RECURSION

CS302 – Introduction to CS302
University of Wisconsin – Madison
Lecture 16

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Recursive Methods

- A recursive method is a method that calls itself.
- Many difficult algorithmic problems have simple solutions that use recursion
- Recursive methods usually take a while to fully wrap your head around so let’s look at some examples

Factorials

- The following method computes the factorial of a number:

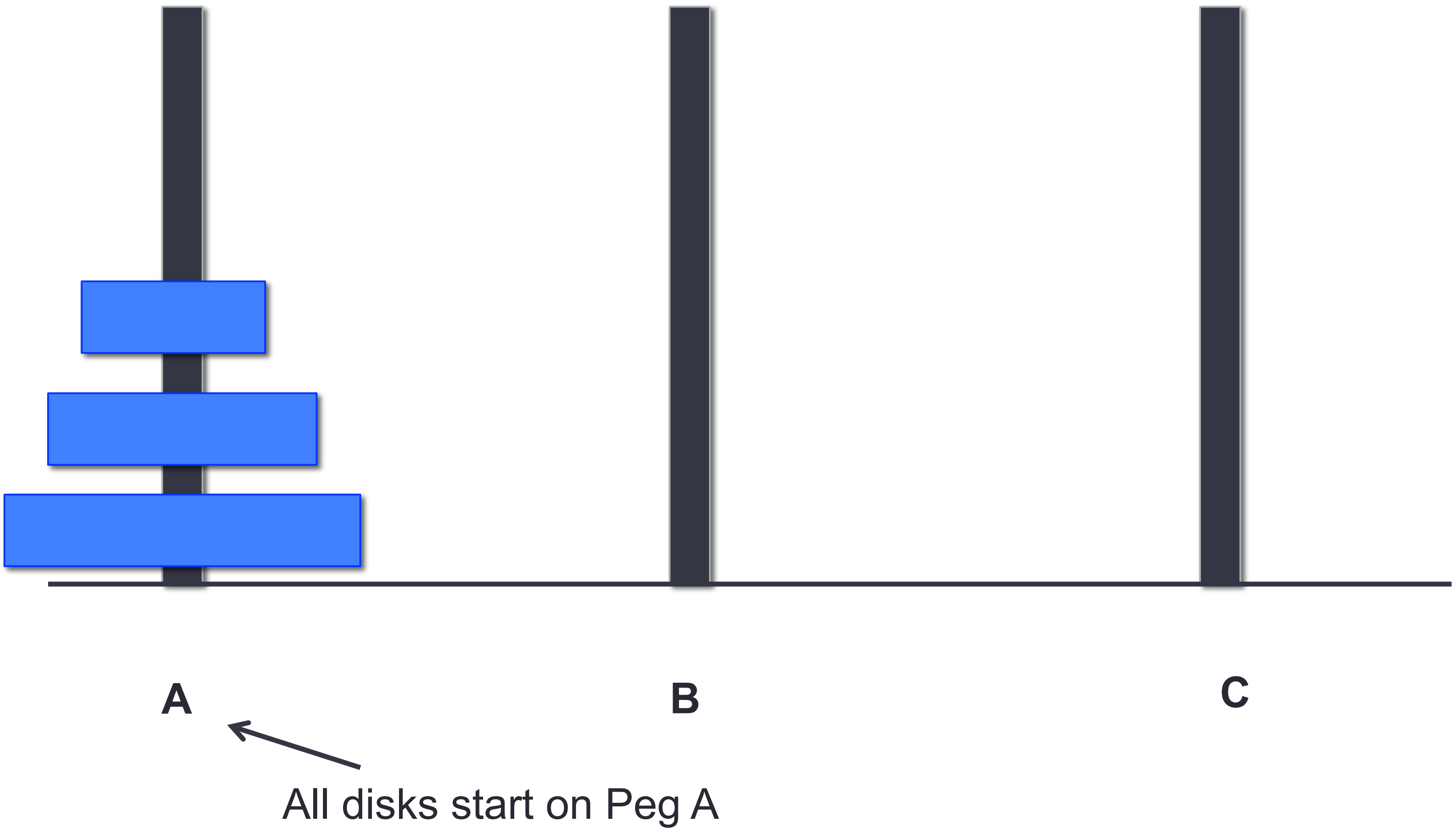
```
public static int factorial(int n)
{
    // stopping criteria
    if (n <= 1) return 1;

    // recursive call
    else return n * factorial(n - 1);
}
```

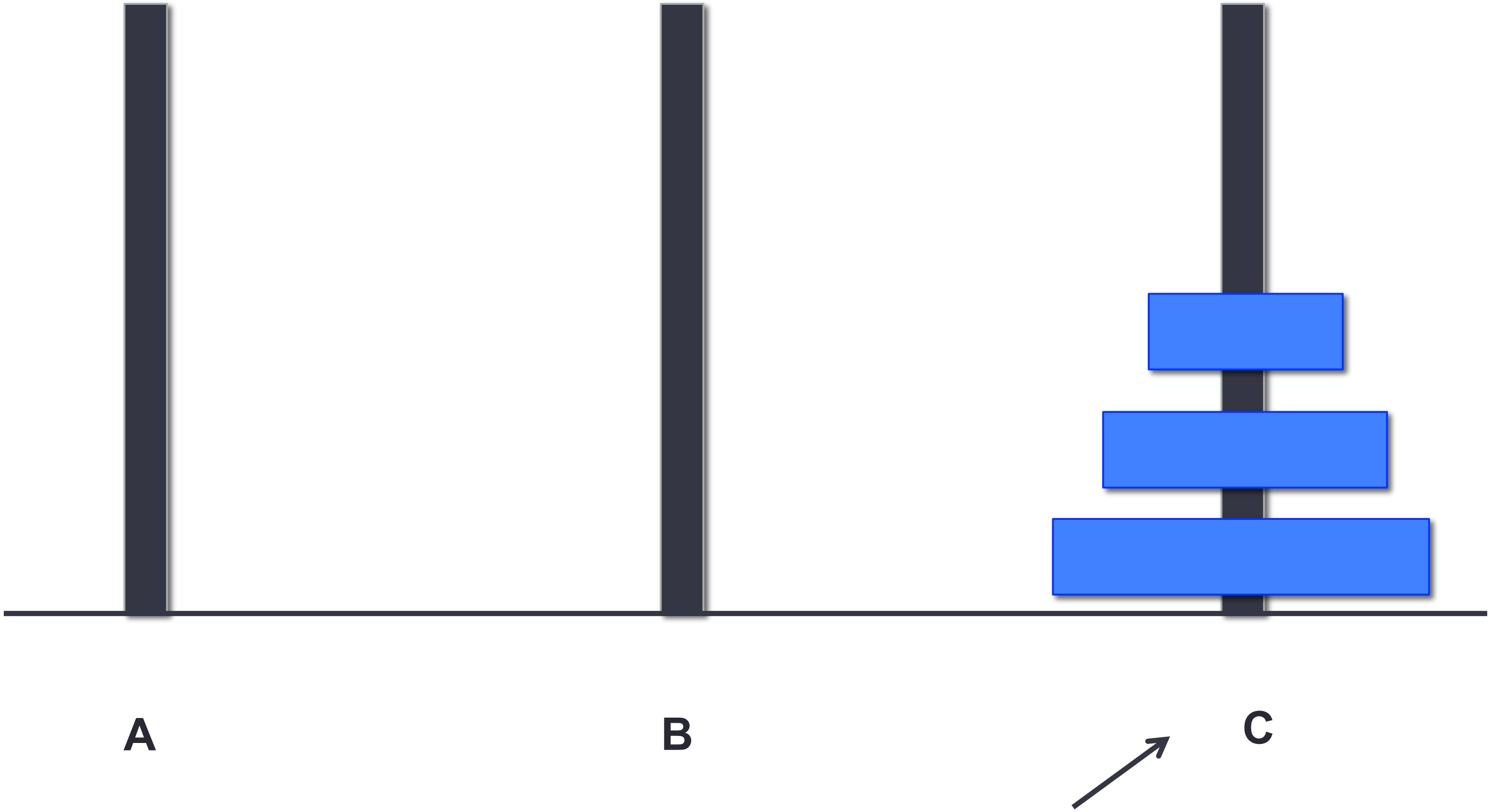
Tower of Hanoi

- Tower of Hanoi is a classic puzzle that has a simple recursive algorithmic solution
- The premise of the game is as follows:
  - We have 3 pegs (named A, B, and C) that we can accommodate disks
  - The game starts with some number of disks on peg A
  - The goal is to move the disks from A to C
  - HOWEVER, we can never put a larger disk on top of a smaller disk

