Problem 1: Future value of money invested at a given interest rate and what it means to solve symbolically first!

\[
TVOM := F = P \cdot (1 + a)^n
\]  

(1)

\[
years := solve(TVOM, n)
\]  

(2)

\[
yearsTo25K := \text{subs}(P = 15000, F = 25000, a = 0.055, years)
\]  

(3)

It will take a little over 9 and a half years to get to $25,000 if you currently have $15,000 and it earns 5.5% interest.

\[
evalf(yearsTo25K)
\]  

(4)

Problem 2: Comparing investment options

Get an expression for computing \( F \), the future value of money, by solving \( TVOM \) for \( F \) or using the right-hand side function, \( \text{rhs()} \)

\[
FVOM := \text{rhs}(TVOM)
\]  

(5)

Option 1: invest $1500 at 5.75% for 3 years, then reinvest at 5.20% for the remaining 3 years

\[
option1_{\text{years1to3}} := \text{subs}(P = 1500, a = 0.0575, n = 3, FVOM)
\]  

(6)

\[
option1_{\text{years4to6}} := \text{subs}(P = option1_{\text{years1to3}}, a = 0.0520, n = 3, FVOM)
\]  

(7)

Option 2: invest $1500 at 5.5% for 6 years

\[
option2 := \text{subs}(P = 1500, a = 0.055, n = 6, FVOM)
\]  

(8)

In hindsight, it would have been better to invest at the guaranteed 5.5% for 6 years than to take the initially higher rate of 5.75% for 3 years.

Problem 3: Present value of money payed out over \( n \) periods of time

\[
PVOM := P = \frac{A \cdot (1 + a)^n - 1}{a \cdot (1 + a)^n}
\]  

(9)

\[
\text{monthlyPayment} := \text{solve}(PVOM, A)
\]
Monthly payment for a $10,000 loan at a rate of 6.9% annual interest and a payment period of 4 years is $239.00

\[
\text{monthlyPayment} := \frac{P \times a \times (1 + a)^n}{(1 + a)^n - 1}
\]  

\text{(10)}

Monthly payment for $10,000 loan at rate of 6.9% annual interest and a payment period of 4 years is $239.00

\[
\text{payment1} := \text{subs}\left(P = 10000, a = \frac{0.069}{12}, n = 48, \text{monthlyPayment}\right)
\]

\[
\text{payment1} := 238.9987698
\]  

\text{(11)}

Monthly payment for $10,000 loan at rate of 5.9% annual interest and a payment period of 4 years is $234.39

\[
\text{payment2} := \text{subs}\left(P = 10000, a = \frac{0.059}{12}, n = 48, \text{monthlyPayment}\right)
\]

\[
\text{payment2} := 234.3920719
\]  

\text{(12)}

\textbf{Problem 4: Shop for a better deal (attempt to solve symbolically)}

\[
\text{monthlyInterestRate} := \text{solve}(PVOM, a)
\]

\[
\text{monthlyInterestRate} := \text{RootOf}\left(-P \times Z \times (1 + Z)^n + A \times (1 + Z)^n - A\right)
\]  

\text{(13)}

The \text{solve} \text{ command didn't work, so we try the allvalues command on the result of the solve command.}

\[
\text{monthlyIntRate} := \text{allvalues}(\text{monthlyInterestRate})
\]

\[
\text{monthlyIntRate} := \text{RootOf}\left(-P \times Z \times (1 + Z)^n + A \times (1 + Z)^n - A\right)
\]  

\text{(14)}

Maple still cannot find a symbolic solution, so we will have to find a numeric solution to this problem.

\textbf{Problem 5: Shop for a better deal (solve numerically)}

First plot the monthly payment vs the interest rate to see approximately what interest rate would give us the lower payment.

We will need an expression representing \( A \) (the monthly payment) in terms of \( a \) (the monthly interest rate) for our loan for $10,000 over 4 years.

Recall that we earlier solved \( PVOM \) for \( A \) and named the result \text{monthlyPayment}

\[
\text{myMonthlyPayment} := \text{subs}(P = 10000, n = 48, \text{monthlyPayment})
\]

\[
\text{myMonthlyPayment} := \frac{10000 \times a \times (1 + a)^{48}}{(1 + a)^{48} - 1}
\]  

\text{(15)}

\[
\text{plot}\left(\text{myMonthlyPayment}, a = 0 .. \frac{0.06}{12}\right)
\]
From the plot we see that a (monthly) interest rate around 0.004 (or 0.4%) will give us a monthly payment of $230.

