Compressed Wideband Spectrum Sensing for Whitespace Networking

Vishnu Vardhan
ECE Dept
Problem Description

- Spectrum is heavily unutilized in many bands (0-6 GHz)
- Whitespace Networking: Cognitive Radios opportunistically exploiting these spectral holes. How?
- UHF Spectrum Map (7th floor, CS dept, UW-Madison)

- Each channel is 6 MHz wide, only 5 out of 40 channels in UHF band are occupied
Compressed Wideband Sensing

- Currently band by band sensing is employed which is time consuming
- Need high rate ADC to sense wideband which is prohibitive
- The Edge Spectrum is compressible hence Compressed Wideband Sensing
A1) Compressed Sensing Architecture

- Proposed by **Yvan Polo, Ying Wang et al**, *Compressive wide-band spectrum sensing* ICASSP, 2009

AIC can be visualized as Nyquist rate sampling ($x_{N \times 1}$) followed by Compressed sensing ($\Phi_{M \times N} x_{N \times 1}$)

\[
y = \Phi x
\]

\[
r_y = \Phi r_x
\]

where $\overline{\Phi}$ is related to $\Phi$
Edge Spectrum is given by
\[ z = \Gamma F W r_x = \Gamma F W (\Phi^{-1} r_y) \]
where \( W = \text{Wavelet Smoothing} \)
\( F = \text{Fourier Transform} \)
\[ \Gamma = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \text{ i.e the 1st order difference} \]

\[ \Rightarrow r_y = \Phi (\Gamma F W)^{-1} z = \Phi G z \]

Recovery:
\[ z = \text{argmin} \| z \|_I \quad \text{s.t.} \quad r_y = (\Phi G) z \]
Samples Required

• Need $M > s \log(n/s)$
• Can we do better?
• In a given location, the Support of TV bands is static, only the Support of MICs is varying, that too slowly
• During initial sensing we try to accurately gauge $K = \{\text{support of TV bands}\}$, and use this knowledge in subsequent sensing
• We explore two approaches in this respect
A2) Modified l-1 recovery


- Recovery:

\[
x = \arg \min_x \| x_{K^c} \|_1 \quad \text{s.t.} \quad y = \Phi x
\]

where \( K \) is known support and \(| K | = k \)

Requires \( m > c'(u \log(ne/u) + (k + u) \log(9/\delta)) \)

\[
\sim O(u \log(n/u) + k)
\]

\[
\sim f(u) + k
\]
## Exact Reconstruction

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>l-0 version</strong></td>
<td>( \delta_{2s} &lt; 1 )</td>
<td>( \delta_{k+2u} &lt; 1 )</td>
</tr>
</tbody>
</table>

|               | \( \delta_s + \delta_{2s} + \delta_{3s} < 1 \) | \( 2\delta_{2u} + \delta_{3u} + \delta_k + \delta^2_{k+u} + 2\delta^2_{k+2u} < 1 \) |
| **l-1 version** |                                    |                        |
A3) Selective Acquisition

- What if I don’t care about the Known Support i.e. filter out the Known Support during signal acquisition

\[
\text{Choose } \overline{\Phi} = \Phi P_{K^\perp} = \Phi (I - \Psi_K (\Psi_K^\prime \Psi_K)^{-1} \Psi_K^\prime) \\
y = \overline{\Phi} x = \overline{\Phi} (x_K + x_{K^\perp}) = \Phi P_{K^\perp} (x_K + x_{K^\perp}) = \Phi P_{K^\perp} x_{K^\perp} = \Phi \tilde{x}
\]

Requires \( m > c'(u \log(ne/u) + u \log(9/\delta)) \)
\[\sim O(u \log(n/u))\]
Performance of the 3 schemes
Total Channels =9, DTVs=3, Analog=1,  M=0.4N, Known = 19,20,23, Unknown=26
Normalised MSE vs m/n
B) Compressed Detection

• In the previous approaches we first recover the Edge Spectrum and pose the detection problem on it.
• In this section we avoid the recovery phase and pose the detection problem directly on the compressed samples.

\[
\text{At Rx : } y = \Phi \left( \sum_{i=1}^{K} x_i + n \right)
\]

where \( x_i \)'s are orthogonal

Detection of Known Sparse Signal (K=1)

\[ H_0 : y = \Phi n \]
\[ H_1 : y = \Phi (x + n) \]
\[ x \in \mathbb{R}^N, \ n \sim N(0, \sigma^2 I_N), \ \Phi \text{ is fixed} \]

\[ H_0 : y \sim N(0, \sigma^2 \Phi \Phi^T), \quad H_1 : y \sim N(\Phi s, \sigma^2 \Phi \Phi^T) \]

