Space = A Directed Graph

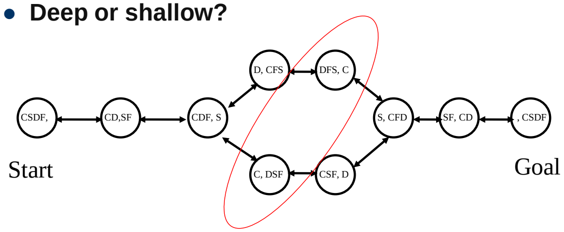




- In general there will be many generated, but un-expanded, states at any given time
- One has to choose which one to "expand" next

Different Search Strategies

- **The generated, but not yet expanded, states define the *Frontier* (aka *Open* or *Fringe*) list**
- **The essential difference is, which one to expand first?**
- **Deep or shallow?**



Formalizing Search in a State Space

State-space search is the process of searching through a state space for a solution by **making explicit a sufficient portion of an implicit state-space graph to include a goal node:** **TREE SEARCH Alg.**

Frontier = {*S*}, where *S* is the start node
Loop do
  if *Frontier* is empty **then return** failure
  pick a node, *n*, from *Frontier*
  if *n* is a goal node **then return** solution
  Generate all *n*'s successor nodes and add them all to *Frontier*
  Remove *n* from *Frontier*

Formalizing Search in a State Space

- **This algorithm does *NOT* detect goal when node is generated**
- **This algorithm does *NOT* detect loops in state space**
- **Each node implicitly represents**
  - a partial solution path from the start node to the given node
  - cost of the partial solution path
- **From this node there may be**
  - many possible paths that have this partial path as a prefix
  - many possible solutions

Uninformed Search on Trees

- **Uninformed means we *only* know:**
  - The goal test
  - The ***successors***() function
- But ***not*** which non-goal states are better
- For now, also assume state space graph is a **tree**
  - That is, we won't encounter (or at least worry about) repeated states
  - We will relax this later
- ***Search strategies differ by what un-expanded node is expanded next***

Uninformed Search Strategies

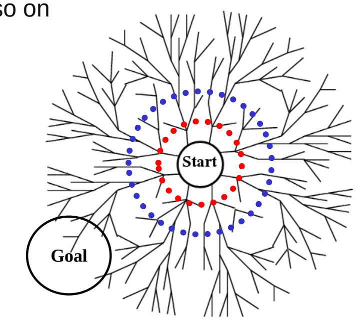
Uninformed Search: strategies that order nodes *without* using any domain specific information, i.e., doesn't use any information stored in a state

- **BFS: breadth-first search**
  - *Queue (FIFO) used for the Frontier list*
  - remove from front, add to back
- **DFS: depth-first search**
  - *Stack (LIFO) used for the Frontier list*
  - remove from front, add to front

Breadth-First Search (BFS)

Expand the shallowest node first:

1. Examine states one step away from the initial states
2. Examine states two steps away from the initial states
3. and so on

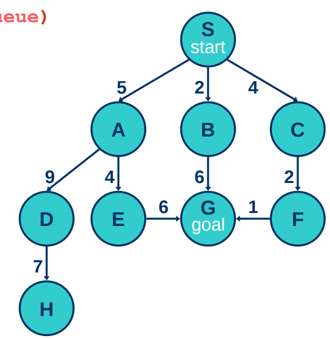


Breadth-First Search (BFS)

```
generalSearch(problem, queue)
```

# of nodes tested: 0, expanded: 0

expnd. node	Frontier list
	{S}

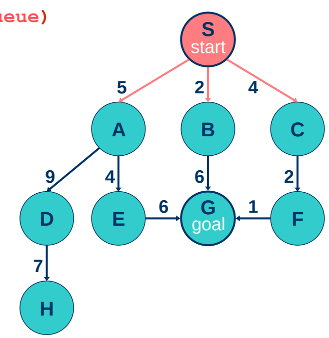


Breadth-First Search (BFS)

```
generalSearch(problem, queue)
```

# of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S}
S not goal	{A,B,C}

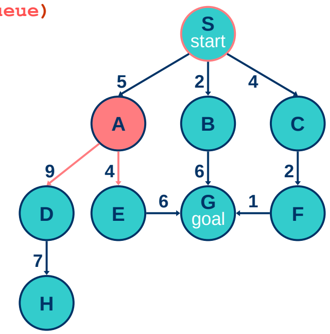


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier list
	{S}
S	{A,B,C}
A not goal	{B,C,D,E}

