Sample Questions for Exam 3

The following are meant to give you some examples of questions that might be asked on the third exam. The sample exam questions are not intended to represent the length or difficulty of the actual exam or to completely cover the range of exam topics. For a list of topics and more information about exam 3, see the Exams page of the course website.

A reference similar to the one below and on the following page will be provided for you on the exam.

List of Sample Maple Commands:

ans := allvalues( result )
ans := diff( expr, x )
ans := diff( f(t), t ) = ...
ans := dsolve( {ode, f(0) = a}, f(x) )
ans := dsolve( {ode1, ode2, f(0) = a, g(0) = b, D(f)(0) = c, D(g)(0) = d}, {f(x), g(x)} )
ans := eval( expr, {x = a, y = b} )
ans := evalf( expr )
ans := fsolve( eqn, x )
ans := fsolve( eqn, x = 1 )
ans := fsolve( eqn, x = 1..2 )
ans := int( expr, x )
ans := int( expr, x = a..b )
ans := lhs( eqn )
ans := rhs( eqn )
ans := simplify( expr )
ans := solve( {eqns}, {unkns} )
ans := subs( x = a, y = b, z = c, {expr_or_eqns} )

assume( -1 < b, b < 0 )
plot( [expr1, expr2], x = a..b )
plot3d( [expr1, expr2], x = a..b, y = c..d )
with(plots):
    implicitplot( [eqn1, eqn2], x = a..b, y = c..d, color = [color1, color2] )
    implicitplot3d( [eqn1, eqn2], x = a..b, y = c..d, z = e..f )

List of Sample MATLAB Commands:

[t, y] = ode45( @func, tspan, ystart )
plot( X, Y, 'r')
disp([ num2str(x), ' text ')

Other Stuff:

Compound interest formula: \( F = P(1+i)^n \), where \( F \) is future value, \( P \) is present value, \( i \) is annual interest rate, \( n \) is number of years.

Area of a right triangle with base = \( b \) and height =\( h \): \( \text{area}_{\text{triangle}} = \frac{1}{2}(b*h) \)

Circumference of a circle with radius = \( r \): \( \text{circ}_{\text{circle}} = 2\pi r \)

Area of a circle with radius = \( r \): \( \text{area}_{\text{circle}} = \pi r^2 \)
Volume of a cylinder with radius \( r \) and height \( h \): 
\[
vol_{cylinder} = \pi r^2 h
\]

Equation of a Line: 
\[ y = mx + b \]
or via the point-slope formula: 
\[ (y_2 - y_1) = m(x_2 - x_1) \]

Distance Formula: 
\[
dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Solution of linear first-order ODE in the form 
\[
df(t) + p(t)f(t) = g(t)
\]
is 
\[
f(t) = \frac{1}{u(t)} \left[ \int_t u(s)g(s)ds + c \right]
\]
where 
\[
u(t) = exp \left[ \int_t p(s)ds \right]
\]

2-point Multiple Choice

**Choose the one best answer after reading all of the choices.** Note: where Maple commands are shown, you should assume that the right-arrow key (\( \rightarrow \)) has been used where needed.

1) Which of the following Maple expressions corresponds to the mathematical formula:

\[
F = \frac{e^t}{(x-b)(1-a)} \left[ \frac{3+x}{b+\cos(t)} \right]
\]

A. \( F = \exp(t)/((x - b)*(1 - a))*(3 + x)/(b + \cos(t)) \)

B. \( F = [\exp(t)/((x - b)*(1 - a))]*[(3 + x)/(b + \cos(t))] \)

C. \( F = (\exp^t)/((x - b)*(1 - a))*(3 + x)/(b + \cos(t)) \)

D. \( F = \exp^t/((x - b)*(1 - a))*(3 + x)/(b + \cos(t)) \)

E. \( F = (\exp^t)/((x - b)*(1 - a))*((3 + x)/(b + \cos(t))) \)

2) Consider the following sequence of Maple commands:

\[
E1 := 5
\]
\[
E2 := 7
\]
\[
E3 := E1 + E2
\]

Which one of the following is the result?

