THE UNDECIDABLE

Basic Papers On Undecidable Propositions, Unsolvable Problems And Computable Functions

Edited by

MARTIN DAVIS, 1928- ed.

Professor of Mathematics
Yeshiva University

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$$\overline{R}(0,\mathfrak{y}) \,\&\, R(n+1,\mathfrak{y}) \,\&\, [\,\chi(n,\mathfrak{y})=0\,].$$

holds.

From this it follows that the function $\chi(n,\mathfrak{y})$ (considered as a function of n) remains 0 until the least value of n for which $R(n,\mathfrak{y})$ holds, and, from there on, is equal to this value (if $R(0,\mathfrak{y})$ already holds, then the corresponding $\chi(n,\mathfrak{y})$ is constant and $=0$). Hence we have:

$$\psi(\mathfrak{x},\mathfrak{y})=\chi(\varphi(\mathfrak{x}),\mathfrak{y})$$
$$S(\mathfrak{x},\mathfrak{y}) \sim R[\psi(\mathfrak{x},\mathfrak{y}),\mathfrak{y}]\,.$$

The relation T can, by negation, be reduced to a case analogous to that of S, thus proving Theorem IV.

The functions $x+y$; $x.y$, x^y and the relations $x<y$, $x=y$ are, as one can easily check, recursive, and we now define, starting from these concepts, a sequence of functions (relations) 1-45, of which each is defined from the preceding ones by the methods indicated in Theorems I-IV. In so doing, several of the definitional steps allowed by Theorems I-IV are often combined into one step. Each of the functions (relations) 1-45, among which occur, for example, the concepts "FORMULA", "AXIOM", "DIRECT CONSEQUENCE", is therefore recursive.

1. $x/y \equiv (Ez)[z \leqslant x \,\&\, x=y\,.\,z]$[33] we'd say $y|x$
x is divisible by y[34].
2. $\mathrm{Prim}(x) \equiv \overline{(Ez)}[z \leqslant x \,\&\, z \neq 1 \,\&\, z \neq x \,\&\, x/z] \,\&\, x>1$
x is a prime number.

33. The symbol $\equiv$ will be used in the sense of "definitional equality", and therefore in definitions it represents either $=$ or $\sim$ (otherwise the symbolism is Hilbert's.).

34. Everywhere in the following definitions where one of the expressions (x), (Ex), ϵx occurs it is followed by a bound for x. This bound serves only to assure the recursive nature of the defined concept (cf. Theorem IV). On the other hand the extension of the defined concept would, in most cases, not be changed by omission of this bound.

3. $0\ Pr\ x \equiv 0$

what if $n >> x$?

$(n+1)\ Pr\ x \equiv \epsilon y\,[y \leqslant x\ \&\ \mathrm{Prim}(y)\ \&\ x/y\ \&\ y > n\ Pr\ x]$

$n\ Pr\ x$ is the n-th prime factor of x (according to magnitude).[34a]

4. $0! \equiv 1$

$(n+1)! \equiv (n+1).n!$

5. $Pr\ (0) \equiv 0$

$Pr\ (n+1) \equiv \epsilon y[\,y \leqslant \{Pr(n)\}! + 1\ \&\ \mathrm{Prim}(y)\ \&\ y > Pr(n)]$ or 0 if none

$Pr(n)$ is the n-th prime number (according to magnitude).

εy = least y such that

$y \leq x$?

6. $n\ Gl\ x \equiv \epsilon y[\,y > x\ \&\ x/(n\ Pr x)^{y}\ \&\ x/(n\ Pr\ x)^{y+1}]$

$n\ Gl\ x$ is the n-th term of the sequence of numbers corresponding to the number x (for $n>0$ and n not greater than the length of this sequence).

7. $l(x) \equiv \epsilon\ y\,[\,y \leqslant x\ \&\ y\ Pr\ x > 0\ \&\ (y+1)\ Pr\ x = 0]$

$l(x)$ is the length of the sequence of numbers correlated with x

8. $$x * y \equiv \epsilon z\Big\{ z \leqslant [Pr\,(l(x)+l(y))]^{x+y}\ \&\ (n)[n \leqslant l(x) \rightarrow n\ Gl\ z = n\ Gl\ x]\ \&\ (n)[0 < n \leqslant l(y) \rightarrow (n+l(x))\ Gl\ z = n\ Gl\ y\,]\Big\}$$ concatenate

$x * y$ corresponds to the operation of juxtaposing two finite sequences of numbers.

9. $R(x) \equiv 2^{x}$

$R(x)$ corresponds to the sequence of numbers consisting of only the number x (for $x > 0$).

10. $E(x) \equiv R(11) * x * R(13)$

$E(x)$ corresponds to the operation of placing in parentheses (11 and 13 are correlated with the primitive symbols "(" and ")").

11. $n\ \mathrm{Var}\ x \equiv (Ez)[\,13 < z < x\ \&\ \mathrm{Prim}(z)\ \&\ x = z^{n}]\ \&\ n \neq 0$

x is a VARIABLE OF THE n-TH TYPE.

12. $\mathrm{Var}(x)\ \equiv (En)[n \leqslant x\ \&\ n \mathrm{Var}\, x]$

x is a VARIABLE.

13. $\mathrm{Neg}(x) \equiv R(5) * E(x)$

$\mathrm{Neg}(x)$ is the NEGATION of x.

34a. For $0 < n \leqslant z$, where z is the number of distinct prime numbers dividing x. Observe that, for $n = z + 1$, $n\ Pr\ x = 0$.