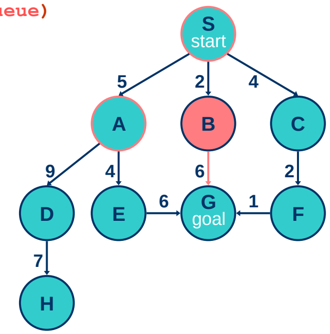


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 3, expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B not goal	{C,D,E,G}

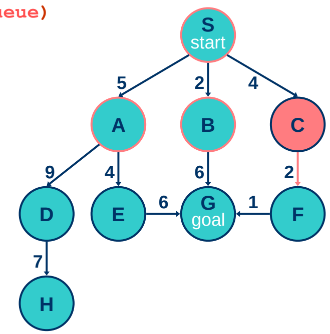


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 4, expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C not goal	{D,E,G,F}

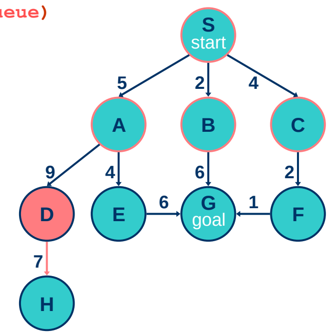


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 5, expanded: 5

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D not goal	{E,G,F,H}

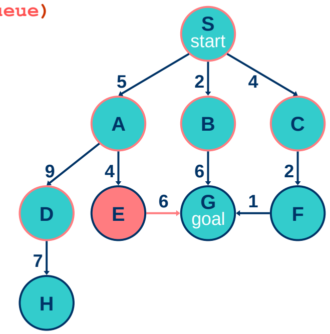


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

\# of nodes tested: 6, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E not goal	{G,F,H,G}

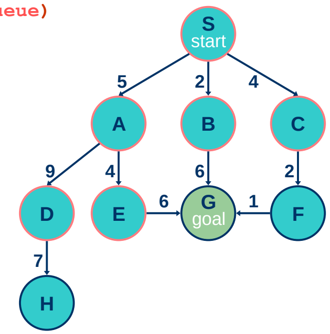


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

\# of nodes tested: 7, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E	{G,F,H,G}
G goal	{F,H,G} no expand

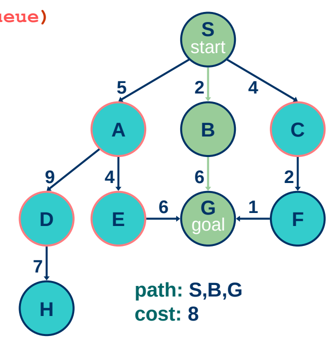


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

\# of nodes tested: 7, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E	{G,F,H,G}
G	{F,H,G}



path: S,B,G
cost: 8

Evaluating Search Strategies

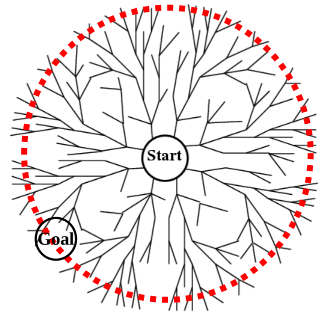
- **Completeness**
  If a solution exists, will it be found?
  - a complete algorithm will find ***a*** solution (not all)
- **Optimality / Admissibility**
  If a solution is found, is it guaranteed to be optimal?
  - an admissible algorithm will find a **solution with minimum cost**

Evaluating Search Strategies

- **Time Complexity**
  How long does it take to find a solution?
  - usually measured for worst case
  - measured by counting **number of nodes expanded**
- **Space Complexity**
  How much space is used by the algorithm?
  - measured in terms of the **maximum size of the *Frontier list*** during the search

What's in the Frontier (Queue) for BFS?

