Compressed Wideband Spectrum Sensing for Whitespace Networking

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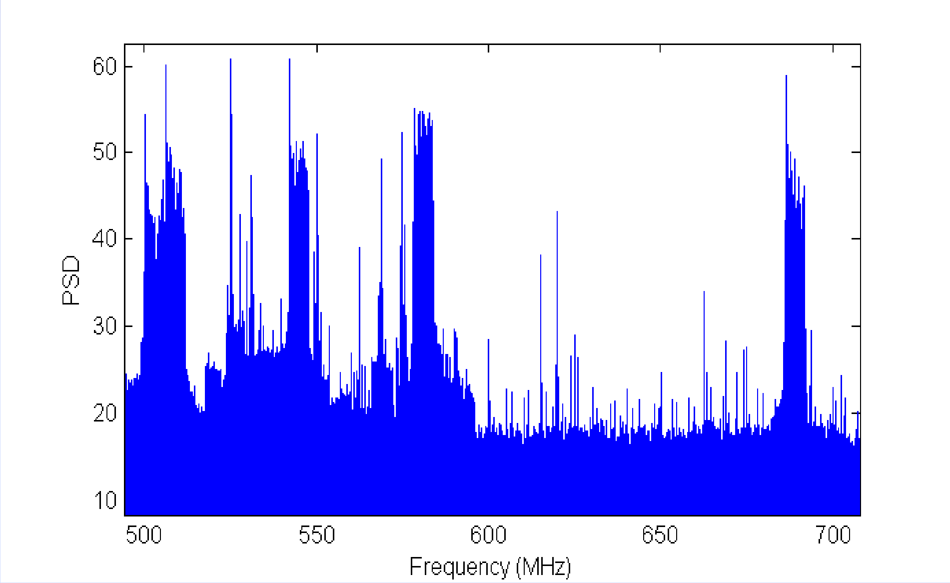
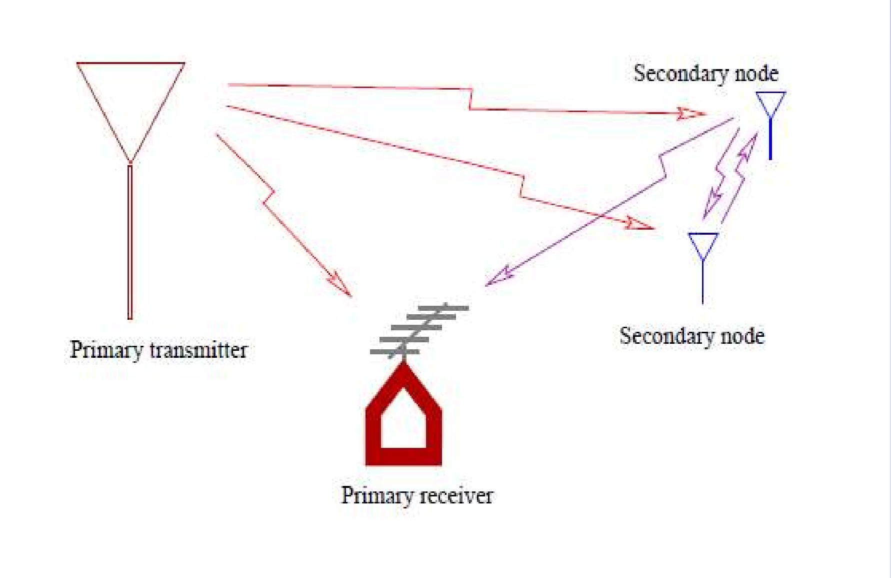
Problem Description

