## Announcements

■ Homework \#2 is assigned, it is due Monday, July 7 (1 week from today)

■ Project proposals are due today

■ Read Chapter 9 in AI: A Modern Approach for next time

| General Logic <br> * Logics are characterized by what they consider to be "primitives" |  |  |
| :---: | :---: | :---: |
| Logic | Primitives | Available Knowledge |
| Propositional | facts | true/false/unknown |
| First-Order | facts, objects, relations | true/false/unknown |
| Temporal | facts, objects, relations, times | true/false/unknown |
| Probability Theory | facts | degree of belief $0 \ldots 1$ |
| Fuzzy | degree of truth | degree of belief $0 \ldots 1$ |
|  |  | 3 |



## PL Review: Inference Rules

| Modus Ponens | $\frac{\alpha \Rightarrow \beta, \alpha}{\beta, \alpha}$ | Given the following knowledge base: |  |
| :---: | :---: | :---: | :---: |
| And-Elimination (AE): | $\frac{\alpha_{1} \wedge \alpha_{2} \wedge \ldots \wedge \alpha_{n}}{\alpha_{1}}$ | 1. $P$ |  |
| And-Introduction (AI): | $\frac{\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}}{\alpha_{1} \wedge \alpha_{2} \wedge \ldots \wedge \alpha_{n}}$ |  |  |
| Or-Introduction (OI): | $\frac{\alpha_{1}}{\alpha_{1} v \alpha_{2} v \ldots v \alpha_{n}}$ | 5. $(P \wedge R) \Rightarrow(S \vee W)$ |  |
| Double-Negation <br> Elimination (DNE): | $\frac{\neg \neg \alpha}{\alpha}$ | Prove $\mathbf{S}$ using natural deduction with these rules. |  |
|  | $\alpha \vee \beta, \neg \beta$ | 6. R | (MP: 1,2) |
| Unit Resolution (UR): | $\alpha$ | 7. 7 W | (MP: 3,6) |
| Resolution (R): | $\alpha \vee \beta, \neg \beta \vee\rangle$ | 8. $\mathrm{P} \wedge \mathrm{R}$ | (AI: 1,6$)$ |
|  | avp | 9. S $\vee$ W | (MP: 5,8) |
| deMorgan's Law (DML): | $\neg(-\alpha \vee \beta)$ | $10 . \mathrm{s}$ | (UR: 7,9) |
|  | ${ }^{\alpha \wedge}{ }^{\text {a }}$ |  |  |

## First-Order Logic

- Propositional logic has advantages
- Simple
- Inference is fast and easy
- But PL is limited in key ways
- Enumerate all facts as separate propositions
- No concept of individuals or objects
- Can't express relationships easily
- First-Order Logic is a logic language designed to remedy these problems


## FOL Syntax: Basic

- A term is used to denote an object in the world
- Constant: Bob, 2, Madison, Green, ...
- Variable: $\mathbf{x}, \mathbf{y}, \mathbf{a}, \mathrm{b}, \mathrm{c}, \ldots$
- Function(term ${ }_{1}, \ldots$, term $_{n}$ ):
- e.g. sqrt (9), distance (Madison, Chicago)
- Maps one or more objects to another object
- Can refer to an unnamed object: e.g. leftLLegof (John)
- Represents a user defined functional relation
- A ground term is a term with no variables


## FOL Syntax: Basic

- An atom is smallest expression to which a truth value can be assigned
- Predicate(term ${ }_{1}, \ldots$, term $_{n}$ ):
- e.g. teacher (Burr, You), lte (sqrt (2), sqrt (7))
- Maps one or more objects to a truth value
- Represents a user defined relation
- Term $_{1}=$ Term $_{2}$ :
- e.g. height (Burr) $=73 i n, 1=2$
- Represents the equality relation when two terms refer to the same object


## FOL Syntax: Basic

- A sentence represents a fact in the world that is assigned a truth value
- Atom
- Complex sentence using connectives: $\wedge \vee \neg \Rightarrow \Leftrightarrow$
- e.g. spouse (Burr, Nat) $\Rightarrow$ spouse (Burr, Nat)
- e.g. less $(11,22) \wedge$ less $(22,33)$
- Complex sentence using quantified variables: $\forall \exists$
- More about these in a bit...