- **If goal is at depth *d*, how big is the frontier (worst case)?**



Breadth-First Search (BFS)

- **Complete**
- **Optimal / Admissible**
  - **Yes**, ***if*** all operators (i.e., arcs) have the same constant cost, or costs are positive, non-decreasing with depth
  - otherwise, not optimal but does guarantee finding solution of shortest *length* (i.e., fewest arcs)

Breadth-First Search (BFS)

- **Time and space complexity: $O(b^d)$ (i.e., exponential)**
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node
- Very slow to find solutions with a large number of steps because must look at all shorter length possibilities first

Breadth-First Search (BFS)

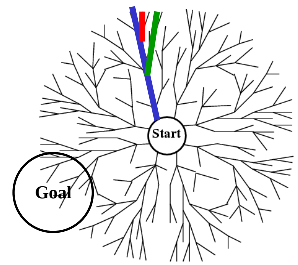
- **A complete search tree has a total # of nodes =** **$1 + b + b^2 + ... + b^d = (b^{(d+1)} - 1) / (b-1)$**
  - d: the tree's depth
  - b: the branching factor at each non-leaf node
- **For example: $d = 12$, $b = 10$**
  $1 + 10 + 100 + ... + 10^{12} = (10^{13} - 1)/9 = O(10^{12})$
  - If BFS expands 1,000 nodes/sec and each node uses 100 bytes of storage, then BFS will take 35 years to run in the worst case, and it will use 111 terabytes of memory!

Depth-First Search

Expand the ***deepest*** node first

1. Select a direction, go deep to the end
2. Slightly change the end
3. Slightly change the end some more…

Use a Stack to order nodes on the *Frontier* list

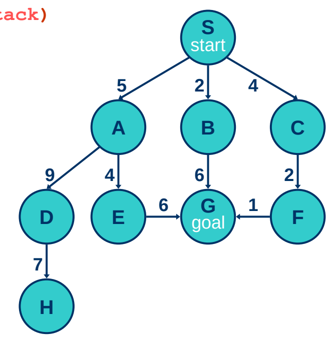


Depth-First Search (DFS)

`generalSearch(problem, stack)`

\# of nodes tested: 0, expanded: 0

expnd. node	Frontier list
	{S}

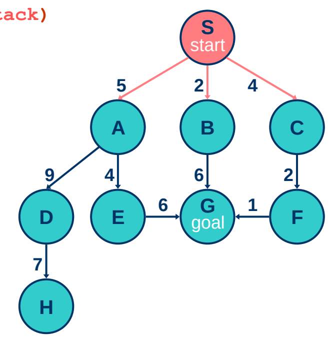


Depth-First Search (DFS)

`generalSearch(problem, stack)`

\# of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S}
S not goal	{A,B,C}

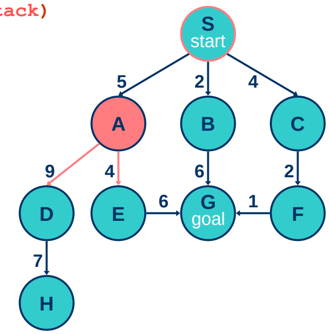


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier list
	{S}
S	{A,B,C}
A not goal	{D,E,B,C}

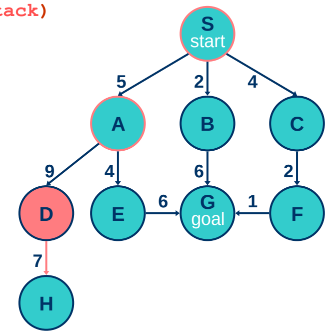


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 3, expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{D,E,B,C}
D not goal	{H,E,B,C}

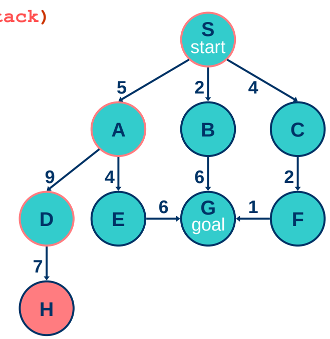


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 4, expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H not goal	{E,B,C}

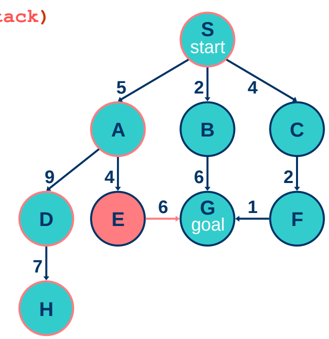


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 5, expanded: 5

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E not goal	{G,B,C}

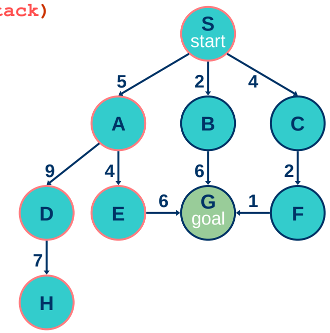


Depth-First Search (DFS)

generalSearch(problem, stack)