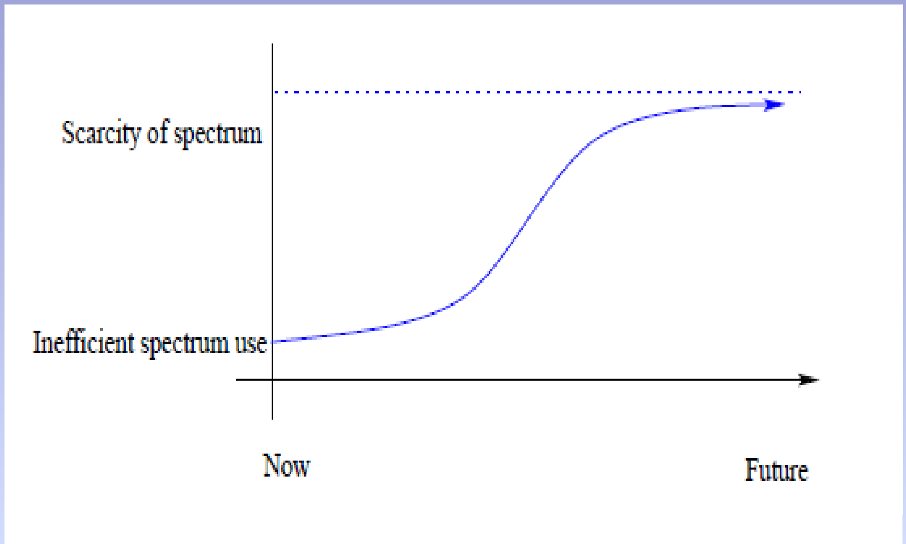
- Spectrum is heavily unutilized in many bands (0-6 GHz)
- Whitespace Networking: Cognitive Radios opportunistically exploiting these spectral holes. How ?
- UHF Spectrum Map (7th floor, CS dept, UW-Madison)





- Each channel is 6 MHz wide, only 5 out of 40 channels in UHF band are occupied

Compressed Wideband Sensing

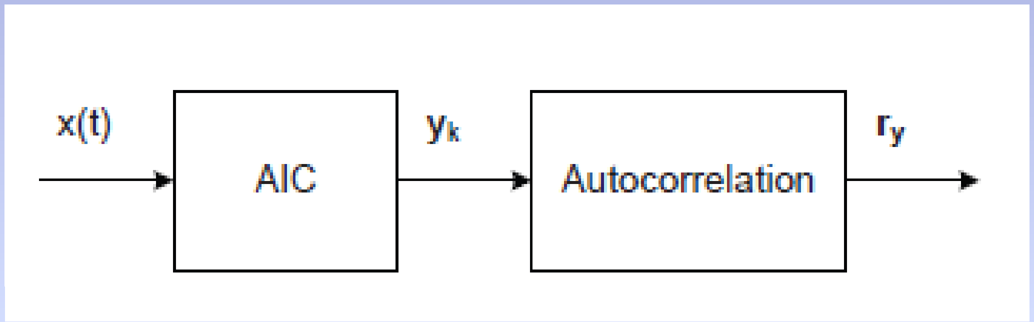


[Borrowed from Anant Sahai's DySPAN '05Tutorial]

- Currently band by band sensing is employed which is time consuming
- Need high rate ADC to sense wideband which is prohibitive
- The Edge Spectrum is compressible hence Compressed Wideband Sensing

A1) Compressed Sensing Architecture

- Proposed by ***Yvan Polo, Ying Wang*** *et al*, *[Compressive wide-band spectrum sensing]* *ICASSP, 2009*



AIC can be visualized as Nyquist rate sampling ($x_{N\times 1}$)

followed by Compressed sensing ($\Phi_{M\times N}\, x_{N\times 1}$)

$$y = \Phi x$$

$$r_y = \overline{\Phi} r_x$$

where $\overline{\Phi}$ is related to Φ

Edge Spectrum is given by

$$z = \Gamma F W r_x = \Gamma F W(\overline{\Phi}^{-1} r_y)$$

where W = Wavelet Smoothing

F = Fourier Transform

$$\Gamma = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ 0 & \cdots & -1 & 1 \end{bmatrix}$$ i.e the 1st order difference

$$\Rightarrow r_y = \overline{\Phi}(\Gamma F W)^{-1} z = \overline{\Phi} G z$$

Recovery :

$$\boldsymbol{z = \underset{z}{argmin} \| z \|_1 \ \ s.t. \ \ r_y = (\overline{\Phi} G) z}$$

Samples Required

- Need $M > s \log(n/s)$
- Can we do better ?
- In a given location, the Support of TV bands is static, only the Support of MICs is varying, that too slowly
- During initial sensing we try to accurately gauge K = {support of TV bands}, and use this knowledge in subsequent sensing
- We explore two approaches in this respect

