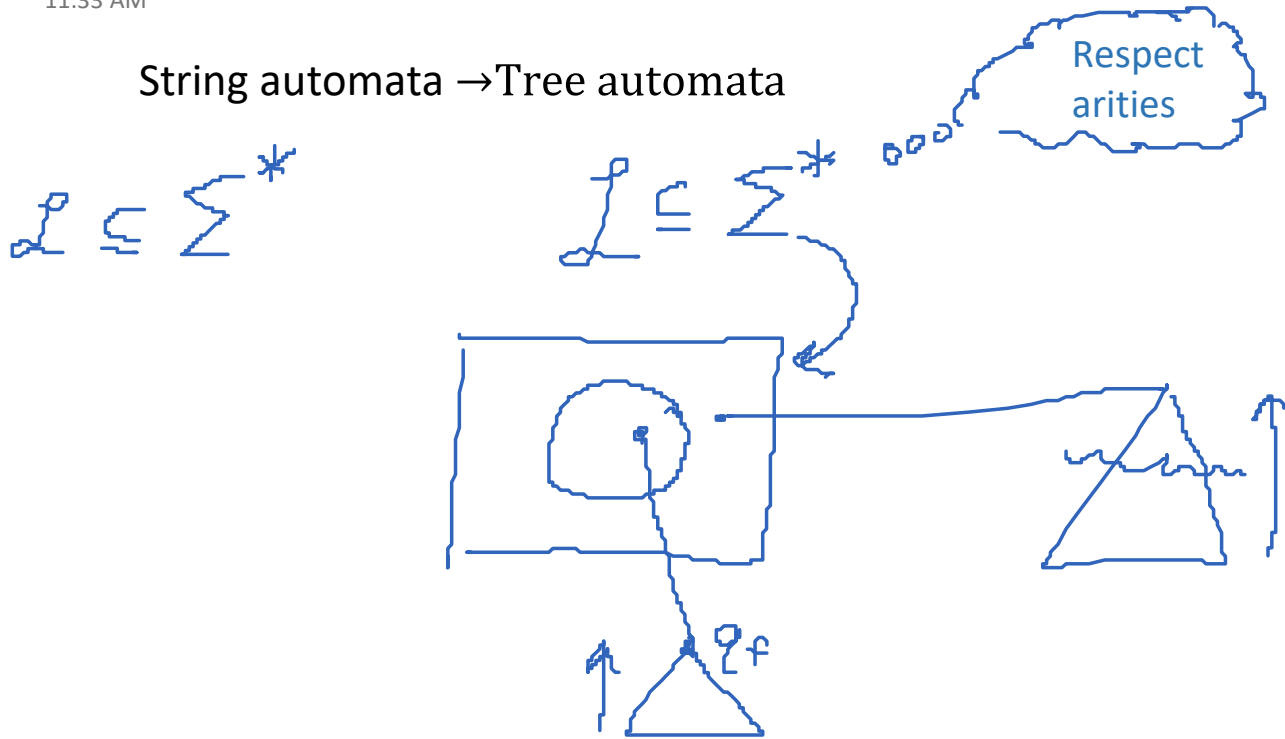


Bottom-up rewrite systems (BURS)

Thursday, March 12, 2020 11:33 AM

String automata → Tree automata



$$\text{Times}(q_1(X_1), q_5(X_2)) \rightarrow q_{17}(\text{Times}(X_1, X_2))$$

$\circ \square$

≈ Rebuilding

Suggests: Why not build something new?