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E	{G,B,C}
G goal	{B,C} no expand

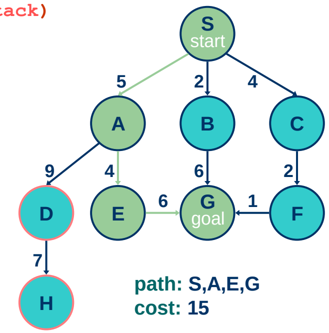


Depth-First Search (DFS)

generalSearch(problem, stack)

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E	{G,B,C}
G	{B,C}



path: S,A,E,G
cost: 15

Depth-First Search (DFS)

- **May not terminate without a depth bound**
  i.e., cutting off search below a fixed depth, *D*
- **Not complete**
  - with or without cycle detection
  - and, with or without a depth cutoff
- **Not optimal / admissible**
- ***Can find long solutions quickly if lucky***

Depth-First Search (DFS)

- **Time complexity: $O(b^d)$ exponential**
  Space complexity: $O(bd)$ linear
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node
- Performs "**chronological backtracking**"
  - i.e., when search hits a dead end, backs up *one* level at a time
  - problematic if the mistake occurs because of a bad action choice near the top of search tree

Uniform-Cost Search (UCS)

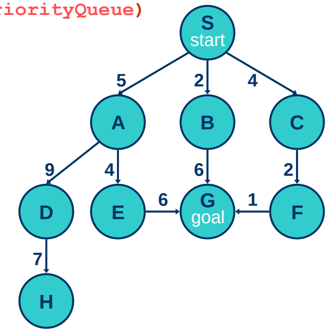
- Use a "**Priority Queue**" to order nodes on the *Frontier* list, sorted by path cost
- Let $g(n)$ = cost of path from start node s to current node n
- Sort nodes by increasing value of g

Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 0, expanded: 0

expnd. node	Frontier list
	{S}

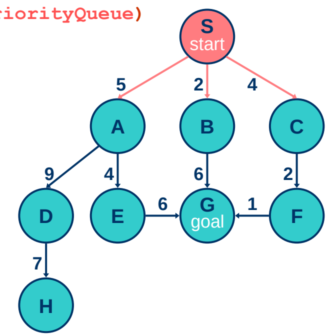


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S:0}
S not goal	{B:2,C:4,A:5}

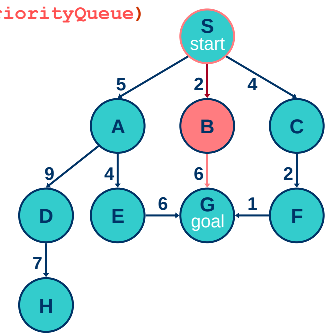


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B not goal	{C:4,A:5,G:2+6}

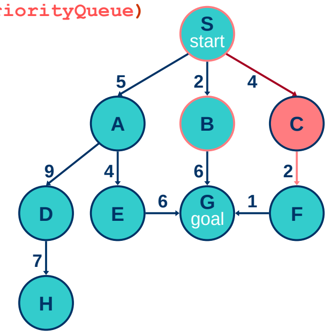


Uniform-Cost Search (UCS)

generalSearch(problem, priorityQueue)

# of nodes tested: 3, expanded: 3

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C not goal	{A:5,F:4+2,G:8}

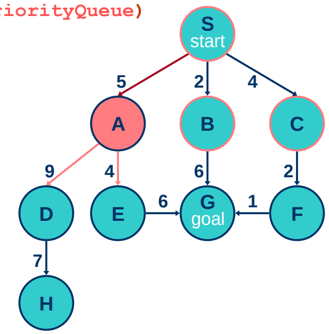


Uniform-Cost Search (UCS)

generalSearch(problem, priorityQueue)

# of nodes tested: 4, expanded: 4

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A not goal	{F:6,G:8,E:5+4, D:5+9}

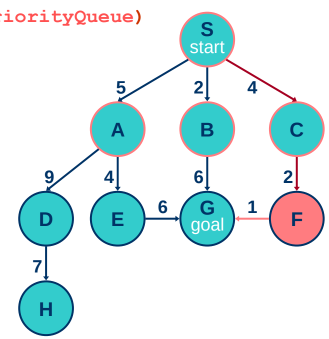


Uniform-Cost Search (UCS)

generalSearch(problem, priorityQueue)