## FOL Semantics: Assigning Truth

* The atom predicate $\left(\right.$ term $_{1}, \ldots$, term $\left._{n}\right)$ is true iff the objects referred to by term $_{1}, \ldots$, term $_{n}$ are in the relation referred to by the predicate
- What is the truth value for $\mathbf{s}(\mathrm{B}, \mathrm{N})$ ?
- Model:
- Objects: Burr, Nat, Thom, Mark
- Relation: spouse \{<Burr,Nat>,<Nat,Burr>\} - Interpretation:
- B means Burr, N means Nat, T means Thom, etc.
- $s\left(\right.$ term $_{1}$, term $\left._{2}\right)$ means term ${ }_{1}$ is the spouse of term


## FOL Syntax: Quantifiers

The universal quantifier: $\forall$

* Sentence holds true for all values of x in the domain of variable x
- Main connective typically $\Rightarrow$ forming if-then rules
- "All humans are mammals" in FOL becomes:
$\forall \mathbf{x}$ human ( $\mathbf{x}$ ) $\Rightarrow$ mammal ( $\mathbf{x}$ )
- Means if $x$ is a human then $x$ is a mammal


## FOL Syntax: Quantifiers

$\forall \mathbf{x}$ human ( $\mathbf{x}$ ) $\Rightarrow$ mammal ( $\mathbf{x}$ )

- Equivalent to the conjunction of all the instantiations of variable $x$ :
(human (Burr) $\Rightarrow$ mammal (Burr)) $\wedge$ (human (Nat) $\Rightarrow$ mammal (Nat)) $\wedge$ (human (Thom) $\Rightarrow$ mammal (Thom)) ^...


## FOL Syntax: Quantifiers

The existential quantifier: $\exists$

* Sentence holds true for some value of x in the domain of variable x
- Main connective typically $\wedge$
- "Some humans are male" in FOL becomes: $\exists \mathrm{x}$ human (x) $\wedge$ male ( x )
- Means $x$ is some human and $x$ is a male


## FOL Syntax: Quantifiers

■ Common mistake is to use $\wedge$ as main connective

- Results in a blanket statement about everything

■ For example: $\forall \mathbf{x}$ human ( $\mathbf{x}) \wedge$ mammal ( $\mathbf{x}$ ) (human (Burr) ^ mammal (Burr)) ^ (human (Nat) $\wedge$ mammal (Nat)) $\wedge$ (human (Thom) ^ mammal (Thom)) ^...

- But this means everything is human and a mammal!


## FOL Syntax: Quantifiers

$\exists \mathrm{x}$ human (x) $\wedge$ male (x)

- Equivalent to the disjunction of all the instantiations of variable $x$ :
(human (Burr) $\wedge$ male (Burr)) $\vee$
(human (Nat) ^ male (Nat)) $\vee$
(human(Thom) ^ male(Thom)) $\vee$...


## FOL Syntax: Quantifiers

■ Properties of quantifiers:

- $\forall \mathbf{x} \forall \mathbf{y}$ is the same as $\forall \mathbf{y} \forall \mathbf{x}$
$-\exists \mathbf{x} \exists \mathbf{y}$ is the same as $\exists \mathbf{y} \exists \mathbf{x}$

■ Why?

- $\forall \mathbf{x} \forall \mathbf{y}$ likes ( $\mathbf{x}, \mathrm{y}$ )
the active voice: "Everyone likes everyone."
- $\forall \mathbf{y} \forall \mathbf{x}$ likes ( $\mathbf{x}, \mathbf{y}$ )
the passive voice: "Everyone is liked by everyone."


## FOL Syntax: Quantifiers

- Properties of quantifiers:
- $\forall \mathbf{x} \exists \mathbf{y}$ is not the same as $\exists \mathbf{y} \forall \mathbf{x}$
$-\boldsymbol{\exists x} \forall \mathbf{y}$ is not the same as $\forall \mathbf{y} \exists \mathbf{x}$

■ Why?

- $\forall x \exists y$ likes ( $x, y$ )
"Everyone has someone they like."
$-\exists y \forall x$ likes ( $\mathbf{x}, \mathrm{y}$ )
"There is someone who is liked by everyone."