Starting Condition

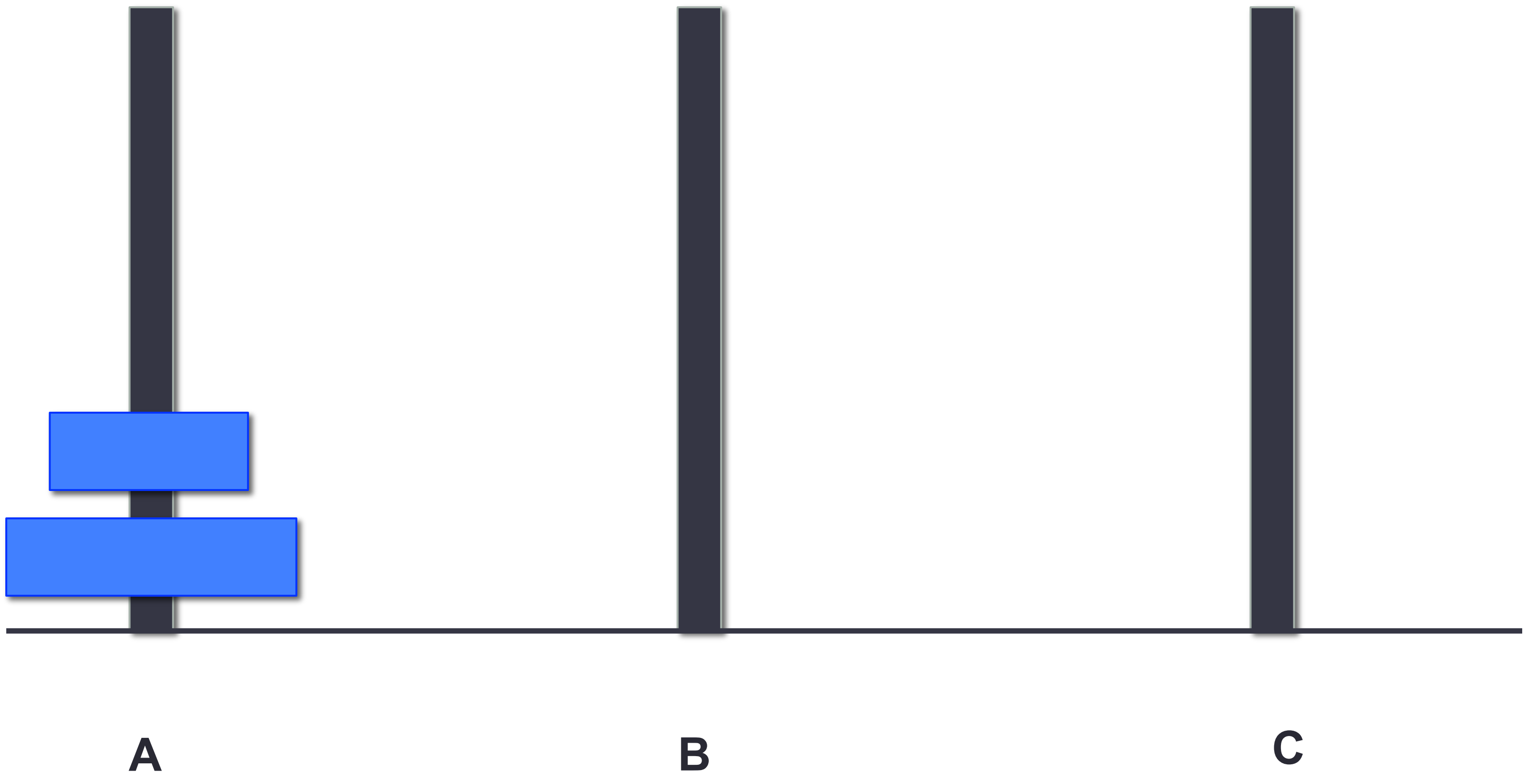


Victory Condition

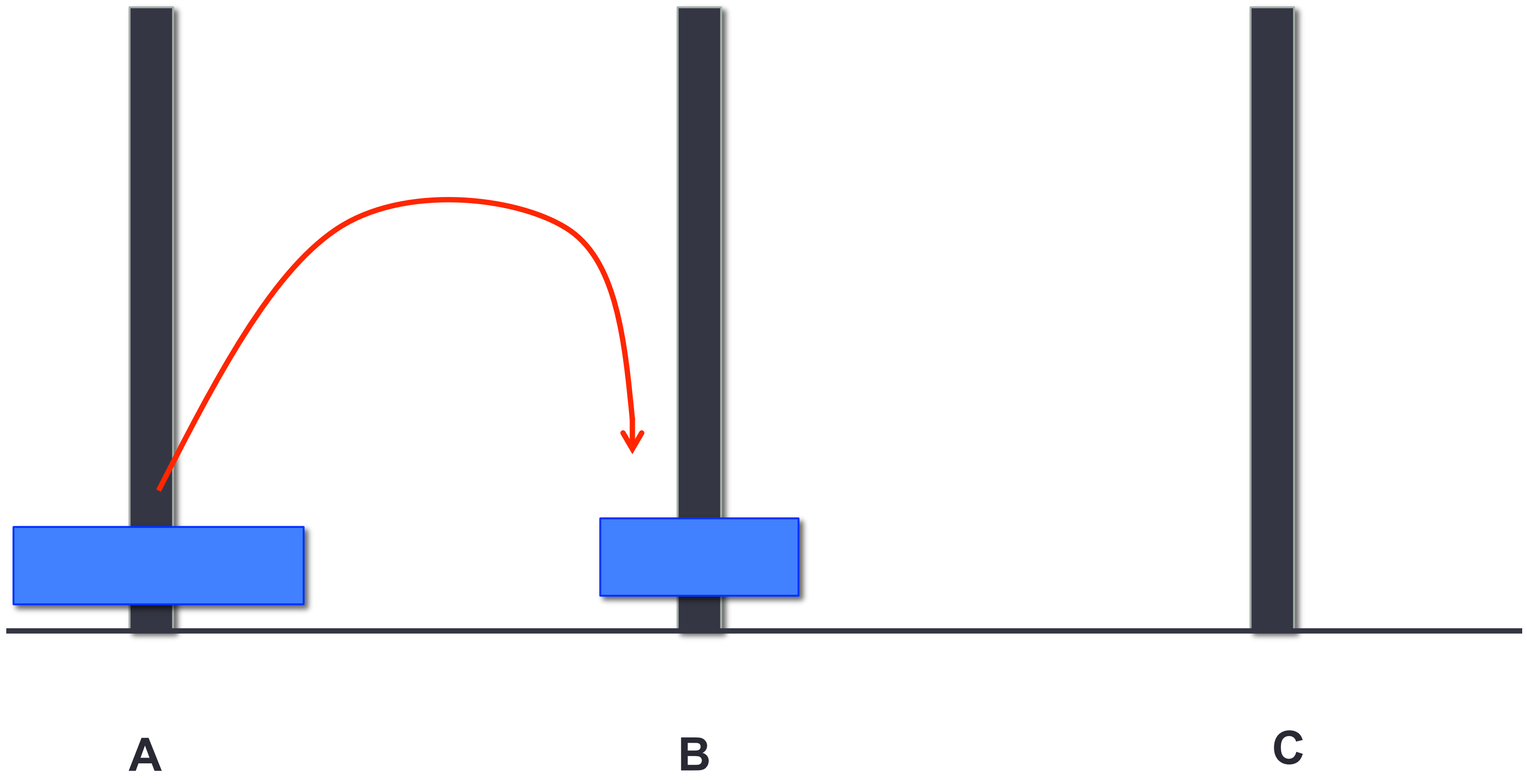


The goal is to move all disks to peg C