We use \texttt{fsolve} to find the exact interest rate

\begin{verbatim}
> desiredMonthlyInterestRate := fsolve(myMonthlyPayment = 230, a = 0.003 .. 0.005)
\end{verbatim}

\begin{equation}
\text{desiredMonthlyInterestRate} := 0.004112756193
\end{equation}

\begin{verbatim}
> desiredAnnualInterestRate := 12 \cdot \text{desiredMonthlyInterestRate}
\end{verbatim}

\begin{equation}
\text{desiredAnnualInterestRate} := 0.04935307432
\end{equation}

We need an annual interest rate of 4.93% or lower to have a monthly payment of $230 or less for a 4 year loan on $10,000

\textbf{Problem 6: The effects of inflation}

First we wish to determine the yearly rate of return when an initial investment of $100,000 doubles in 5 years.

\begin{verbatim}
> TVOMa := solve(TVOM, a)
\end{verbatim}

\begin{equation}
TVOMa := e^{\frac{\ln\left(F/P\right)}{n}} - 1
\end{equation}

\begin{verbatim}
> yrlyRateOfReturn := subs(P = 100000, F = 200000, n = 5, TVOMa)
\end{verbatim}

\begin{equation}
yrlyRateOfReturn := e^{\frac{1}{5} \ln(2)} - 1
\end{equation}
So, the yearly rate of return, ignoring inflation, is 14.9%

Now we take inflation into consideration.

Substitute $c$ in for $a$ in the formula for the future value of an investment

$$TVOMc := F = P \ (1 + c)^n$$

and formula for the present value of money borrowed over time

$$PVOMc := P = \frac{A \ ((1 + c)^n - 1)}{c \ (1 + c)^n}$$

Solve the new future value of money formula for $c$, the combined interest-inflation rate

$$cRateTVOM := solve(TVOMc, c)$$

$$cRateTVOM := \frac{\ln\left(\frac{F}{P}\right)}{n} - 1$$

$$rateEqn := (1 + c) = (1 + i) \cdot (1 + r)$$

$$rateEqn := 1 + c = (1 + i) \ (1 + r)$$

Solve the equation that relates $c$, $i$, and $r$ for $c$

$$cRateIR := solve(rateEqn, c)$$

$$cRateIR := i \ r + i + r$$

Set the two expressions for $c$ equal to each other and solve for $r$

$$cRateEqn := cRateTVOM = cRateIR$$

$$cRateEqn := \frac{\ln\left(\frac{F}{P}\right)}{n} - 1 = i \ r + i + r$$

$$realReturn := solve(cRateEqn, r)$$

$$realReturn := \frac{\ln\left(\frac{F}{P}\right)}{n} - i - 1 \ \frac{1}{1 + i}$$

$$myRealReturn := subs(P = 100000, F = 200000, n = 5, i = 0.028, realReturn)$$

$$myRealReturn := 0.9727626459 \ e^{\frac{1}{5} \ln(2)} - 1.000000000$$

$$evalf(myRealReturn)$$

0.117410851

My real return on the investment when inflation is 2.8% is 11.7%

What if inflation is 5%?

$$myRealReturn5 := subs(P = 100000, F = 200000, n = 5, i = 0.05, realReturn)$$

$$myRealReturn5 := 0.9523809524 \ e^{\frac{1}{5} \ln(2)} - 1.000000000$$

$$evalf(myRealReturn5)$$

0.093998433

Now the real return is only 9.39%.

What if inflation is 8%?
Problem 7: What if there are complications?

Suppose it takes 9 years instead of 5?

```plaintext
> myRealReturn9yrs := subs(P = 100000, F = 200000, n = 9, i = 0.028, realReturn)

myRealReturn9yrs := 0.9727626459 e^{\frac{1}{9} \ln(2)} - 1.000000000
```

```plaintext
> evalf(myRealReturn9yrs)

0.050641769
```

So just a 5.06% return on $100,000 dollars. Let's compare this to investing the money in a CD at 5.5% for 5 years.

```plaintext
> FinvestCD := subs(P = 100000, a = 0.055, n = 5, TVOM)

FinvestCD := F = 1.306960006 \times 10^5
```

```plaintext
> myRealReturnCD := subs(P = 100000, FinvestCD, i = 0.028, n = 5, realReturn)

myRealReturnCD := 0.9727626459 e^{\frac{1}{5} \ln(1.306960006)} - 1.000000000
```

```plaintext
> evalf(myRealReturnCD)

0.026264591
```

We would have $130,696.00 from our CD after 5 years, but the real return, if inflation is 2.8%, is only 2.62%.

Moral of the story:
High inflation rates definitely encourage businesses (and individuals) to spend now instead of save or invest for a future return. But, taking the safe way and just saving the money is still likely to lead to less real return than investing (if you can double your money in 5 years).