A. \( E3 := E1 + E2 \)

B. \( E3 := 5 + E2 \)

C. \( E3 := E1 + 7 \)

D. \( E3 := 5 + 7 \)

E. \( E3 := 12 \)
3) What is printed out after the third command of the following sequence of commands?

\[
E_1 := (q - r)^2 - 5*(p + s) \\
\text{subs}(r = -1, s = 3, E_1) \\
\text{subs}(q = 3*r, E_1)
\]

A. \(-11 - 5p\)  
B. \((-3 - r)^2 - 5p - 5s\)  
C. \((3r + 1)^2 - 5p - 15\)  
D. \(4 - 5p - 5s\)  
E. \(4r^2 - 5p - 5s\)

4) This question concerns plotting the two curves given by

\[
y = 4(x-1)^2 + x^3 \\
y = x^4 - 5x^2 + 8
\]

The following commands define the two curves and add additional plot options, if needed.

\[
y1 := 4*(x - 1)^2 + x^3 \\
y2 := x^4 - 5*x^2 + 8 \\
\text{with(plots):}
\]

Which other command is needed to plot the curves?

A. \(\text{plot( y1, y2, x = -5..5 )}\)  
B. \(\text{plot( ( y1, y2 ), x = -5..5 )}\)  
C. \(\text{plot( [ rhs(y1), rhs(y2) ], x = -5..5 )}\)  
D. \(\text{implicitplot( [ y1, y2 ], x = -5..5, y = -10..500 )}\)  
E. \(\text{implicitplot( [ y = y1, y = y2 ], x = -5..5, y = -10..500 )}\)

5) Which code choice finds the symbolic expression for the point in time where the maximum height of this trajectory occurs? Assume that Maple can solve this symbolically.

\[
> \text{trajectory} := y = y0 + v0 \cdot \sin(\theta0) \cdot t - \frac{g \cdot t^2}{2}
\]

A. \(\text{maxH := solve( diff(rhs(trajectory), y0) = 0, t)}\)  
B. \(\text{maxH := solve( diff(rhs(trajectory), y0) = 0, y0)}\)  
C. \(\text{maxH := solve( diff(trajectory, y) = 0, t)}\)  
D. \(\text{maxH := solve( diff(rhs(trajectory), t) = 0, t)}\)  
E. \(\text{maxH := solve( diff(rhs(trajectory), t) = 0, y0)}\)
6) Consider the curve
\[ r(z) = \frac{z(5-z)}{3+z^2} \]
and the surface it generates by being revolved around the \( z \)-axis between \( z = 0 \) and \( z = H \). The curve is defined in Maple with
\[ rz := z*(5 - z)/(3 + z^2) \]
The Maple command for computing the volume of revolution of the curve between \( z = 0 \) and \( z = H \) is given by
A. \( \text{vol} := \text{int}( 2\pi rz^2, z = 0..H ) \)
B. \( \text{vol} := \text{int}( 2\pi rz z, z = 0..H ) \)
C. \( \text{vol} := \text{int}( \pi rz^2, z = 0..H ) \)
D. \( \text{vol} := \text{int}( \pi rz^2, z = [0, H] ) \)
E. \( \text{vol} := \text{int}( rz, z = 0..H ) \)

7) For the volume defined by the previous problem, which of these Maple commands will find the value of \( H \) between 0 and 5 where the volume is 4.3?
A. \( \text{eval( vol, H = 4.3 )} \)
B. \( \text{evalf( vol, H = 4.3 )} \)
C. \( \text{fsolve( diff(vol, H) = 4.3, H = 0..5 )} \)
D. \( \text{fsolve( vol, H = 0..5 ) = 4.3} \)
E. \( \text{fsolve( vol = 4.3, H = 0..5 )} \)