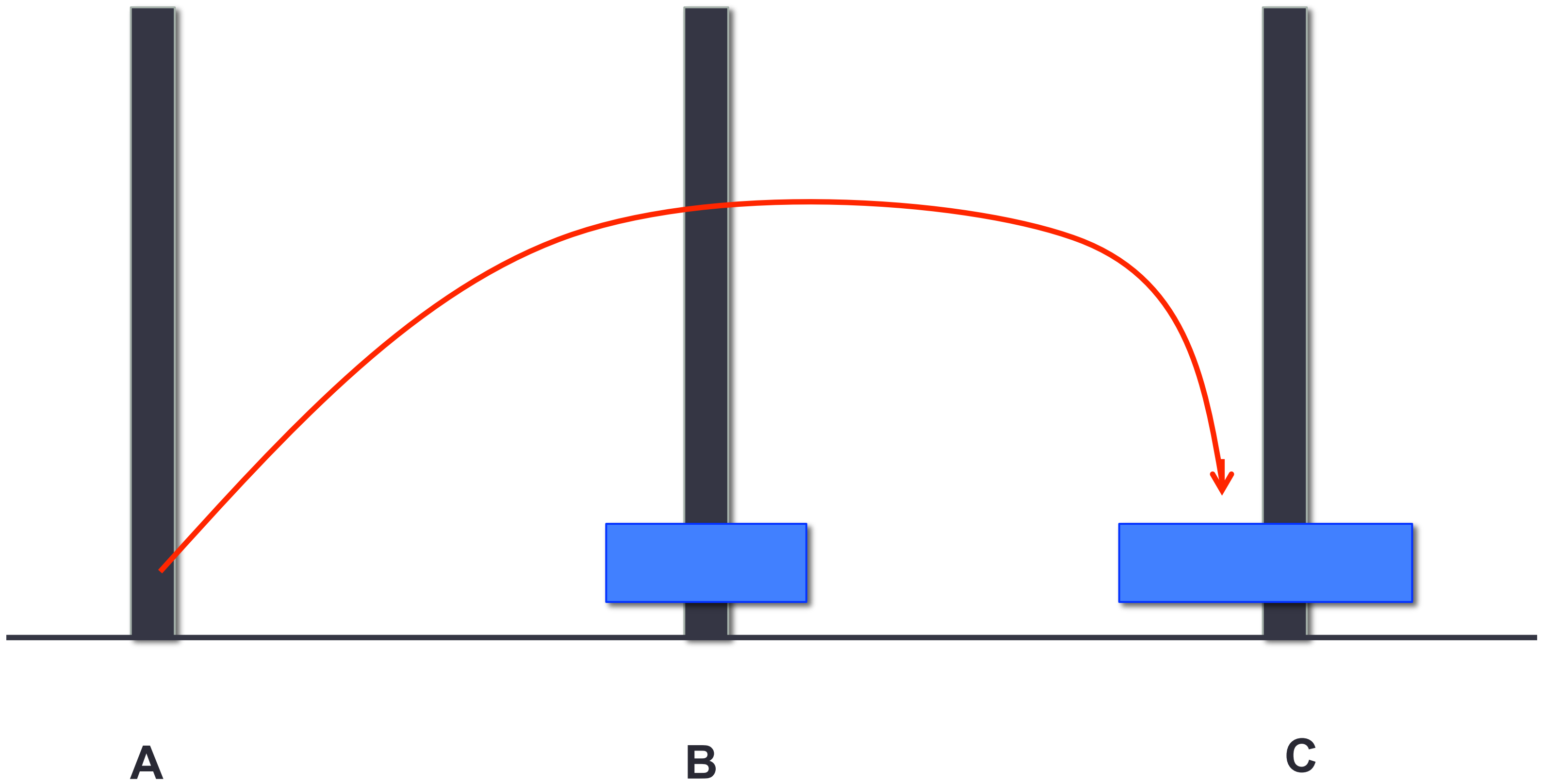
Let's First Solve It With 2 Disks



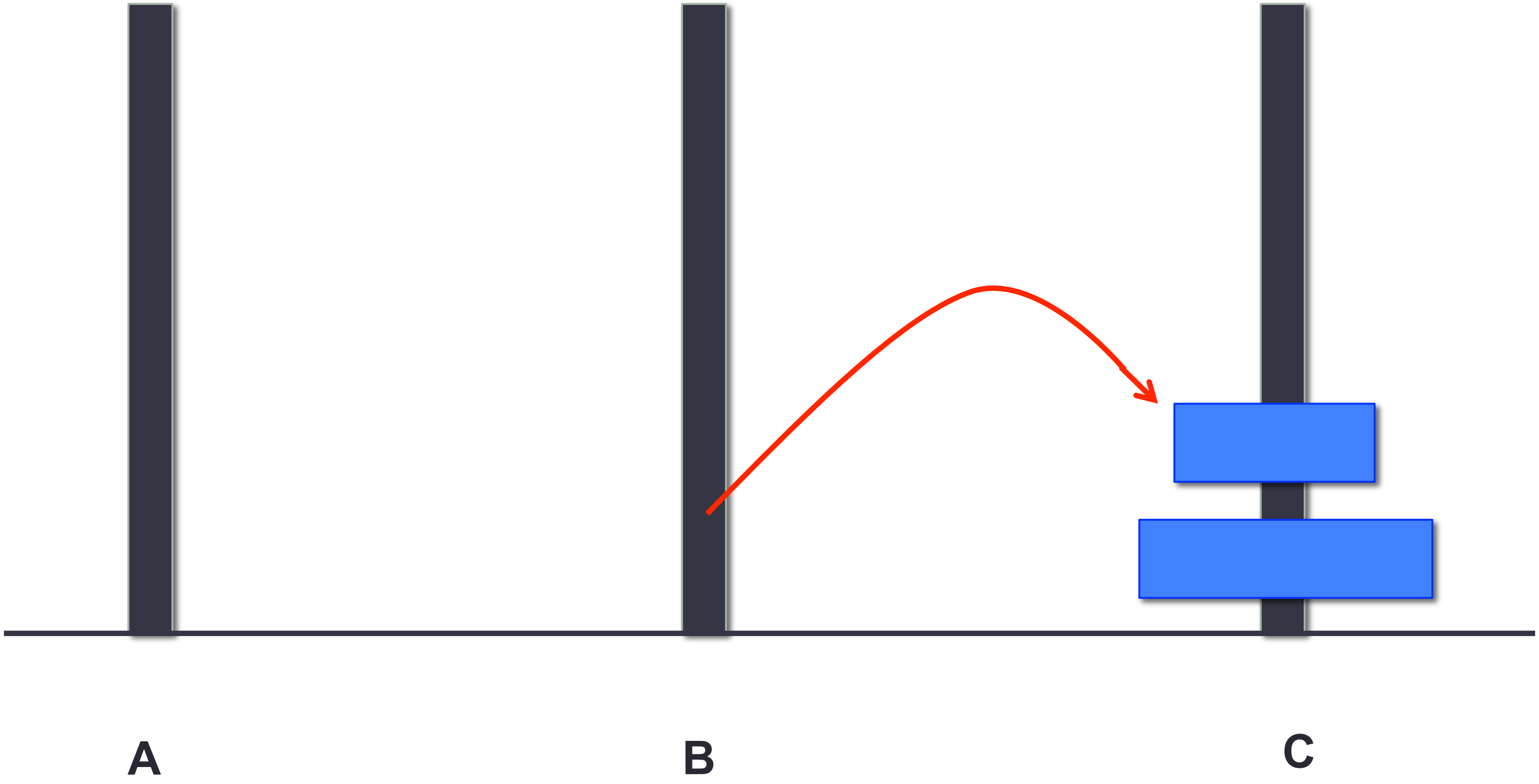
Step 2



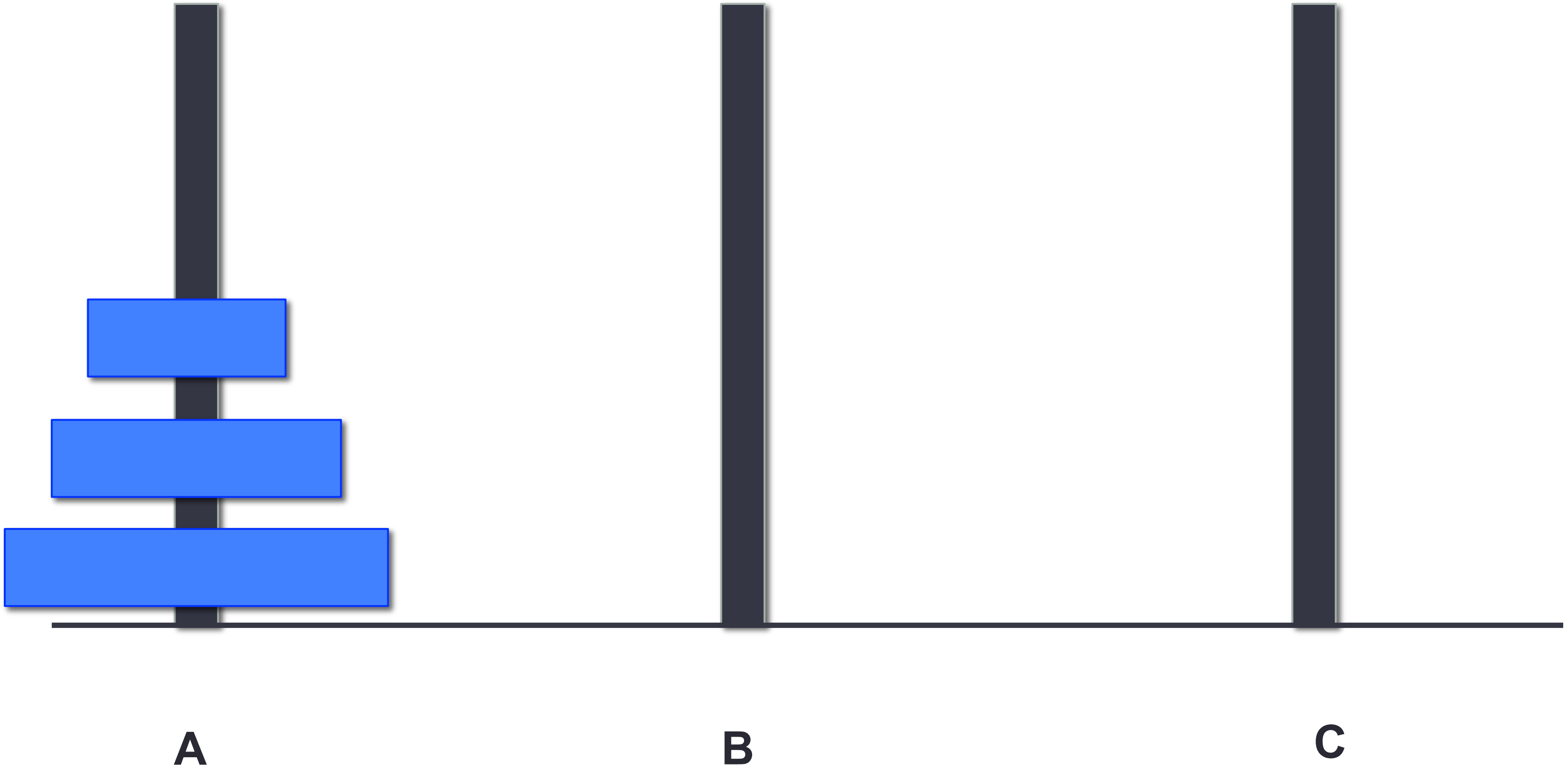
Step 3