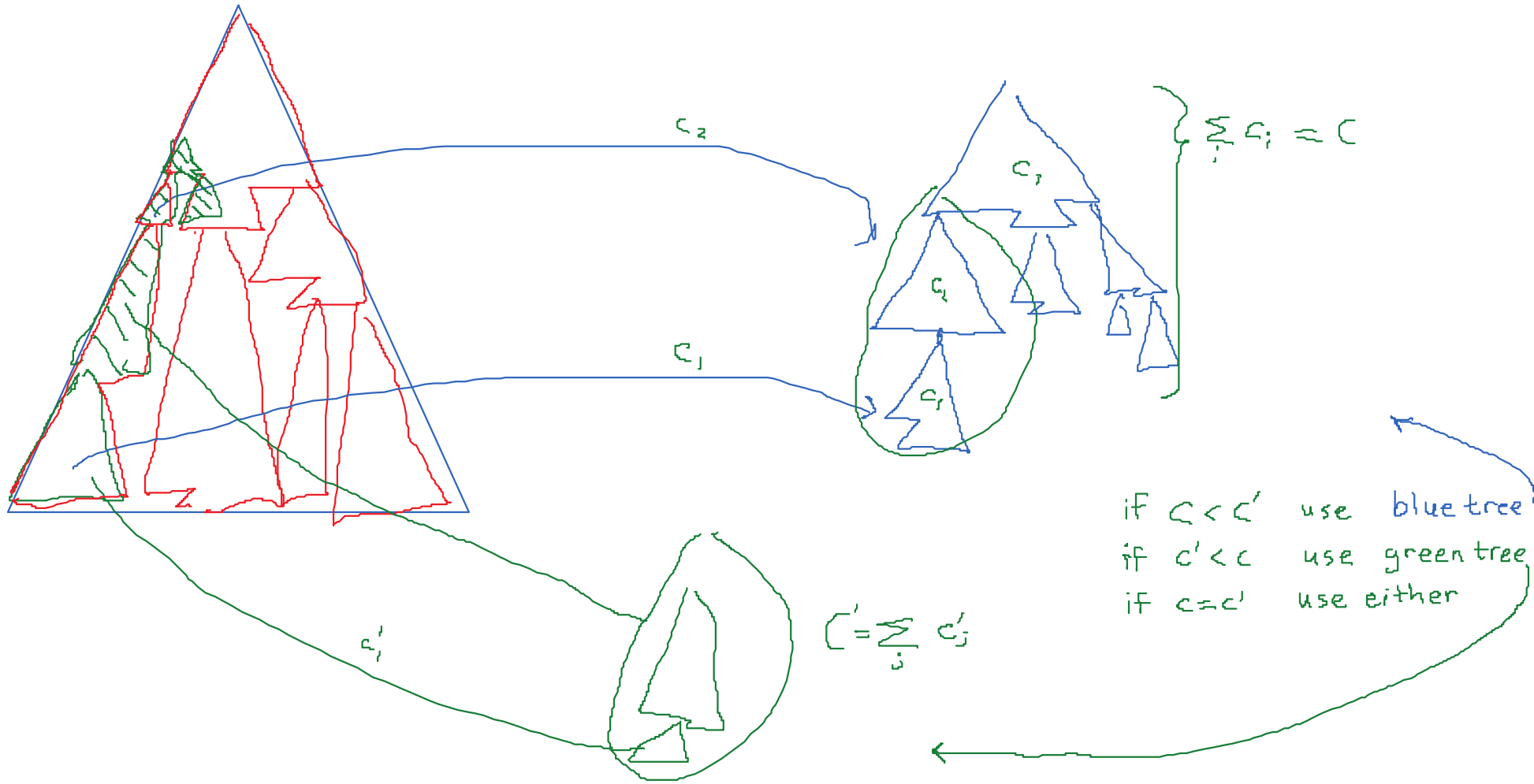
$$\text{Times}(q_{nz}(X_1), q_z(X_2)) \rightarrow q_z(0)$$

L_1
(e.g., IR language)

L_2
(e.g., Code language)

Dynamic programming (1)

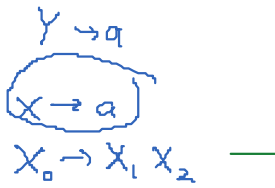
Friday, March 13, 2020 11:11 AM



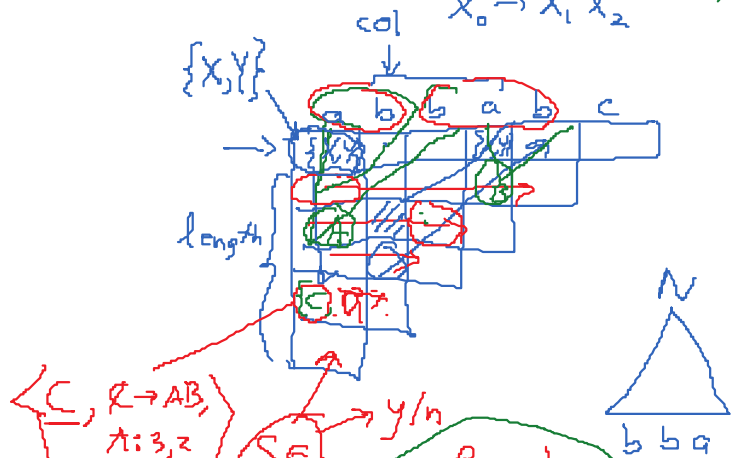
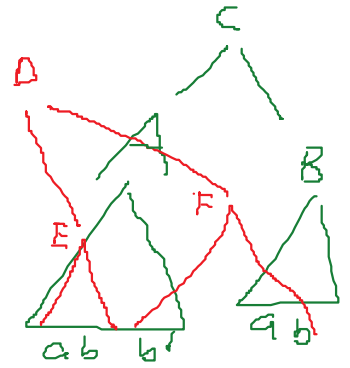
Dynamic programming (2)

Friday, March 13, 2020 11:19 AM

CYK Algorithm: $\omega \in L(G)$,
where G is a context-free grammar



$D \rightarrow EF$
 $C \rightarrow AB$



$C, R \rightarrow AB,$
 $A: 3, 2$
 $B: 3, 4$

witness

		B b	
		c a, b b b	
a			
b		1	3
a	a	2	3
a			
b			

(1) $\alpha \rightarrow \beta + C_1$ to convert "a" to "b"
 (2) $\alpha \rightarrow \beta + C_2$ to append "b"
 with (3) $\alpha \rightarrow \beta + C_3$ to delete "a"