## FOL Syntax: Quantifiers

- Properties of quantifiers:
- $\forall x P(x)$ when negated is $\exists x \neg P(x)$
$-\exists \mathbf{x} \mathbf{P}(\mathbf{x})$ when negated is $\forall \mathbf{x} \neg \mathbf{P}(\mathbf{x})$
- Why?
- $\forall \mathbf{x}$ sleeps ( x )
"Everybody sleeps."
- $\exists \mathrm{x}$ नsleeps ( x ) negated: "Somebody doesn't sleep."


## Summary So Far

- Constants: Bob, 2, Madison, ...
- Variables: $\quad x, y, a, b, c, \ldots$
- Functions: Income, Address, Sqrt, ..
- Predicates: Teacher, Sisters, Even, Prime...

■ Connectives: $\wedge \vee \neg \Rightarrow \Leftrightarrow$

- Equality: =
- Quantifiers: $\forall \exists$


## FOL Syntax: Quantifiers

- Properties of quantifiers:
$-\forall \mathbf{x} \mathbf{P}(\mathbf{x})$ is the same as $\neg \boldsymbol{\exists x} \neg \mathrm{P}(\mathbf{x})$
$-\exists \mathbf{x} \mathbf{P}(\mathbf{x})$ is the same as $\neg \forall \mathbf{x} \neg \mathbf{P}(\mathbf{x})$

■ Why?

- $\forall \mathbf{x}$ sleep ( $\mathbf{x}$ )
"Everybody sleeps."
- $\neg \exists \mathrm{x}$ नsleep ( x )
double negative: "Nobody don't sleep."


## FOL Syntax: Basics

■ A free variable is a variable that isn't bound by a quantifier

- i.e. $\exists \mathrm{y}$ Likes $(\mathbf{x}, \mathrm{y}): \mathbf{x}$ is free, y is bound

■ A well-formed formula is a sentence where all variables are quantified (none are free)

## Summary So Far

- Term: Constant, variable, or function... denotes an object in the world (a ground term has no variables)
- Atom: Is smallest expression assigned a truth value - e.g. Predicate $\left(\right.$ term $_{1}, \ldots$, term $\left._{n}\right)$, term $_{1}=$ term $_{2}$
- Sentence: An atom, quantified sentence with variables, or complex sentence using connectives; assigned a truth value
- Well-Formed Formula (wff): A sentence where all variables are quantified


## Thinking in Logical Sentences

## Convert the following sentences into FOL:

■ "Bob is a fish."

- What is the constant?
- Bob
- What is the predicate?
- is a fish
- Answer: fish (Bob)

■ "Burr and Mark are grad students."
■ "Burr, Mark, or Nat is not a rat."

## Thinking in Logical Sentences

## Now let's think about quantification:

- "Burr likes everything."
- What is the constant?
- Burr
- How are they variables quantified?
- All/universal
- Answer: $\forall x$ likes (Burr, x)
- i.e. likes (Burr, IceCream) $\wedge$ likes (Burr, Nat) $\wedge$ likes (Burr, Armadillos) ^ ...
■ "Burr likes something."
■ "Somebody likes Burr."


## Thinking in Logical Sentences

## We can also have multiple quantifiers:

- "Somebody heard something."
- What are the variables?
- somebody and something
- How are they quantified?
- both are at least one/existential
- Answer: $\exists \mathbf{x}, \mathrm{y}$ heard $(\mathrm{x}, \mathrm{y})$

■ "Everybody heard everything."
■ "Somebody did not hear everything."

## Thinking in Logical Sentences

## We can also do this with relations:

■"America bought Alaska from Russia."

- What are the constants?
- America, Alaska, Russia
- What are the relations?
- bought
- Answer: bought (America, Alaska, Russia)

■ "Warm is between cold and hot."
■ "Burr and Nat are married."

## Thinking in Logical Sentences

- All
- Things: anything, everything, whatever
- Persons: anybody, anyone, everybody, everyone, whoever

■ Some (at least one)

- Things: something
- Persons: somebody, someone
- None
- Things: nothing
- Persons: nobody, no one


## Thinking in Logical Sentences

## Let's allow more complex quantified relations:

■ "All stinky shoes are allowed."

- How are ideas connected?
- being a shoe and being stinky implies that it is allowed
- Answer: $\forall \mathrm{x}$ shoe ( x ) $\wedge$ stinky ( x ) $\Rightarrow$ allowed ( x )

■ "No stinky shoes are allowed."

