1. Prove the following set is not regular.

\[ L = \{ a^n b^m c^{n+m} \mid n, m \geq 1 \}. \]

Here \( \Sigma = \{a, b, c\} \) is the alphabet set.

2. Prove the following set is not regular.

\[ L = \{ 0^n 1^{2n} \mid n \geq 1 \}. \]

3. Prove the following set is not regular.

\[ L = \{ a^i b^j \mid \gcd(i, j) = 1 \}. \]

(Hint: For any \( N \) given by an Adversary, choose \( z = a^{(N+1)!+1} b^{(N+1)!} \). Then the Adversary must give us a decomposition as guaranteed by the “regular version” of the PL, \( z = uvw \), where \( 0 < |v| \) and \( |uw| \leq N \), and so \( u,v \in a^* \). In particular \( |v| \leq N \). Then the pumped version is \( a^{(N+1)!+1+i|v|} b^{(N+1)!} \). This \( j \) is \( i-1 \) in the PL. Now choose \( i \) suitably...)

4. Here is a “strong version” of the Pumping Lemma for regular sets:

If \( L \) is regular, then there exists an \( N \geq 1 \), such that for every decomposition of \( x \in L \) and for every decomposition of \( x = x_1 x_2 x_3 \), where \( |x_2| \geq N \), there exists a further decomposition of \( x_2 = uvw \), where \( 0 < |v| \leq N \), such that for all \( i \geq 0 \), \( x_1 u^i v^i w x_3 \in L \).

Give a proof of this “strong version” of the Pumping Lemma.

Prove that the “strong version” of the Pumping Lemma implies the “regular version” of the Pumping Lemma.

5. In homework 2, you did problem 1.33 from the book.

Apply the algorithm given in class to find the minimum state DFA equivalent to the one you found in 1.33.

6. 1.35.

7. 1.46 part (a).

8. Find the equivalent classes of the relation \( R_L \) for the \( L \) defined in 1.46 part (a).

Recall that for any language \( L \), not necessarily a regular language, we defined the equivalence relation \( R_L \) on \( \Sigma^* \) as follows: \( x R_L y \) iff \( \forall z \in \Sigma^*, [xz \in L \iff yz \in L] \).
9. (extra credit) 1.57.

Note: You should get on to your homework as soon as possible. Don’t delay to the last minute.