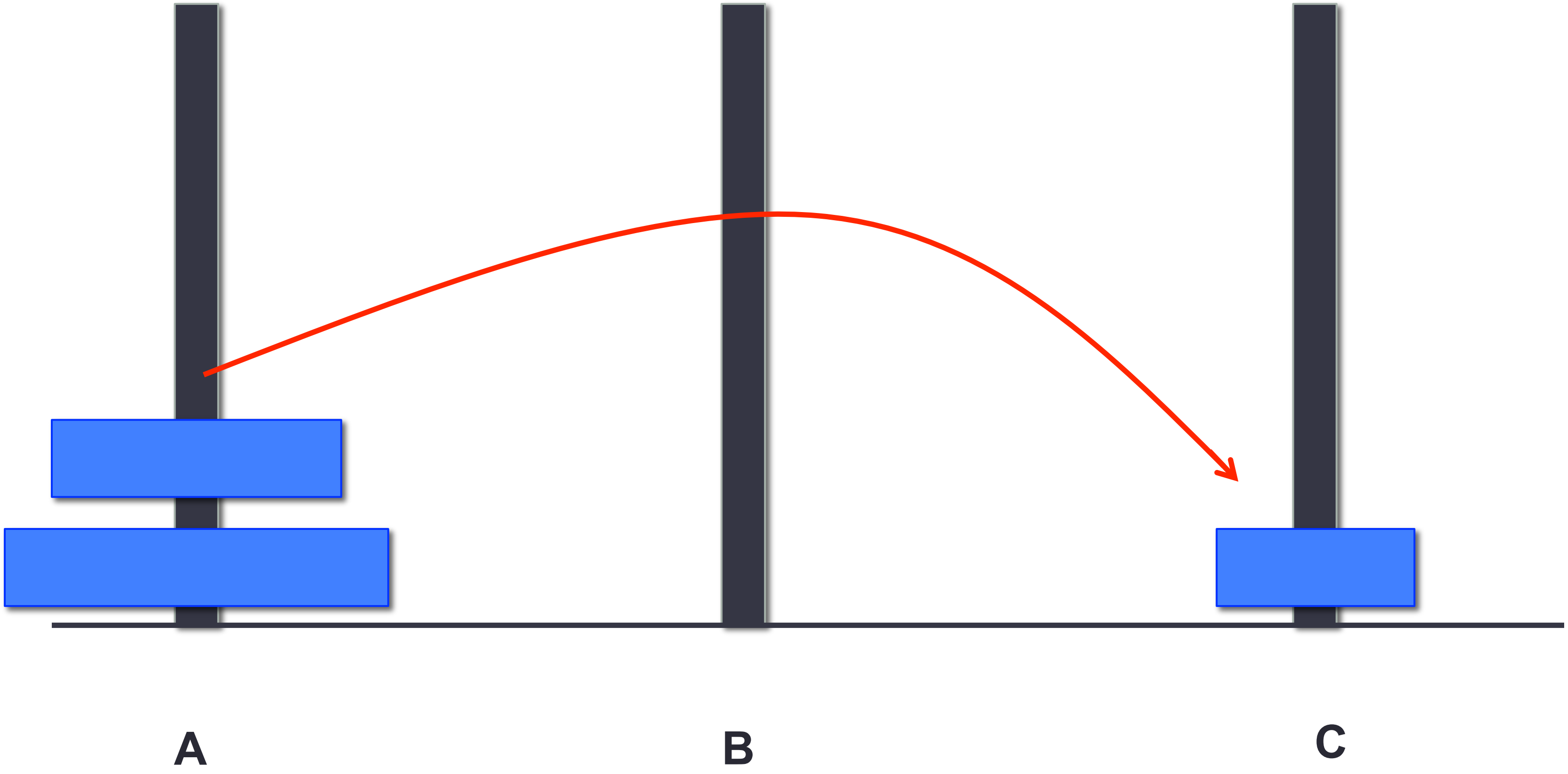
Step 4: Victory



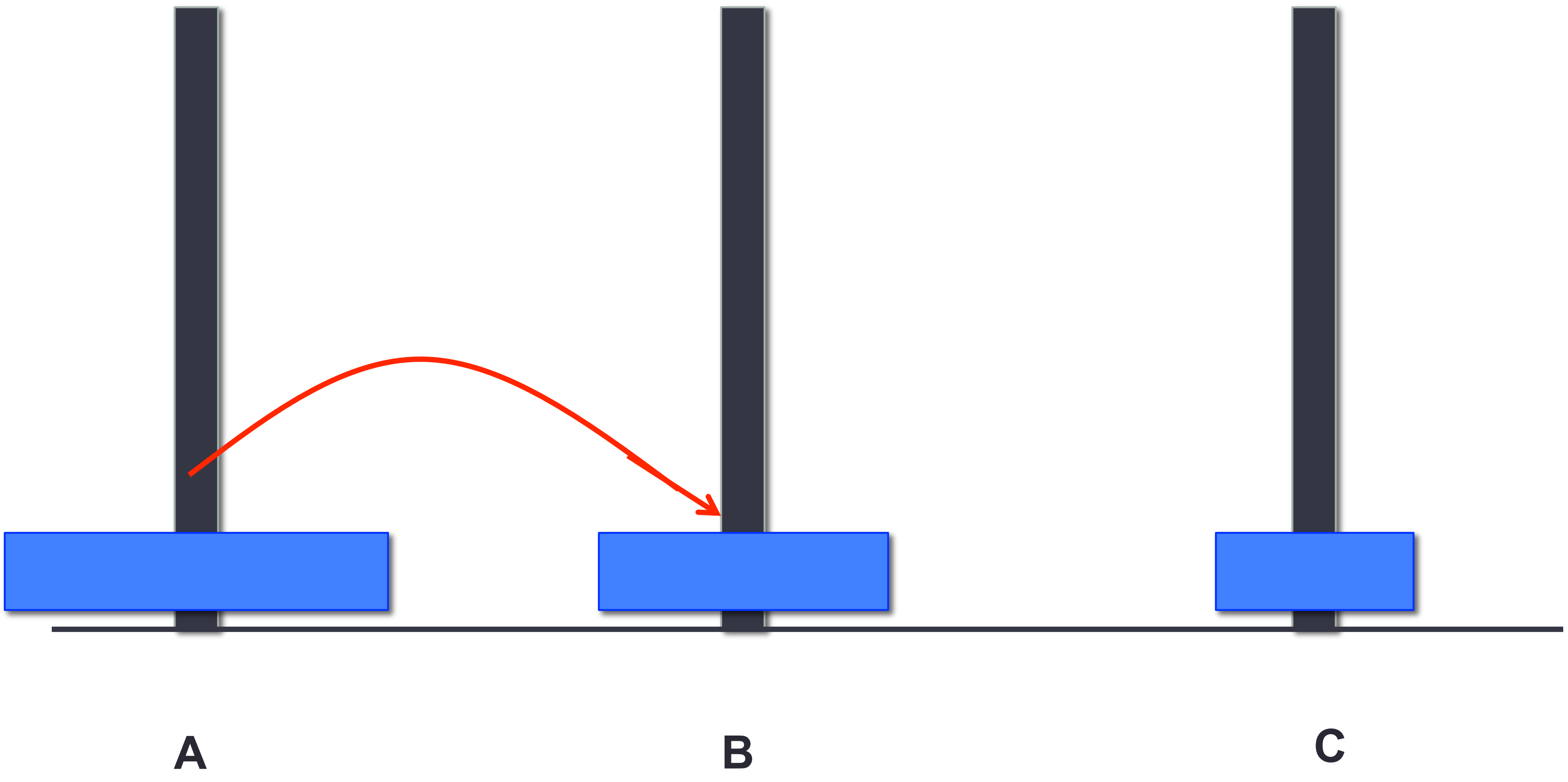
Let's Solve it with 3 Disks



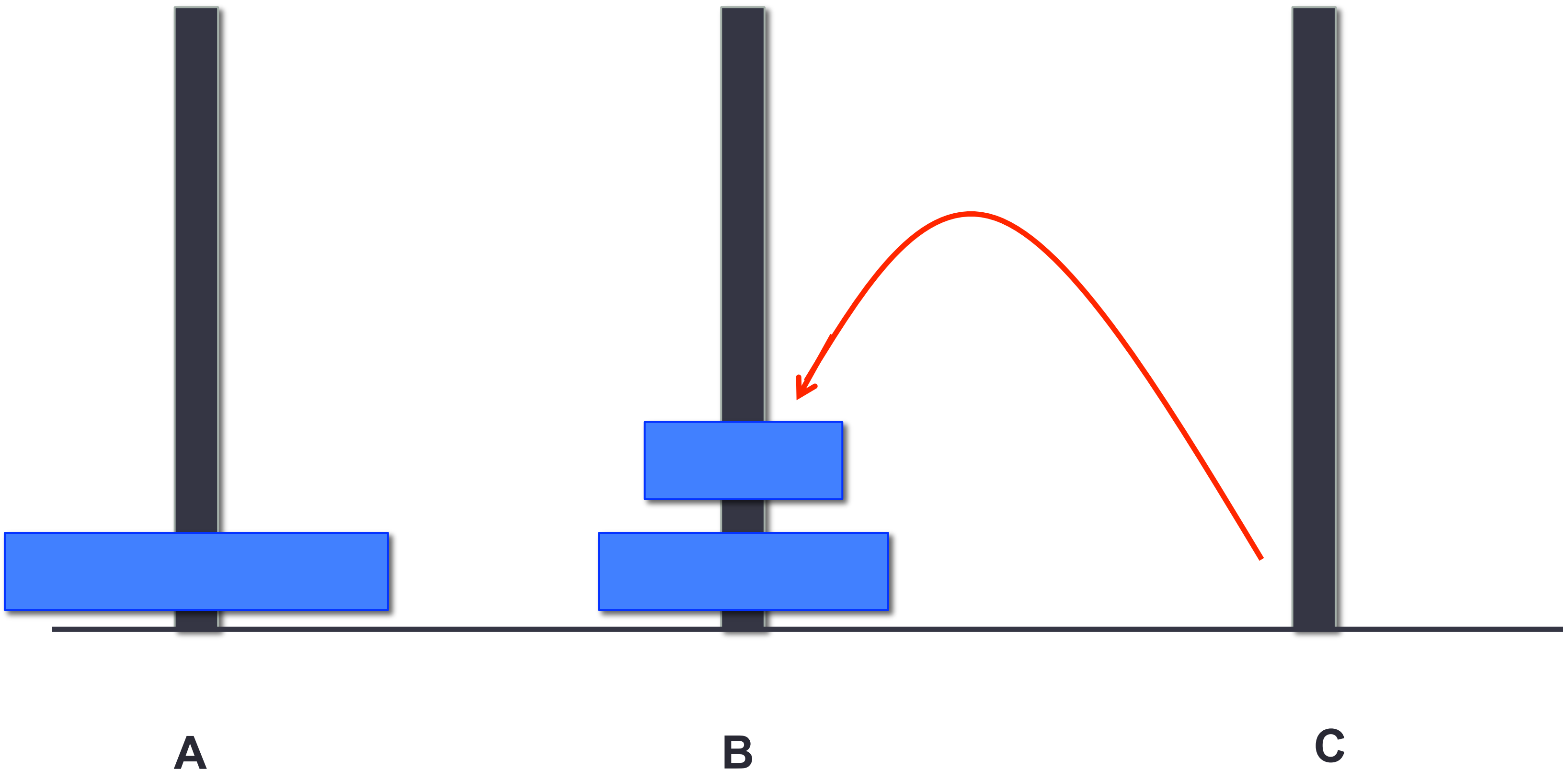
Step 1



