1. Do Exercise 3-4-2.

2. Do Exercise 3-4-3.

3. Solve the problem in Exercise 4-2-2 by adding dual labels to the tableau and applying Phase I and Phase II in the usual way. (Hint: If you need to add a row and column for Phase I, just use the usual addrow and addcol commands; the dual labels for the row and column will be left blank, which is OK.)

4. Do Exercise 4-4-3.

5. Consider the standard form LP

   \[
   \begin{array}{l}
   \text{minimize} \\
   \text{subject to}
   \end{array}
   \begin{array}{l}
   p^T x \\
   Ax \geq b \\
   x \geq 0.
   \end{array}
   \]

   Let \( u \in \mathbb{R}^m, u \geq 0 \).

   (a) Prove that if \( x \) is feasible for the LP, then it also satisfies the inequality \( u^T Ax \geq u^T b \).

   (b) Prove that for any \( u \geq 0 \), the optimal value of the LP

   \[
   \begin{array}{l}
   \text{minimize}_x \\
   \text{subject to}
   \end{array}
   \begin{array}{l}
   p^T x \\
   (A^T u)^T x \geq b^T u \\
   x \geq 0.
   \end{array}
   \]

   is less than or equal to the optimal value of (1).

*Hard copy to be submitted in class on the due date. No late homework accepted.
(c) Show that (2) is bounded below if $A^T u \leq p$.

(d) **EXTRA CREDIT:** Derive a necessary condition on $u$ such that (2) is bounded below.

(e) **EXTRA CREDIT:** When the LP is bounded, derive an expression for the optimal value of (2). Your expression will depend on the vector $u$.

(f) **EXTRA CREDIT:** Formulate the problem of finding the best such bound, by maximizing the lower bound over $u \geq 0$ subject to the conditions when the LP (2) is bounded.

(g) **EXTRA CREDIT:** How does the optimal value of the resulting optimization in part (f) problem compare to the optimal value of LP (1)?