CS726 - Nonlinear Optimization I - Homework 8

November 7, 2012

This assignment is due at the beginning of class on November 19.

1. Compute the subdifferentials of the following functions

(a) $f_1(x) = \max_{1 \le i \le m} |a_i^T x + b_i|$.

(b) $f_2(x) = \sup_{0 \le t \le 1} p(t)$, where $p(t) = x_1 + x_2 t + \cdots + x_n t^{n-1}$.

(c) $f_3(x) = \|x\|_2$

2. Suppose we are minimizing the convex function f by the subgradient method:

$$x_{k+1} = x_k - \alpha_k g_k$$

where $g_k \in \partial f(x_k)$. Let f_* denote the optimal value of f. After N iterations, let

$$f_{\text{best}} = \min_{1 \le i \le N} f(x_i)$$

We then have the estimate

$$f_{\text{best}} - f_* \le \frac{D^2 + G^2 \sum_{i=1}^N \alpha_i^2}{2 \sum_{i=1}^N \alpha_i}$$

where D denotes the distance from x_1 to the optimal region and G is an upper bound on the norm of the subgradients of f. Prove that if you choose $\alpha_k = \rho / k^\beta$ for $\beta \ge 1/2$, then $f_{\text{best}} - f_* \le C N^{\beta - 1}$ for some $C > 0$. What is the best value of ρ?

3. Write down the dual ascent updates for the solving the linear program

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b \\ & x \in [0,1]^n \end{array} .$$

In your solution, *only dualize with respect to the equality constraint*. Then, at each step, you will minimize the resulting Lagrangian over $[0,1]^n$.

4. Compute the proximal point mappings associated with the following convex functions

(a) $P_1(x) = \frac{1}{2} \min_{\|z\|_2 \le B} \|x - z\|_2^2$

(b) $P_2(x) = -\sum_{k=1}^n \log(x_k)$

(c) $P_3(x) = \mathbb{I}_{[-1,1]^d}(x)$. (The indicator function for the set $[-1,1]^d$.