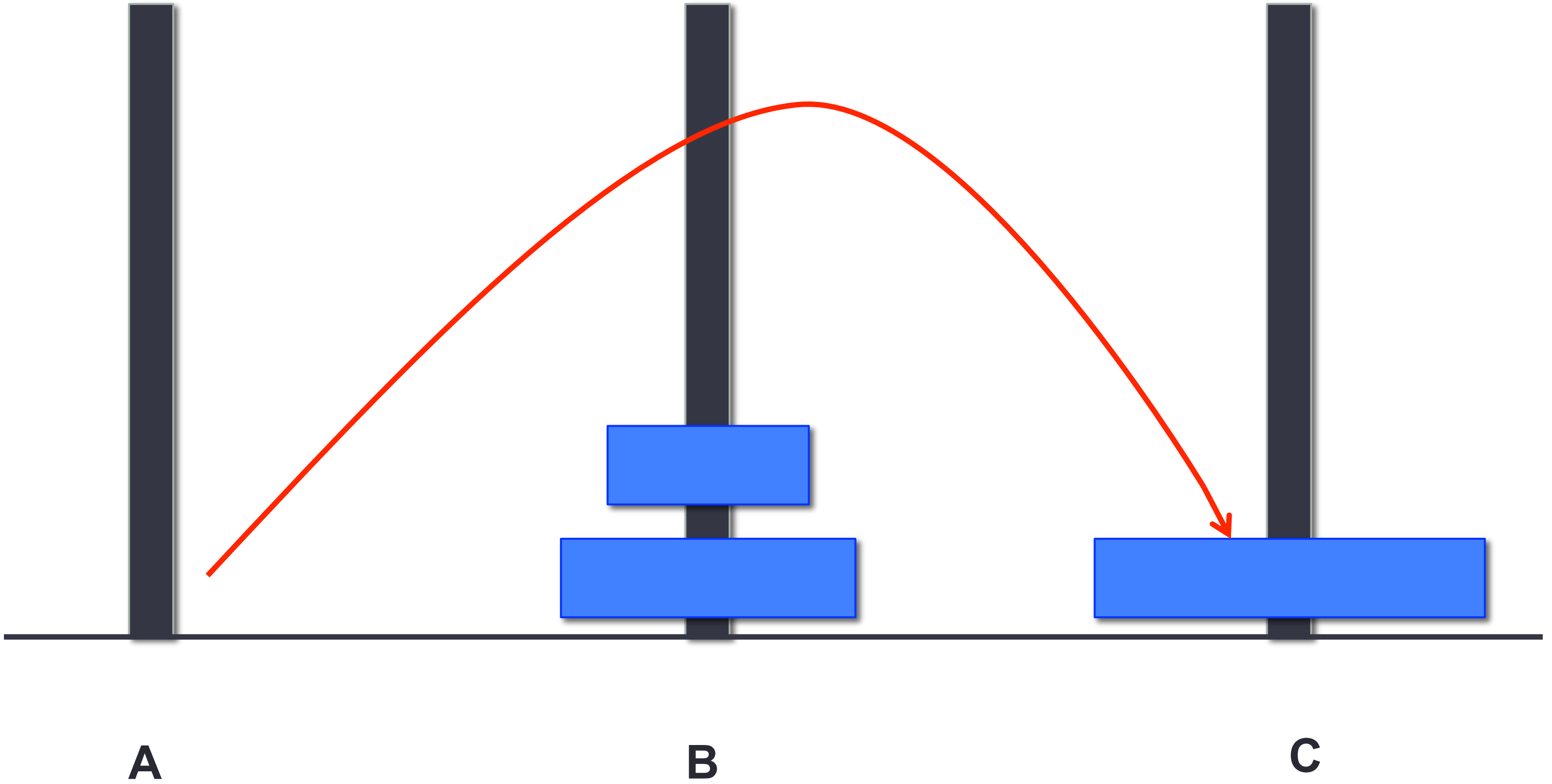
Step 2



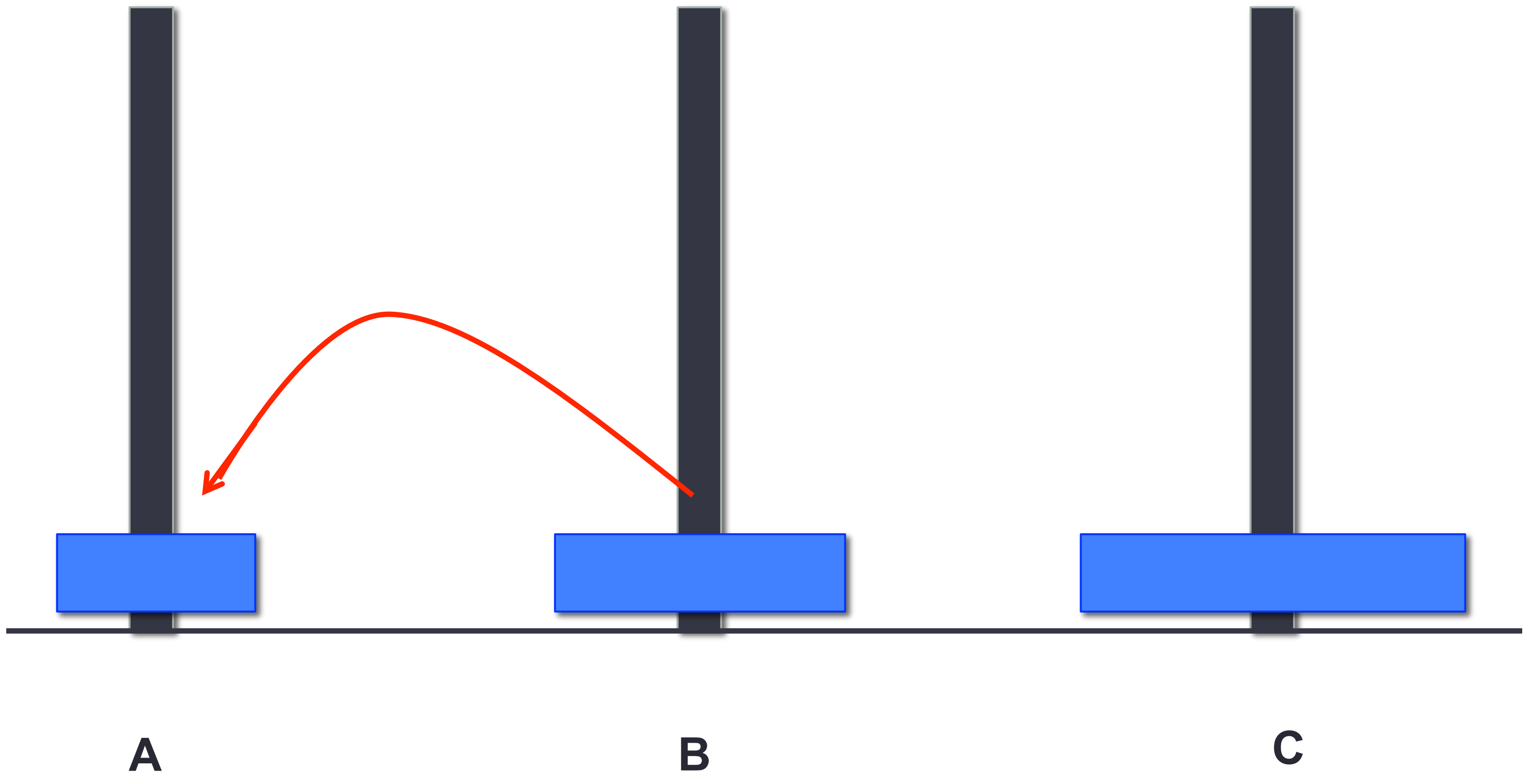
Step 3



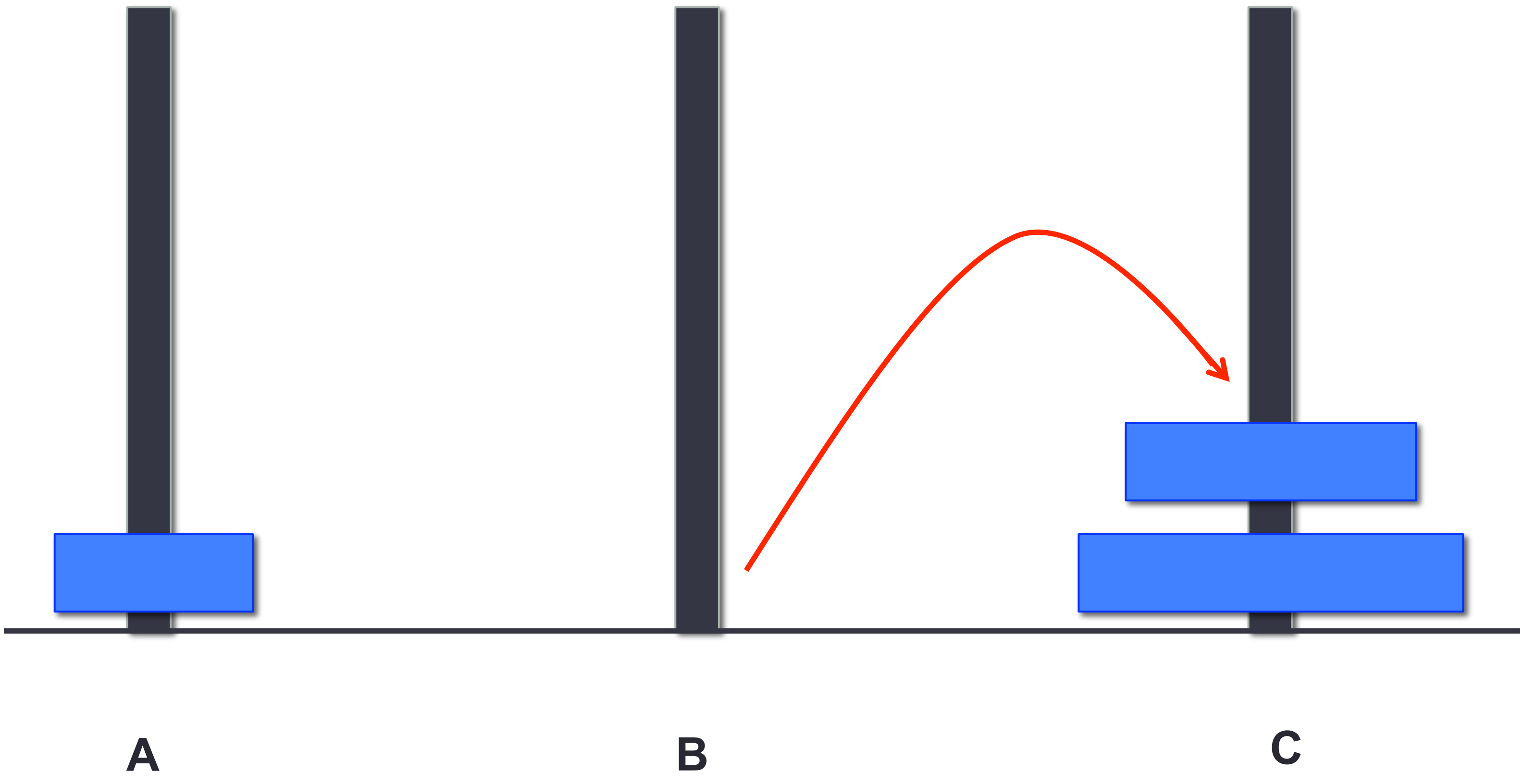
Step 4



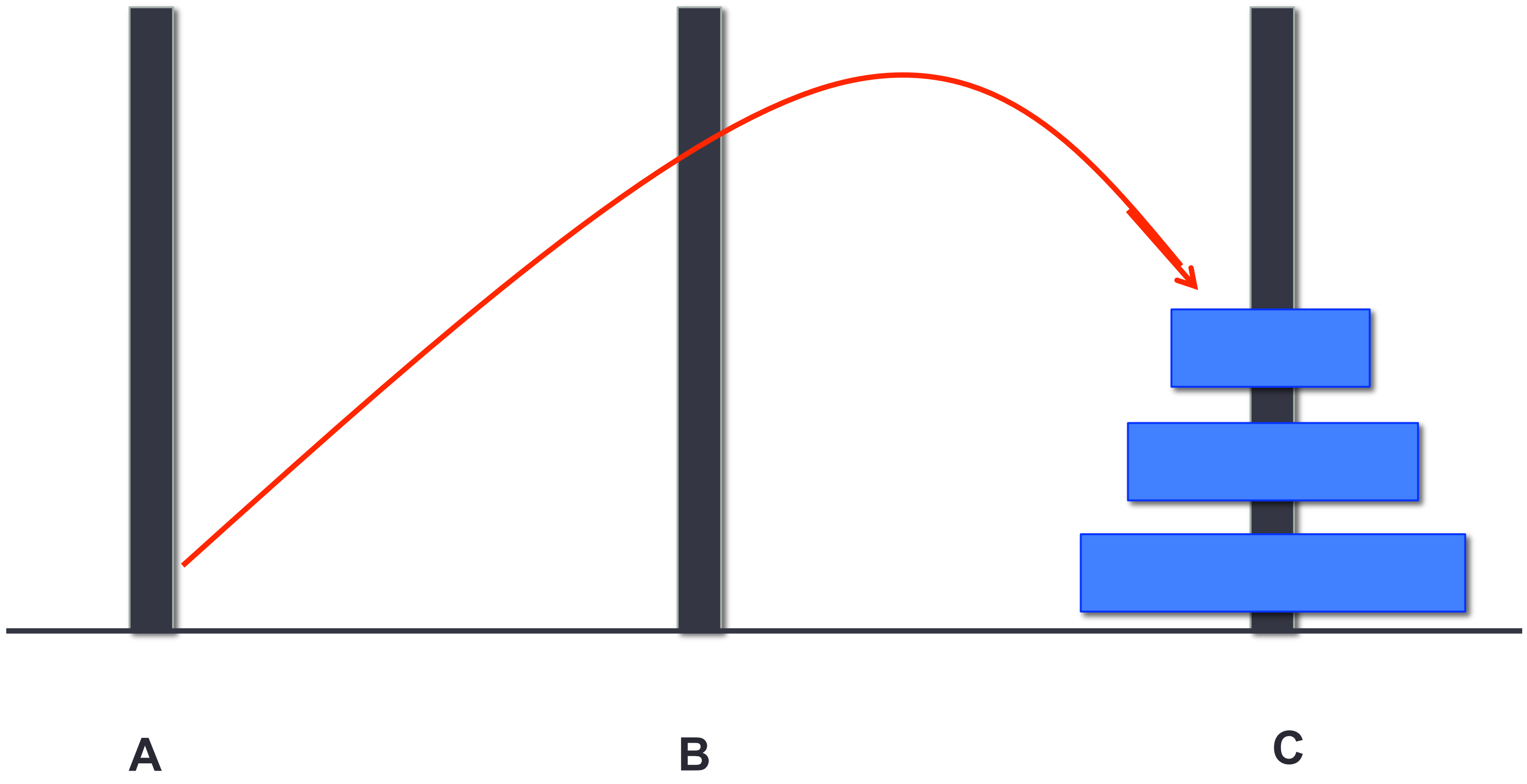