# of nodes tested: 5, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F not goal	{G:4+2+1,G:8,E:9, D:14}

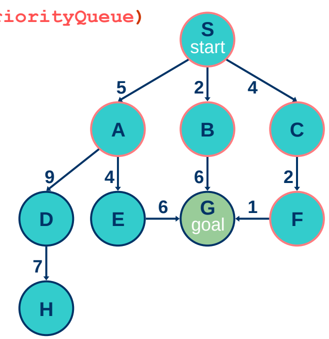


Uniform-Cost Search (UCS)

generalSearch(problem, priorityQueue)

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F	{G:7,G:8,E:9,D:14}
G goal	{G:8,E:9,D:14} no expand

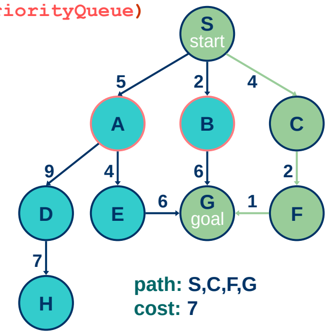


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F	{G:7,G:8,E:9,D:14}
G	{G:8,E:9,D:14}



Uniform-Cost Search (UCS)

- **Called *Dijkstra's Algorithm* in the algorithms literature**
- **Similar to *Branch and Bound Algorithm* in Operations Research literature**

- **Complete**
- **Optimal / Admissible**
  - requires that the goal test is done when a node is ***removed*** from the *Frontier* list rather than when the node is generated by its parent node

Uniform-Cost Search (UCS)

- **Time and space complexity: $O(b^d)$ (i.e., exponential)**
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node

- **More precisely, time and space complexity is $O(b^{C^*/\varepsilon})$ where all edge costs $\geq \varepsilon > 0$, and C^* is the best goal path cost**

Iterative-Deepening Search (IDS)

- **requires modification to DFS search algorithm:**
  - do DFS to depth 1 and treat all children of the start node as leaves
  - if no solution found, do DFS to depth 2
  - repeat by increasing "depth bound" until a solution found

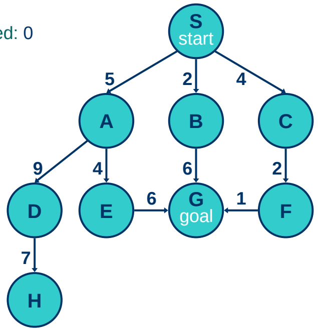
- **Start node is at depth 0**

Iterative-Deepening Search (IDS)

`deepeningSearch(problem)`

depth: 1, # of nodes expanded: 0, tested: 0

expnd. node	Frontier list
	{S}

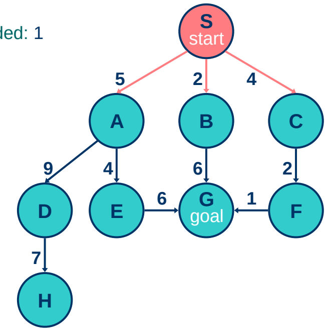


Iterative-Deepening Search (IDS)

`deepeningSearch(problem)`

depth: 1, # of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S}
S not goal	{A,B,C}

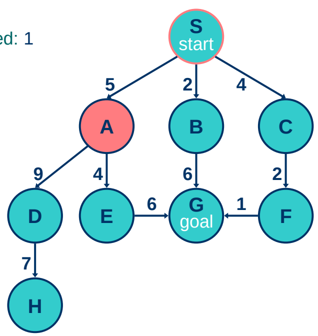


Iterative-Deepening Search (IDS)

`deepeningSearch(problem)`

depth: 1, # of nodes tested: 2, expanded: 1

expnd. node	Frontier list
	{S}
S	{A,B,C}
A not goal	{B,C} no expand

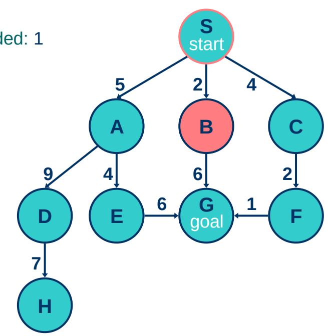


Iterative-Deepening Search (IDS)