- Answer: $\neg \exists \mathrm{x}$ shoe ( x ) $\wedge$ stinky ( x ) $\wedge$ allowed ( x )
- The equivalent:
"Stinky shoes are not allowed."
- Answer: $\forall x$ shoe ( $x$ ) $\wedge$ stinky ( $x$ ) $\Rightarrow$ ᄀallowed ( $x$ )


## Thinking in Logical Sentences

## And some more complex relations:

■ "No one sees everything."

- What are the variables and quantifiers?
- nothing and everything
- not one (i.e. not existential) and all (universal)
- Answer: $\neg \exists \mathbf{x} \forall \mathrm{y}$ sees $(\mathrm{x}, \mathrm{y})$
- Equivalent:
"Everyone doesn't see something."
- Answer: $\forall \mathrm{x} \exists \mathrm{y}$ नsees $(\mathrm{x}, \mathrm{y}$ )

■"Everyone sees nothing."

- Answer: $\forall \mathrm{x} \neg \exists \mathrm{y}$ sees $(\mathrm{x}, \mathrm{y})$


## Thinking in Logical Sentences

We can throw in functions and equalities, too:
■ "Burr and Nat are the same age."

- Are functional relations specified?
- Are equalities specified?
- Answer: age (Burr) = age (Nat)

■ "There are exactly two shoes."

- Are quantities specified?
- Are equalities implied?
- Answer: $\exists \mathrm{x}$ ヨy shoe ( x$) \wedge$ shoe ( y$) \wedge \neg(\mathrm{x}=\mathrm{y}) \wedge$ $\forall z($ shoe $(z) \Rightarrow(x=z) \vee(y=z))$


## Thinking in Logical Sentences

## These are pretty tricky:

- "Assume $x$ is above $y$ if $x$ is directly on the top of $y$, or else there is a pile of one or more other objects directly on top of on another starting with $x$ and ending with $y$."
- Answer: $\forall \mathrm{x} \forall \mathrm{y}$ above $(\mathrm{x}, \mathrm{y}) \Leftrightarrow[\mathrm{OnTop}(\mathrm{x}, \mathrm{y}) \vee$

- President Lincoln: "You can fool some of the people all of the time, and you can fool all of the people some of the time, but you cannot fool all of the people all of the time!"


## Thinking in Logical Sentences

## And some really complex relations:

■ "Any good amateur can beat some professional."

- Lets break this down:
- $\forall x$ [ $(x$ is a good amateur $) \Rightarrow(x$ can beat some professional $)]$
- ( $x$ can beat some professional) is really:
$\exists y[(y$ is a professional $) \wedge(x$ can beat $y)]$
- $\forall x[(x$ is a good amateur $) \Rightarrow$ $\exists y[(y$ is a professional) $\wedge(x$ can beat $y)]$
- Answer: $\forall \mathbf{x}[\{\operatorname{amateur}(x) \wedge \operatorname{good}(x)\} \Rightarrow$
$\exists y$ \{professional $(\mathrm{y}) \wedge$ beat $(\mathbf{x}, \mathrm{y})\}$ ]
■ "Some professionals can beat all amateurs."
- Answer: $\exists x$ [professional (x) ^
$\forall y ~\{a m a t e u r(y) \Rightarrow$ beat $(x, y)\}]$


## Thinking in Logical Sentences

■ Interesting words: always, sometimes, never

- "Good people always have friends."
$\forall \mathbf{x}$ person ( $\mathbf{x}$ ) $\wedge \operatorname{good}(x) \Rightarrow \exists y(f r i e n d(x, y))$
- "Busy people sometimes have friends."
$\exists \mathrm{x}$ person( x ) ^ busy $(\mathrm{x}) \wedge \exists \mathrm{y}(\mathrm{friend}(\mathrm{x}, \mathrm{y}))$
_ "Bad people never have friends."
$\forall x$ person $(x) \wedge \operatorname{bad}(x) \Rightarrow \neg \exists y(f r i e n d(x, y))$


## First-Order Inference

- Recall that with PL, inference is pretty easy
- Enumerate all possibilities (truth tables)
- Apply sound inference rules on facts

■ But in FOL, we have concepts of variables, relations, and quantification

- This complicates things quite a bit!
- Next time, we'll discuss how inference procedures for first-order logic work