A2) Modified l-1 recovery

- Proposed by ***Namrata Vaswani and Wei Lu,*** *Modified-CS: Modifying Compressive Sensing for Problems with Partially Known Support, to appear in IEEE Trans. Signal Processing,2010.*

- Recovery:

$$x = \arg\min_x \| x_{K^c} \|_1 \quad \text{s.t.} \quad y = \Phi x$$

$$\text{where K is known support and } |K| = k$$

$$\text{Requires} \;\; m > c'\left(u \log(ne/u) + (k+u)\log(9/\delta)\right)$$

$$\sim O(u \log(n/u) + k)$$

$$\sim f(u) + k$$

Exact Reconstruction

	Normal	Modified
l-0 version	$\delta_{2s} < 1$	$\delta_{k+2u} < 1$
l-1 version	$\delta_s + \delta_{2s} + \delta_{3s} < 1$	$2\delta_{2u} + \delta_{3u} + \delta_k + \delta^2_{k+u} + 2\delta^2_{k+2u} < 1$

A3) Selective Acquisition

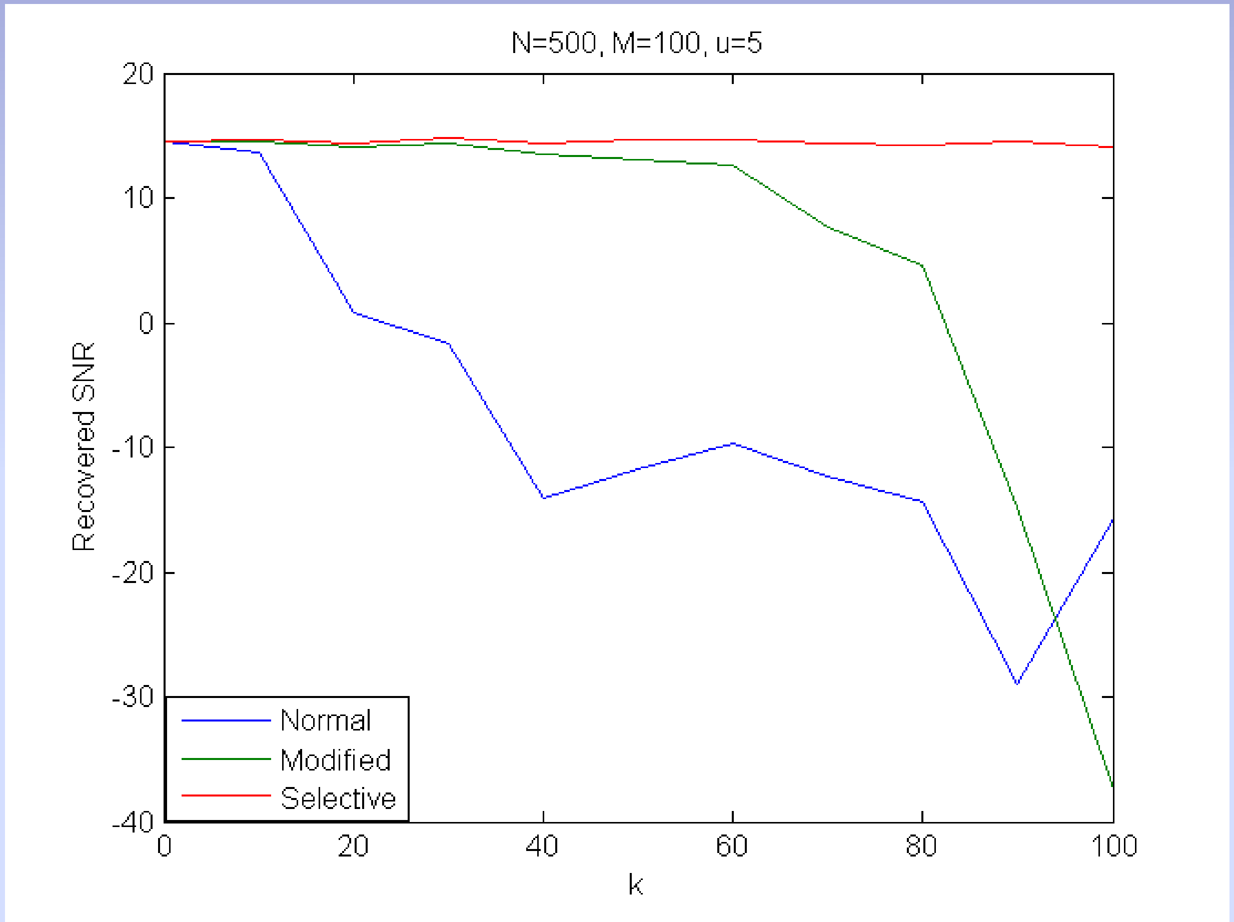
- What if I don’t care about the Known Support i.e. filter out the Known Support during signal acquisition

