AVERSION TO UNCERTAINTY & REVENUE MAXIMIZATION

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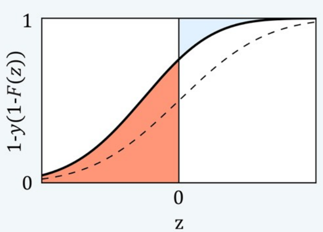
BEHAVIOR MODEL

We initiate the study of revenue maximization under cumulative prospect theory [Tversky and Kahneman, '92], a model developed to explain risk-averse behaviors not captured by traditional expected utility theory (EUT).

The **risk-averse expectation** of a random variable is the expectation of its CDF after transformation by a **probability weighting function** $y : [0,1] \to [0,1]$:

$$\mathrm{E}_y[Z] = \int_0^\infty y(1-F(z))\,dz - \int_{-\infty}^0 \left(1 - y(1-F(z))\right) dz$$



A CDF (dotted), and the transformed CDF. The risk-averse expectation is equal to the blue area minus the red.

COMPARISON AGAINST OTHER RISK MODELS

Model	Utility	\$16 wp ½	\$8 wp 1
Risk Neutral	$x \cdot v$	\$8	\$8
EUT (Nonlinear in value)	$x \cdot v^{1/2}$	\$2	$\$\sqrt{8}$
Our Model (Nonlinear in probability)	$x^2 \cdot v$	\$4	\$8

MECHANISM DESIGN

- Buyer has private value $v \sim F$, public F.
- Seller posts a menu of lotteries (X, P)
  - $X \in \{0,1\}$ – random allocation
  - $P \in \mathbb{R}$ – random payment.
- Buyer selects a lottery to maximize

$$\mathbb{E}[vX - P].$$

- Objective: maximize expected payment.

Standard result for risk-neutral buyer: the optimal mechanism is a fixed price.

EX: SELLING ONE ITEM

Suppose a buyer has utility $v \cdot y(x)$ for an outcome which pays v with probability x.

Let $v \sim U[0,1]$. The optimal price is

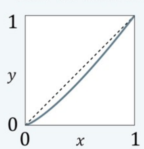
$$\arg\max p(1-p) = 1/2$$

with expected revenue $1/4$.

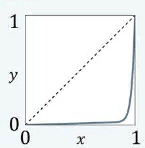
If $y = x^2$, adding a lottery which w.p. $1/2$ allocates at price $3/8$, or else does nothing, increases expected revenue to $25/96$.

RISK ATTITUDES

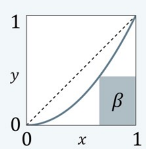
Convex probability weighting functions give rise to risk-averse behavior.



Mild risk aversion.

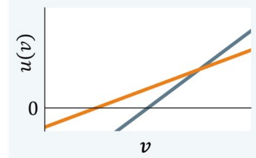


Extreme risk aversion.

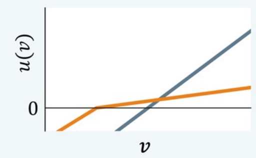


β-bounded risk aversion.

LOTTERIES & REVENUE



Risk-neutral utility curve for the optimal price (gray) and a cheap lottery (orange). Some high-value buyers deviate to the lottery, negating gains from low-value buyers.



Risk-averse utility curves for the same options. The reduced slope of the orange curve means fewer high-value buyers choose the cheap lottery, increasing revenue.

(NEARLY) OPTIMAL MECHANISMS

THM. The optimal mechanism is a menu of simple lotteries.

THM. If the buyer is extremely risk averse, optimal revenue tends to $\mathbb{E}[v]$.

Full extraction of the buyer's value is unrealistic; however, when risk aversion is bounded, so is the gap between random and deterministic mechanisms.

THM. For any β-bounded weighting function, a fixed price gives a β-approximation.

Because the revenue of a fixed price does not depend on the buyer's risk attitude, this result implies a **robust approximation:** the guarantee does not require precise knowledge of the buyer's risk attitude.

DYNAMIC MECHANISMS

- Seller has one item to sell in each of two stages.
- Buyer has $v_1 \sim F_1$ in first stage, $v_2 \sim F_2$ in second.
- Buyer and seller know F_1, F_2.
- Buyer chooses first-stage option before learning v_2.

Risk-neutral buyer: the seller can extract significant extra revenue by charging more in the first stage in exchange for a lower (expected) second-stage payment [Ashlagi et al., '16; Mirrokni et al., '16].

We focus on **posted-price mechanisms:** on the first stage, the seller offers a menu of prices (p_1, p_2).

THM. No robust posted-price mechanism can obtain a constant approximation to the optimal posted-price mechanism, even if all weighting functions are β-bounded.

CONCLUSIONS

Single-shot setting: the optimal mechanism is randomized.

Single-shot setting: the optimal fixed price is a robust constant approximation.

Dynamic setting: no robust constant approximation is possible.

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Full paper: **https://arxiv.org/abs/1703.08607**