Step 5



Step 6

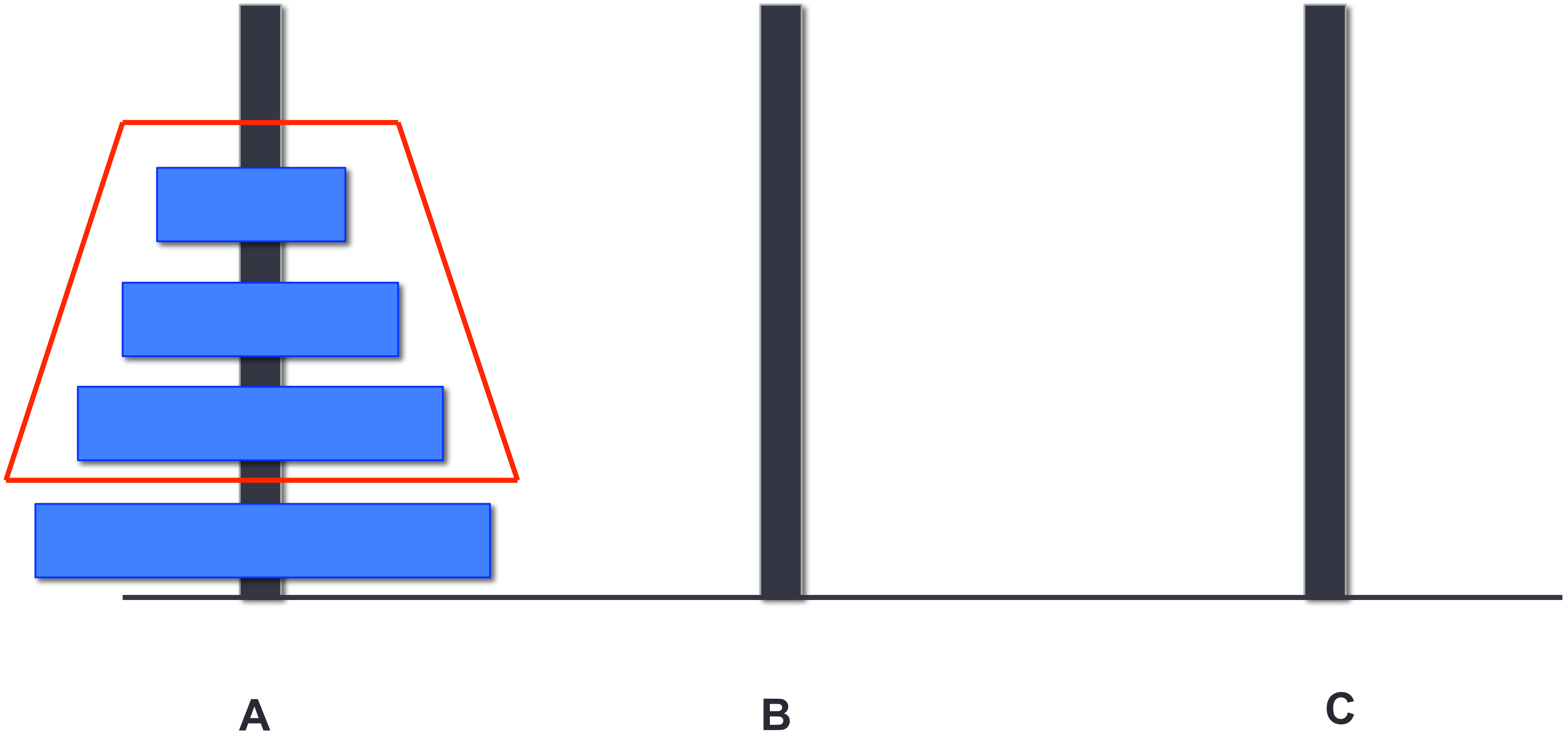


Step 7: Victory



Key Insight

In order to move the entire pyramid of disks to peg C, we first must move the