$$Choose \quad \overline{\Phi} = \Phi P_{K^{\perp}} = \Phi(I - \Psi_K(\Psi_K{}'\Psi_K)^{-1}\Psi_K{}')$$

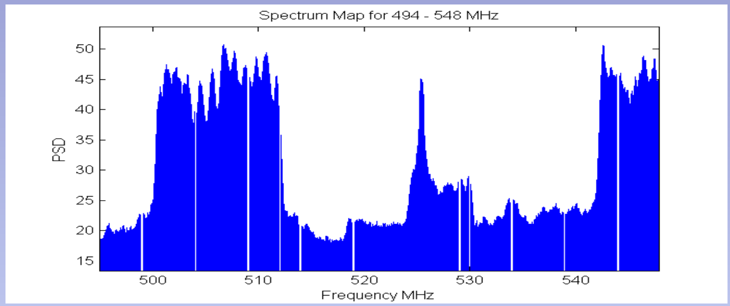
$$y = \overline{\Phi}x = \overline{\Phi}(x_K + x_{K^{\perp}}) = \Phi P_{K^{\perp}}(x_K + x_{K^{\perp}}) = \Phi P_{K^{\perp}} x_{K^{\perp}} = \Phi\tilde{x}$$

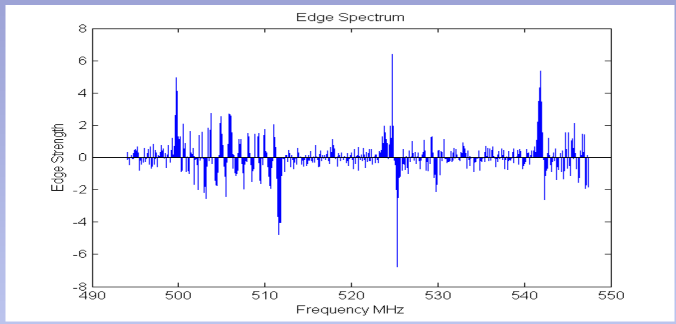
$$\text{Requires } \; m > c'\left(u\log(ne/u) + u\log(9/\delta)\right)$$
$$\sim O(u\log(n/u))$$