8) The solution of the ordinary differential equation
\[ \frac{dx(t)}{dt} = x(t) + 4t - 3 \]
with initial condition \( x(0) = -2 \) is
A. \( x(t) = -5e^t - 4t + 3 \)
B. \( x(t) = e^t - 4t - 1 \)
C. \( x(t) = e^t + 4t - 3 \)
D. \( x(t) = e^t + 2t^2 - 3t - 3 \)
E. \( x(t) = 3e^t - 4t - 1 \)
9) Given the following ordinary differential equation and initial conditions:

\[ \frac{d^2}{dt^2} y(t) + \omega^2 y(t) = 0 \]
\[ \omega = \frac{1}{5}, \quad y(5) = \frac{1}{3} \pi, \quad \frac{d}{dt} y(t)(5) = 0 \]

Suppose that the following Maple command has been executed:

\[ \text{ode} := \text{diff}(y(t),t^2)+\omega^2*y(t)=0 \]

Which of the Maple commands below will solve for \( y(t) \) symbolically?

A. \( \text{os} := \text{dsolve( ode, diff(y(t),t)(5)=0, y(t) )} \)
B. \( \text{os} := \text{dsolve( ode, D(y(5)=Pi/3,t)=0, y(t) )} \)
C. \( \text{os} := \text{dsolve({ode, D(y(t),t)(5)=0}, y(t) )} \)
D. \( \text{os} := \text{dsolve({ode, D(y(5)=0, y(5)=Pi/3}, y(t) )} \)
E. \( \text{os} := \text{dsolve({ode, D(y(t),t)(5)=0, y(5)=Pi/3}, y(t) )} \)

10) Which of the following are steady states of the differential equation \( \frac{dx(t)}{dt} = 3x(t)^2 + 3x(t) - 18 \)

i. \( x(t) = 0 \)
ii. \( x(t) = 2 \)
iii. \( x(t) = 3 \)

A. i only B. ii only C. i and ii only D. ii and iii only E. i, ii, and iii

Written Questions

Write any Maple commands exactly as you would type them without using the context-menu (right-click menu). Use an asterisk (*) or a clearly written dot (·) for multiplication. You do not have to indicate where you hit the right-arrow (→) or the <Enter> key. A list of Maple commands is provided for your reference. You are not required to comment your code, but comments may receive partial credit if they are correct and complete, and the commands are not. Use a # sign to indicate a comment and use a > symbol at start of a line to indicate Maple command statements. Use the variable names shown for full credit.

If a command produces more than one result, you may assume any ordering of the result values and use result[1], result[2], and so on to indicate which of the possible results are used in further calculations.
11) Consider the parabola

\[ y = Px^2 + Qx + R \]

Give the Maple commands to find the values of \( P \), \( Q \), and \( R \) necessary so that the points (1, 7), (3, 2), and (6, 5) are all on the parabola.

12) Suppose the number of bacteria in a petri dish is given by the equation:

\[ pop(t) = 3 - 4 \frac{\cos \left( \frac{t}{2} \right) \sin \left( \frac{t}{5} \right)}{1 + t} \]

where \( t \) is the time (in hours) and \( pop(t) \) is the number of bacteria (in thousands) at time \( t \). A plot of \( pop(t) \) is given below:

Give the Maple commands to determine the maximum population the bacteria in the petri dish reached.
13) We have the following three equations along with the plot of the three equations:

\[ Bx - y = 2 \]
\[ y = \frac{2}{2x - 1} \]
\[ y = \frac{1}{3}(x - 1)^2 + 2 \]
\[ x = B \]

Give the Maple commands to find the value of \( B \) between 3 and 4 necessary so that the wedge-shaped region between the curves has an area of 5.3.

14) The position \( p(t) \) of an object after \( t \) seconds is given by the ordinary differential equation

\[ \frac{dp(t)}{dt} = 1.2p(t) - 0.0001(p(t))^2 \]

with initial condition \( p(0) = 200 \).

Give the MATLAB commands to first solve this ODE numerically and then give the position of the object at time \( t = 8 \).

15) The position \( p(t) \) of an object after \( t \) seconds is given by the ordinary differential equation

\[ \frac{dp(t)}{dt} = 1.2p(t) - 0.0001(p(t))^2 \]

with initial condition \( p(0) = 200 \).

Give the Maple commands to first solve this ODE symbolically and then give the position of the object at time \( t = 8 \).