pyramid above the bottom disk to peg B in order to move the largest disk to peg C



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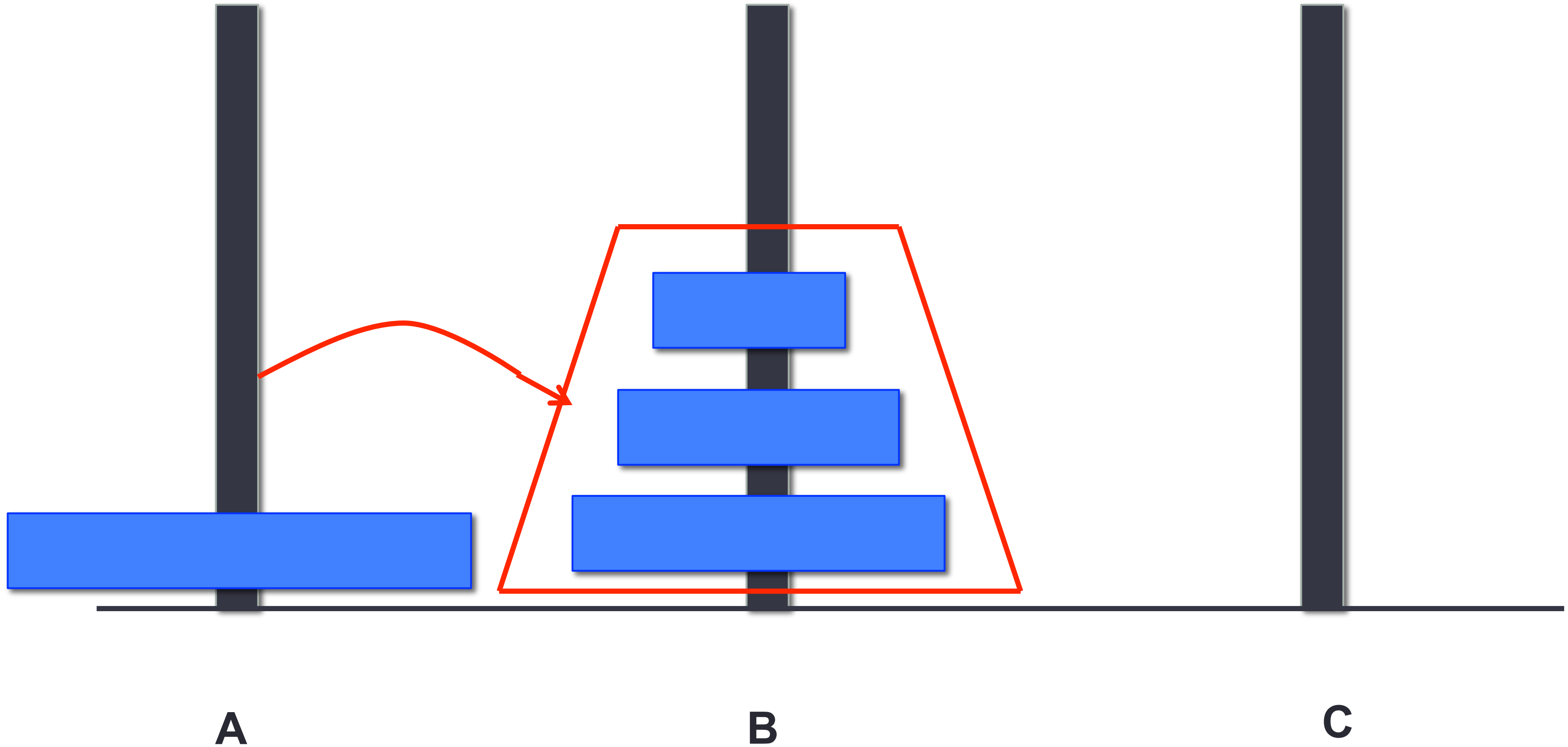