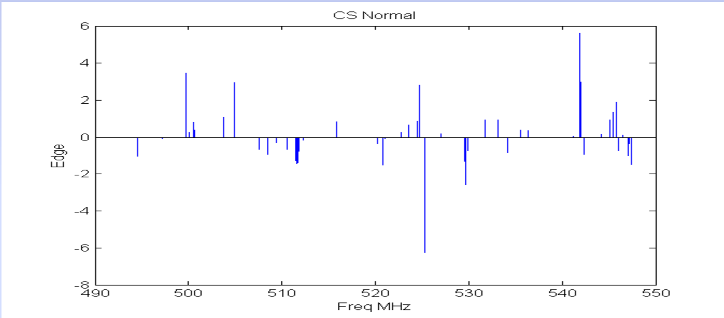
Performance of the 3 schemes

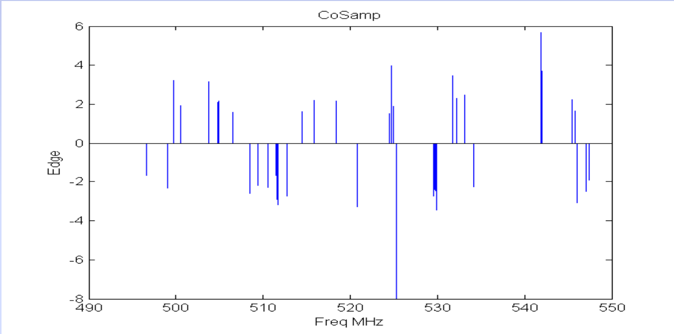


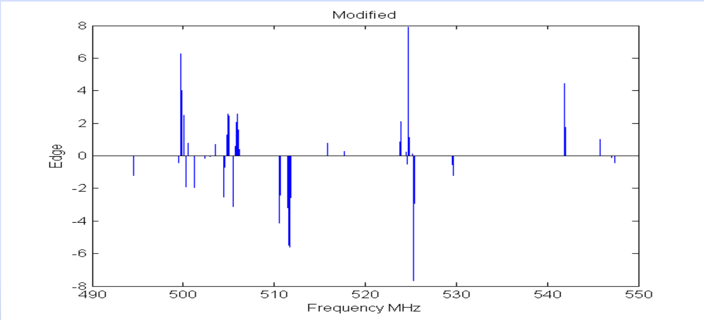
Results with Spectrum Data

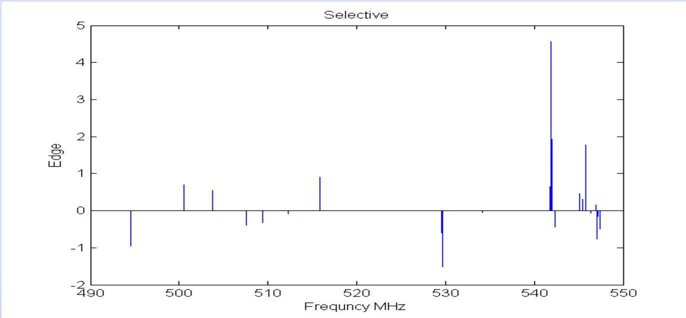






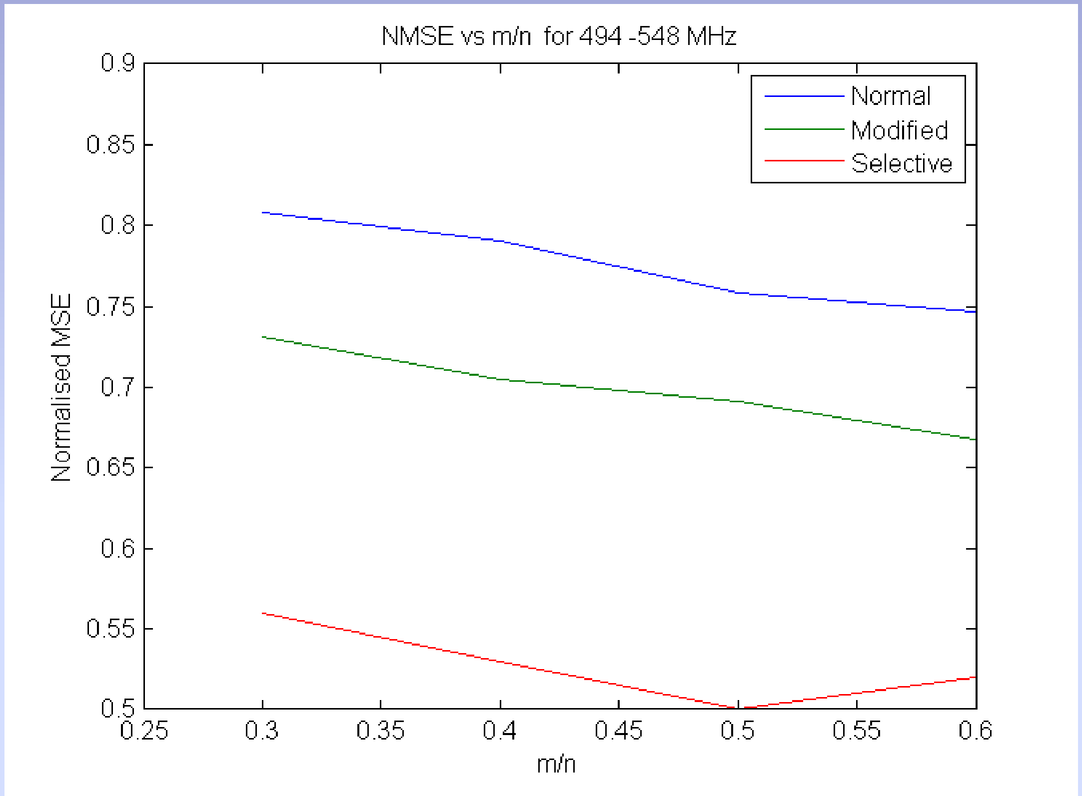






Total Channels =9, DTVs=3,Analog=1, M=0.4N, Known = 19,20,23, Unknown=26

Normalised MSE vs m/n



B) Compressed Detection

- In the previous approaches we first recover the Edge Spectrum and pose the detection problem on it.
- In this section we avoid the recovery phase and pose the detection problem directly on the compressed samples.

$$\text{At Rx} \quad : \quad y = \Phi\left(\sum_{i=1}^{K} x_i + n\right)$$

$$\text{where} \quad x_i\text{'s are orthogonal}$$

- Following approach (for K=1) developed by *M. A. Davenport, P. T. Boufounos, M. B. Wakin, and R. G. Baraniuk, "Signal processing with compressive measurements," IEEE Journal of Selected Topics in Signal Processing, April 2010.*

Detection of Known Sparse Signal (K=1)

$$H_0 : y = \Phi n$$

$$H_1 : y = \Phi(x + n)$$

$$x \in R^N,\ n \sim N(0, \sigma^2 I_N), \Phi \text{ is fixed}$$

