CS726 - Nonlinear Optimization I - Homework V

October 17, 2012

This assignment is due at the beginning of class on October 24.

This assignment should be submitted electronically using learn@uw. The assignment name is hwk5 and your code should be runnable from exactly one script called `run_hwk5.m` (or `run_hwk5.py` if running python). That is, I should be able to type `run_hwk5.m` (or `python run_hwk5.py`) and have all of your experiments executed and appropriate plots produced. Also, make a file `hwk5_commentary.pdf` with all of the plots and write-ups of your findings.

The goal of this assignment is to solve the unconstrained optimization problem

$$\text{minimize } f(x) := -\sum_{i=1}^{n-1} \log(x_{i+1} - x_i - 1) - \sum_{i=1}^{n} \log(n^2 - x_i^2)$$

with respect to $x \in \mathbb{R}^n$. In your algorithms below, initialize x so that

$$x_k = 2k - n - 1 \text{ for } k = 1, \ldots, n$$

The minimizer of this function is called the *analytic center* of the polyhedron

$$\begin{array}{ll} x_{i+1} - x_i \geq 1 & i = 1, \ldots, n-1 \\ |x_i| \leq n & i = 1, \ldots, n \end{array}$$

Write code to minimize f using the algorithms

1. Nesterov's optimal method with backtracking line search for determining the step-size

2. BFGS with bisection line search

3. Barzilai-Borwien

Terminate when

$$\|\nabla f(x_k)\|_\infty \leq \tau(1 + |f(x_k)|)$$

for $\tau = 10^{-5}$.

In all cases (except Barzilai-Borwein), tune the line search parameters to minimize the wall-clock time. Plot the objective function and step length versus iteration number. Determine the optimal value $p^\star$ to high accuracy, also plot $f - p^\star$ versus iteration. Carry these experiments out for $n = 10$, 100, 1000, and 5000.

Finally, create whatever optimization procedure you would like that uses only function and gradient evaluations to minimize f. Describe your algorithm in detail (using pseudo-code if you'd like). Describe how you tuned the parameters. Try to minimize the number of function and gradient evaluations, and wall clock time to optimality.