`deepeningSearch(problem)`

depth: 1, # of nodes tested: 3, expanded: 1

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B not goal	{C} no expand

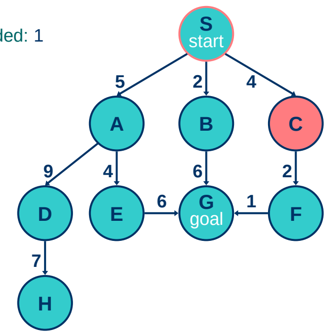


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 1, # of nodes tested: 4, expanded: 1

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C not goal	{ } no expand-FAIL

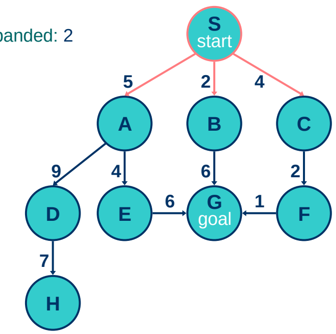


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 4(1), expanded: 2

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S no test	{A,B,C}

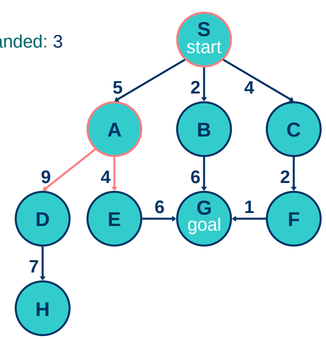


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 4(2), expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A no test	{D,E,B,C}

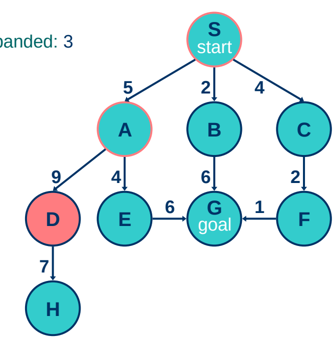


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 5(2), expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A	{D,E,B,C}
D not goal	{E,B,C} no expand

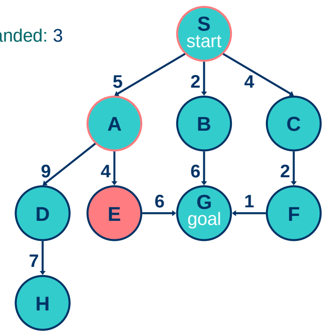


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 6(2), expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A	{D,E,B,C}
D	{E,B,C}
E not goal	{B,C} no expand

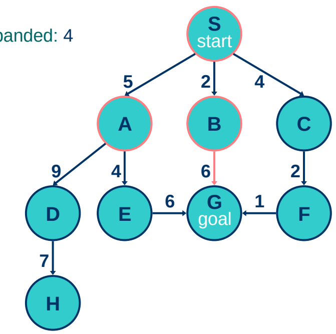


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 6(3), expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A	{D,E,B,C}
D	{E,B,C}
E	{B,C}
B no test	{G,C}

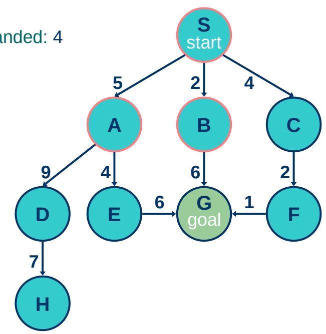


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 7(3), expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A	{D,E,B,C}
D	{E,B,C}
E	{B,C}
B	{G,C}
G goal	{C} no expand

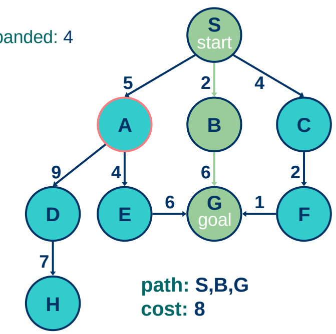


Iterative-Deepening Search (IDS)

deepeningSearch(problem)

depth: 2, # of nodes tested: 7(3), expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C}
B	{C}
C	{ }
S	{A,B,C}
A	{D,E,B,C}
D	{E,B,C}
E	{B,C}
B	{G,C}
G	{C}



path: S,B,G
cost: 8

Iterative-Deepening Search (IDS)

- **Has advantages of BFS**
  - completeness
  - optimality as stated for BFS
- **Has advantages of DFS**
  - limited space
  - in practice, even with redundant effort it still finds longer paths more quickly than BFS

Iterative-Deepening Search (IDS)

- **Space complexity: $O(bd)$ (i.e., linear like DFS)**
- **Time complexity is a little worse than BFS or DFS**
  - because nodes near the top of the search tree are generated multiple times (redundant effort)
- **Worst case time complexity: $O(b^d)$ exponential**
  - because most nodes are near the bottom of tree

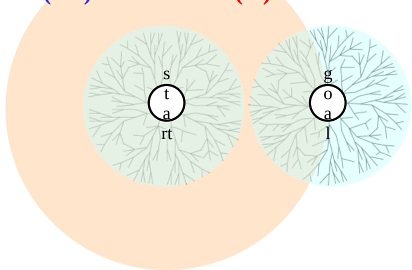
Iterative-Deepening Search (IDS)

How much redundant effort is done?