$$H_0 : y \sim N(0, \sigma^2 \Phi\Phi^T), \qquad H_1 : y \sim N(\Phi s, \sigma^2 \Phi\Phi^T)$$

Log Likelihood Ratio test is :

$$\Lambda(y) = \log\left(\frac{f_1(y)}{f_0(y)}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

which simplifies to

$$t := y^T (\Phi\Phi^T)^{-1} \Phi s \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

Detection of Known Sparse Signal (K=1)

$$H_0 : t \sim N(0, \sigma^2 \| P_{\Phi^T} s \|^2),$$
$$H_1 : t \sim N(\| P_{\Phi^T} s \|^2, \sigma^2 \| P_{\Phi^T} s \|^2)$$
$$where \quad P_{\Phi^T} = \Phi^T (\Phi\Phi^T)^{-1}\Phi$$

$$\text{we set } P_F = \alpha \text{ resulting in}$$
$$P_D(\alpha) = Q\left(Q^{-1}(\alpha) - \| P_{\Phi^T} s \| / \sigma\right)$$

$$We\ know\ that\ if\ \sqrt{N/M}\ P_{\Phi^T}\ provides\ a\ \delta - stable\ embedding\ of\ S, then$$

$$for\ any\ s \in S, \quad (1-\delta)\frac{M}{N}\| s \|^2 \le \| P_{\Phi^T} s \|^2 \le (1+\delta)\frac{M}{N}\| s \|^2$$

$$Thus\ P_D(\alpha) \approx Q\left(Q^{-1}(\alpha) - \sqrt{M/N}\sqrt{SNR}\right)$$

- Since M<N, Compressed Detector has lower Detection Probability, for e.g. if M=0.5N then performance loss is 3 dB in SNR

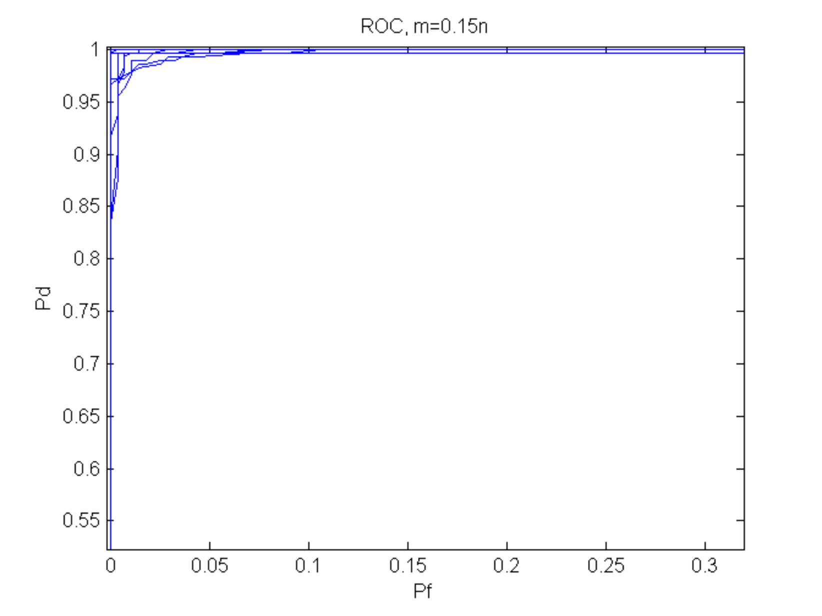
Detection of Known Sparse Signal (K=1)