Log Likelihood Ratio test is:

\[ \Lambda(y) = \log \left( \frac{f_1(y)}{f_0(y)} \right) \]
\[ H_0 \quad \sim \quad H_1 \]

which simplifies to

\[ t := y^T (\Phi \Phi^T)^{-1} \Phi s \quad H_1 \quad \sim \quad H_0 \]
Detection of Known Sparse Signal (K=1)

\[ H_0 : t \sim N(0, \sigma^2 \| P_{\Phi^T} s \|^2) , \]
\[ H_1 : t \sim N(\| P_{\Phi^T} s \|^2, \sigma^2 \| P_{\Phi^T} s \|^2) \]

where \( P_{\Phi^T} = \Phi^T (\Phi \Phi^T)^{-1} \Phi \)

we set \( P_F = \alpha \) resulting in
\[ P_D(\alpha) = Q(Q^{-1}(\alpha) - \| P_{\Phi^T} s \| / \sigma) \]

We know that if \( \sqrt{N/M} P_{\Phi^T} \) provides a \( \delta \) – stable embedding of \( S \), then

for any \( s \in S \), \( (1 - \delta) \frac{M}{N} \| s \|^2 \leq \| P_{\Phi^T} s \|^2 \leq (1 + \delta) \frac{M}{N} \| s \|^2 \)

Thus \( P_D(\alpha) \approx Q(Q^{-1}(\alpha) - \sqrt{M/N} \sqrt{\text{SNR}}) \)

• Since \( M < N \), Compressed Detector has lower Detection Probability, for e.g. if \( M = 0.5N \) then performance loss is 3 dB in SNR
Detection of Known Sparse Signal (K=1)

Upon further simplification it can be shown that

$$P_D(\alpha) \geq 1 - c_2 e^{-c_1 M/N}$$

- Thus for fixed SNR and N the probability of detection approaches 1 exponentially fast with M

- Some Results (Channel = 19, 500-506 MHz, SNR~ 20-25 dB)
Different ‘Phi’s, fixed ‘m’ and ‘Threshold’
Known Signals: K>1

For the \( j^{th} \) channel

Detection Rule is \( t_j := y^T (\Phi \Phi^T)^{-1} \Phi s_j \)

The test statistic, \( t_j = (\Phi (\sum_{i=1}^K x_i + n))^T (\Phi \Phi^T)^{-1} \Phi s_j \)

\[
= x_j^T P_{\Phi^T} s_j + (\sum_{i=1, \neq j}^K x_i)^T P_{\Phi^T} s_j + n^T P_{\Phi^T} s_j
\]

\[
= x_j^T P_{\Phi^T} s_j + \sum_{i=1, \neq j}^K < P_{\Phi^T} x_i, P_{\Phi^T} s_j > + n^T P_{\Phi^T} s_j
\]

\[
= \tilde{s} + \tilde{i} + \tilde{n}
\]
\[ H_{0,j} : t_j \sim N(0, \sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j), \]
\[ H_{1,j} : t_j \sim N(\| P_{\Phi^T} s_j \|^2, \sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j) \]

where \( I_j = E[\tilde{i}^2] \) is the interference power from other transmissions

\[ P_{D,j}(\alpha) = Q \left( Q^{-1}(\alpha) - \frac{\| P_{\Phi^T} s_j \|^2}{\sqrt{\sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j}} \right) \]

Thus Interference further degrades the detector performance
Unknown Signals

For $j$th channel

Detection Rule is $t_j := y^T (\Phi \Phi^T)^{-1} \tilde{y}_j \overset{H_1}{\underset{H_0}{>}} \gamma_j$

- It is an energy detector and its performance takes another 3 dB loss in both normal and compressed cases.

- Not enough data available for wideband case (ie Multi Primary Users)
Compressed Spectrum Detection

- Step 1: UHF has about 40 TV bands with fixed bandwidth (6 MHz) and frequencies out of which only few are occupied. Identify the unused bands.
- Step 2: MICs can be present at any frequencies in the unused bands and have a bandwidth of 200 kHz. Divide the unused spectrum into narrow bands to detect them.

Finally need to compare the Detection Performance and Running Time of ‘Recovery+Detection’ vs ‘Compressed Detection’ vs ‘Uncompressed Detection’
Thank You