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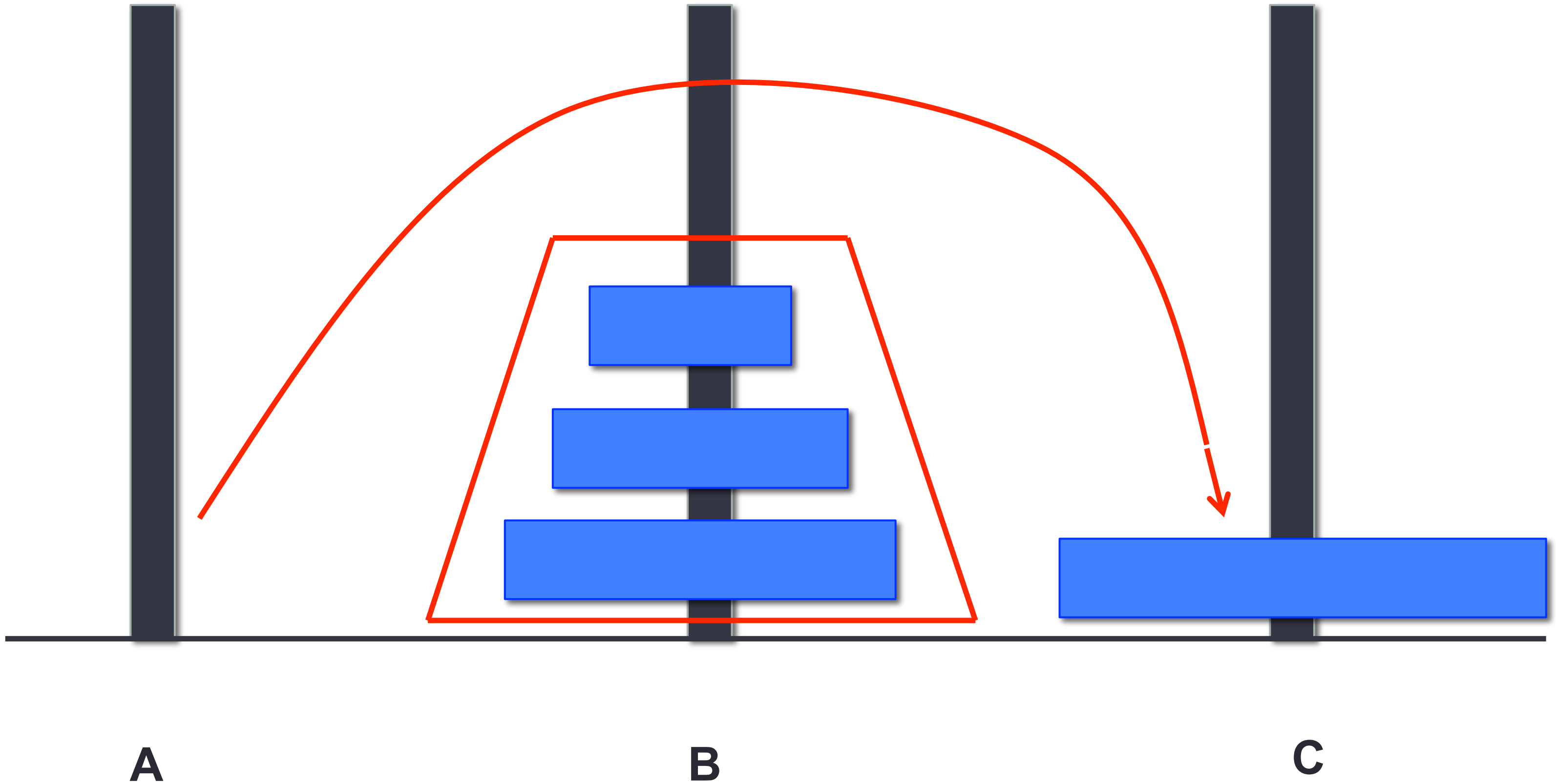
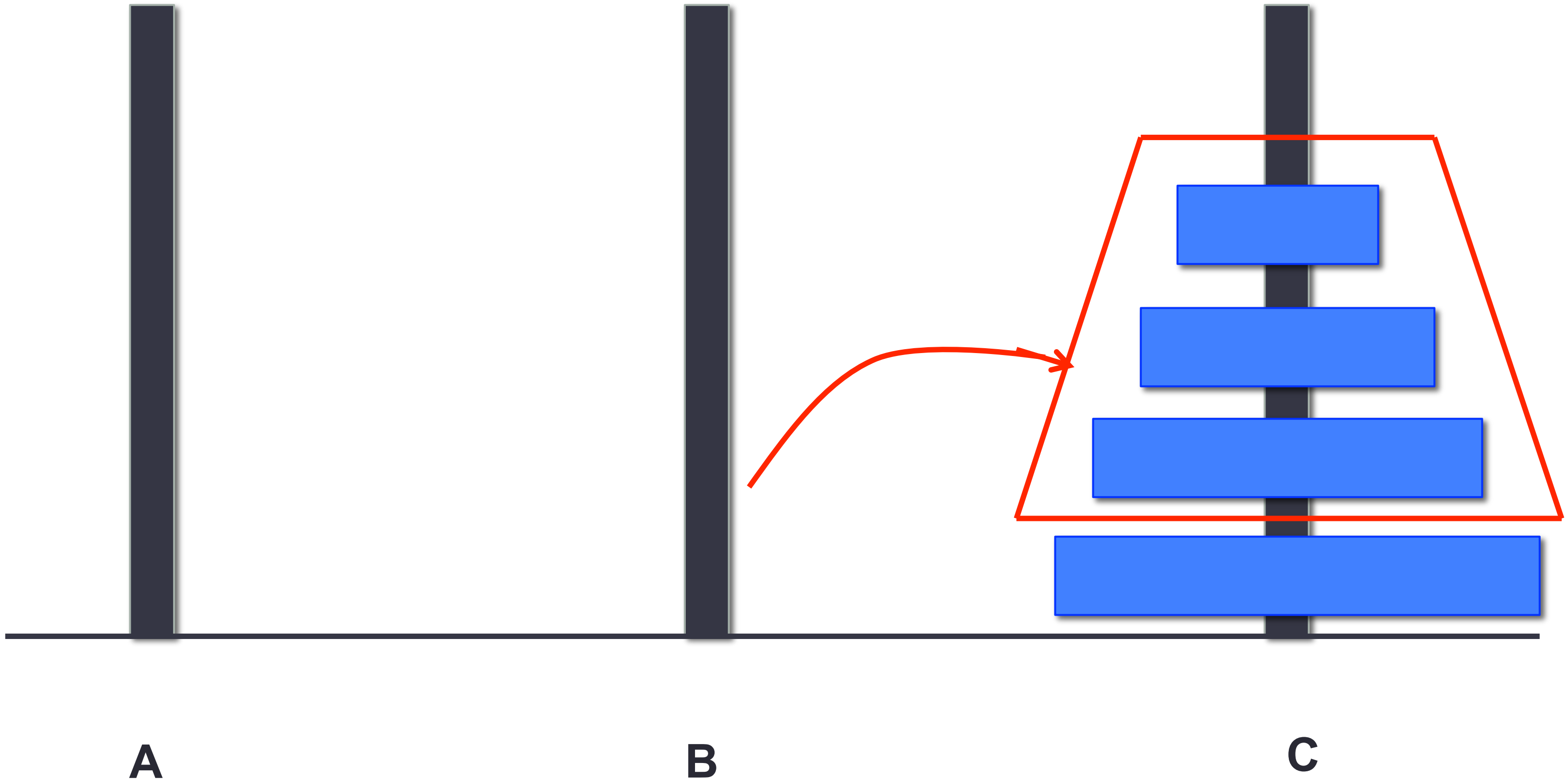
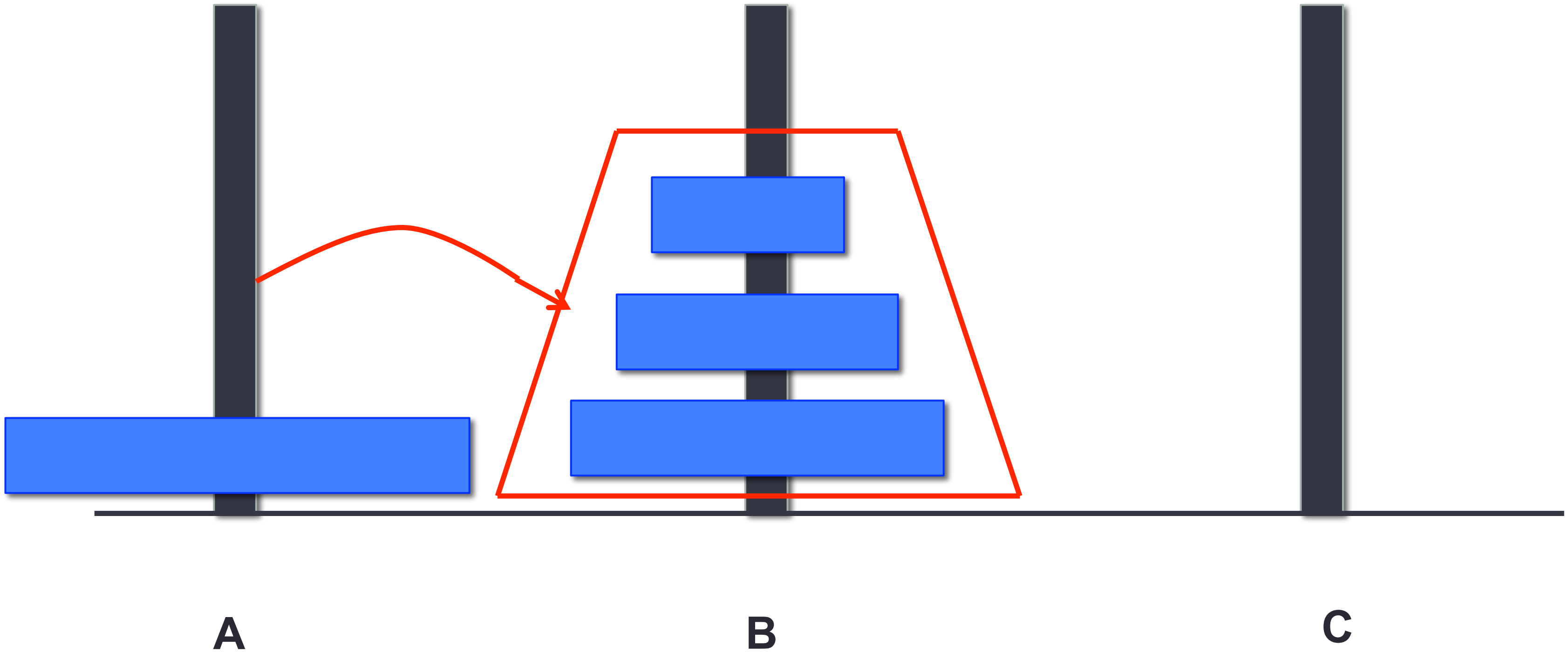


Key Insight



Key Insight

- We see that we are solving the same puzzle on the smaller pyramid as we are on the entire pyramid except for the fact that the destination peg (i.e. victory peg) is now peg B instead of C.



Recursive Solution

- If we create a method that solves the puzzle for a pyramid of size *N* disks from some origin peg to some destination peg, then we recursively call this method on *N*-1 disks (i.e. all disks except for the bottom peg), so that we free up the button peg to move to our destination.
- On these recursive calls, we change which pegs we use for the origin, destination, and placeholder
- See next slide for psuedocode…

Psuedocode

Method used for moving some set of disks

```
moveTower(disks, origin, placeholder, destination)
{
        if numDisks == 0:
                return;

        else:
                moveTower(disks – 1, origin, destination, placeholder);
                move disk from origin to destination
                moveTower(disks – 1, placeholder, origin, destination);
}

moveTower(allDisks, A, B, C);
```

Stopping Condition

Two recursive calls

Call the method on all of the disks

Cool CS Link of the Day

- Proving Fermat's Last Theorem
- http://www.youtube.com/watch?v=7FnXgprKgSE

For integers $n > 2$ the equation

$$a^n + b^n = c^n$$

cannot be solved with positive integers a, b, c.