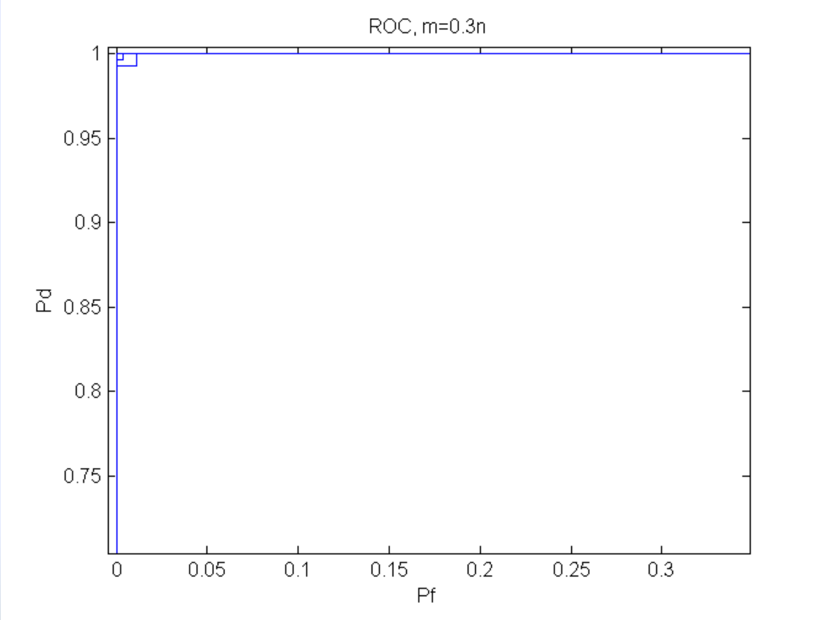
Upon further simplification it can be shown that

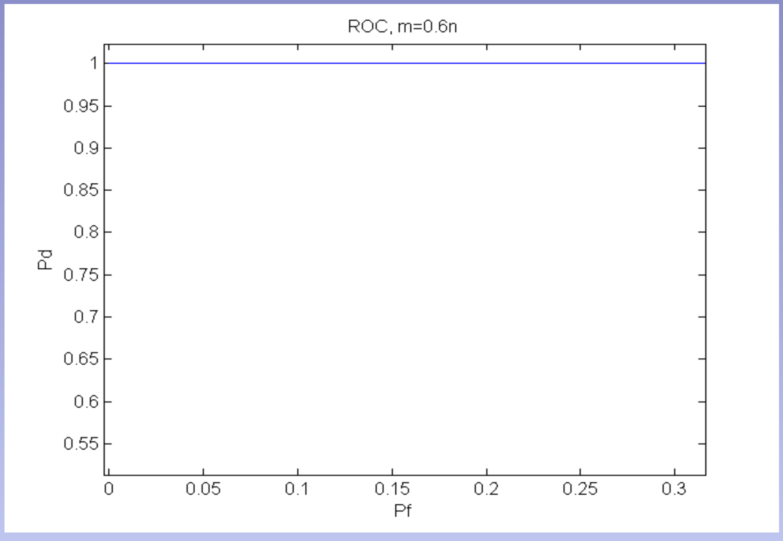
$$P_D(\alpha) \geq 1 - c_2 e^{-c_1 M/N}$$

- Thus for fixed SNR and N the probability of detection approaches 1 exponentially fast with M