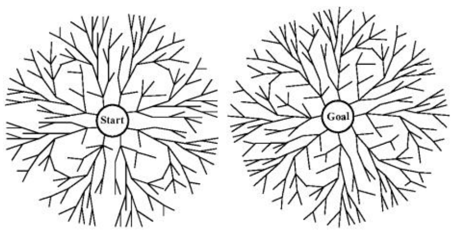
- **The number of times the nodes are generated:**
  $1b^d + 2b^{(d-1)} + \ldots + db \leq b^d / (1 - 1/b)^2 = O(b^d)$
  - d: the solution's depth
  - b: the branching factor at each non-leaf node
- **For example:** $b = 4$
  $4^d / (1 - ¼)^2 = 4^d / (.75)^2 = 1.78 \times 4^d$
  - in the worst case, 78% more nodes are searched (redundant effort) than exist at depth d
  - as b increases, this % decreases

Bidirectional Search

- **Breadth-first search from both start and goal**
- **Frontiers meet**
- **Generates $O(b^{d/2})$ instead of $O(b^d)$ nodes**



Which Direction Should We Search?



Our choices: Forward, backwards, or bidirectional

The issues: How many start and goal states are there?
Branching factors in each direction
How much work is it to compare states?

Performance of Search Algorithms on Trees

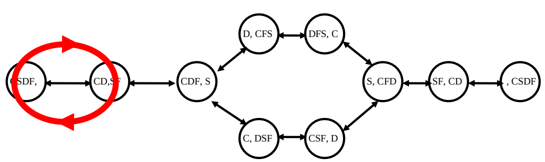
b: branching factor (assume finite) d: goal depth m: graph depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if [1]	$O(b^d)$	$O(b^d)$
Uniform-cost search[2]	Y	Y	$O(b^{C^*/\varepsilon})$	$O(b^{C^*/\varepsilon})$
Depth-first search	N	N	$O(b^m)$	O(bm)
Iterative deepening	Y	Y, if [1]	$O(b^d)$	O(bd)
Bidirectional search[3]	Y	Y, if [1]	$O(b^{d/2})$	$O(b^{d/2})$

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \varepsilon > 0$. C* is the best goal path cost
3. both directions BFS; not always feasible

If State Space is *Not* a Tree

- **The problem: repeated states**

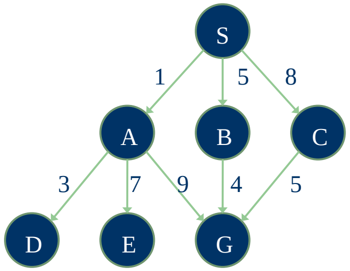


- **Ignoring repeated states: wasteful (BFS) or impossible (DFS). Why?**
- **How to prevent these problems?**

If State Space is *Not* a Tree

- **We have to remember already-expanded states (called *Explored* (aka *Closed*) list) too**
- **Why?**
- **When we pick a state from *Frontier***
  - Remove it from *Frontier*
  - Add it to *Explored*
  - Expand node, generating all successors
  - For each successor, *child*,
    - If *child* is in *Explored*, throw *child* away
    - Otherwise, check whether *child* is in *Frontier*
      - If no, add it to *Frontier*
      - If yes and path-cost(*child*) < path-cost of node already in *Frontier*, then replace that *Frontier* node with *child*

Example



How are nodes expanded by

- Depth First Search
- Breadth First Search
- Uniform Cost Search
- Iterative Deepening

Are the solutions the same?

Nodes Expanded by:

- **Depth-First Search: S A D E G**
  Solution found: S A G
- **Breadth-First Search: S A B C D E G**
  Solution found: S A G
- **Uniform-Cost Search: S A D B C E G**
  Solution found: S B G
- **Iterative-Deepening Search: S A B C S A D E G**
  Solution found: S A G