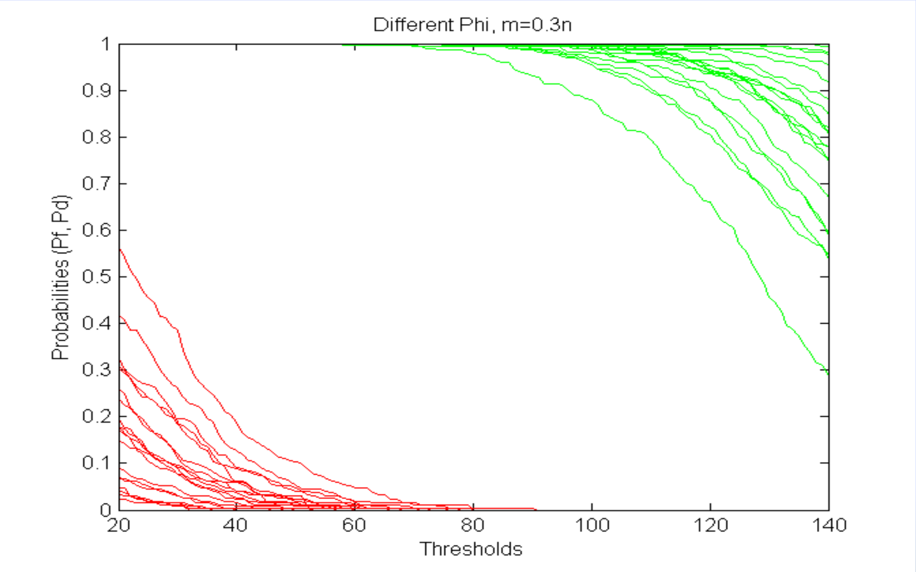
- Some Results (Channel = 19, 500-506 MHz, SNR~ 20-25 dB)







Different ‘Phi’s, fixed ‘m’ and ‘Threshold’



Known Signals: K>1

For j[th] channel

$$\text{Detection Rule is } t_j := y^T(\Phi\Phi^T)^{-1}\Phi s_j \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_j$$

$$\text{The test statistic, } t_j = (\Phi(\sum_{i=1}^{K} x_i + n))^T(\Phi\Phi^T)^{-1}\Phi s_j$$

$$= x_j^T P_{\Phi^T} s_j + (\sum_{i=1,\neq j}^{K} x_i)^T P_{\Phi^T} s_j + n^T P_{\Phi^T} s_j$$

$$= x_j^T P_{\Phi^T} s_j + \sum_{i=1,\neq j}^{K} < P_{\Phi^T} x_i, P_{\Phi^T} s_j > + n^T P_{\Phi^T} s_j$$

$$= \tilde{s} + \tilde{i} + \tilde{n}$$

$$H_{0,j} : t_j \sim N(0, \sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j),$$

$$H_{1,j} : t_j \sim N(\| P_{\Phi^T} s_j \|^2, \sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j)$$

where $I_j = E[\tilde{i}^2]$ is the interference power from other transmissions

$$P_{D,j}(\alpha) = Q\left(Q^{-1}(\alpha) - \frac{\| P_{\Phi^T} s_j \|^2}{\sqrt{\sigma^2 \| P_{\Phi^T} s_j \|^2 + I_j}} \right)$$

Thus Interference further degrades the detector performance

Unknown Signals

For j[th] channel

$$\text{Detection Rule is } t_j := y^T (\Phi\Phi^T)^{-1} \tilde{y}_j \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_j$$

- It is an energy detector and its performance takes another 3 dB loss in both normal and compressed cases.

- Not enough data available for wideband case (ie Multi Primary Users)

Compressed Spectrum Detection

- Step 1: UHF has about 40 TV bands with fixed bandwidth (6 MHz) and frequencies out of which only few are occupied. Identify the unused bands.
- Step 2: MICs can be present at any frequencies in the unused bands and have a bandwidth of 200 kHz. Divide the unused spectrum into narrow bands to detect them.

- Finally need to compare the Detection Performance and Running Time of ‘Recovery+Detection’ vs ‘Compressed Detection’vs ’Uncompressed Detection’

Thank You