String-to-string correction:
What is the cost of converting, e.g.,
abaab into cabbb (given costs c_1 , c_2 , and c_3
for converting one letter to another, appending
a letter, and deleting a letter, respectively)

BURS for Code Generation

Thursday, March 12, 2020 10:59 PM

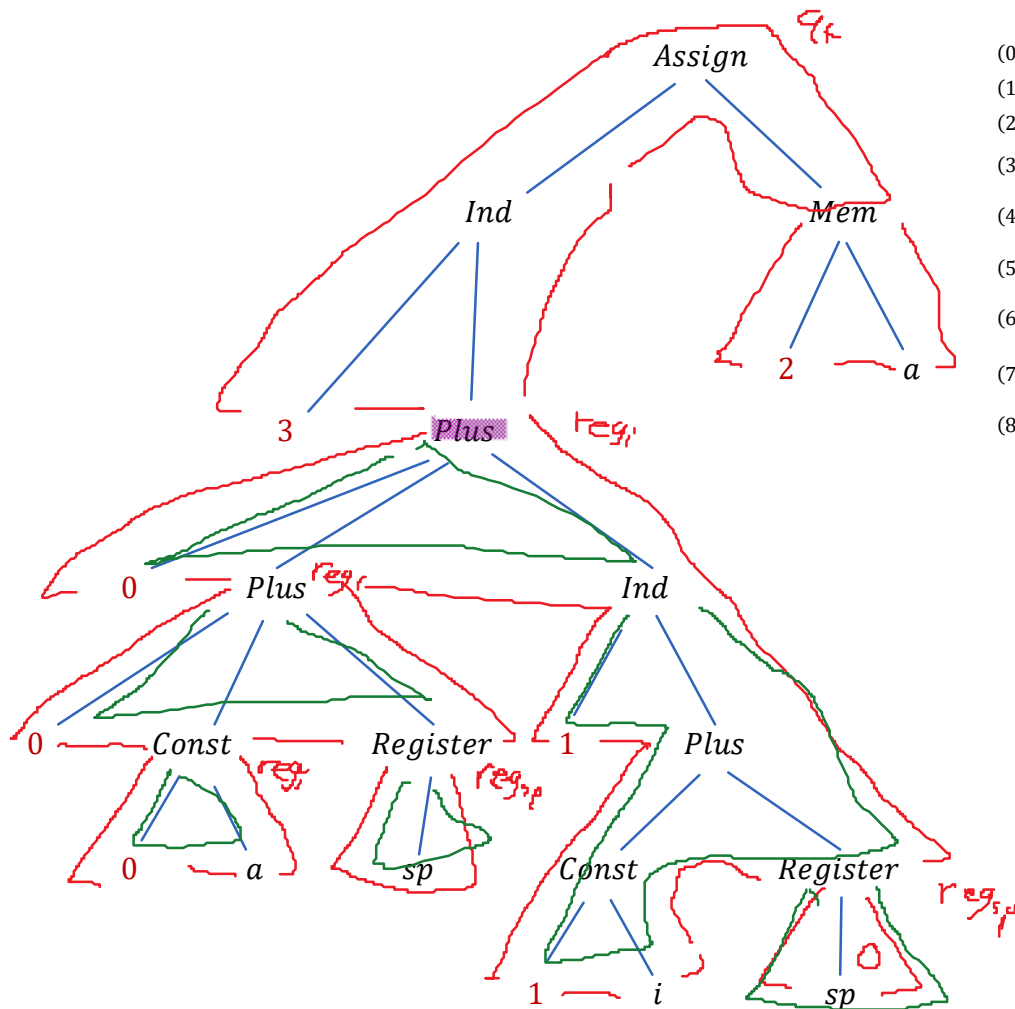
$$\begin{aligned}\Sigma_{IR} &= \{Assign^2, Mem^2, Const^2, Plus^3, Ind^2, Register^1, \dots, constants^0, \dots addresses^0\} \\ \Sigma_{Code} &= \{NoOp^0, R^1, Imm^1, Indirect^1, Mov^2, Cons^2, \dots, constants^0, addresses^0, \dots\} \\ Q &= \{reg_i, q_f\}\end{aligned}$$

In language *Code*, $Add(R(i), R(j))$ adds $R(i)$ to $R(j)$, and leaves the result in register $R(i)$

- (0) $Register(i) \xrightarrow{[0]} reg_i(NoOp())$
- (1) $Const(i, c) \xrightarrow{[2]} reg_i(Mov(R(i), Imm(c)))$
- (2) $Mem(i, a) \xrightarrow{[2]} reg_i(Mov(R(i), Indirect(a)))$
- (3) $Assign(Mem(*, a), reg_i) \xrightarrow{[2]} q_f(Cons(reg_i, Mov(Indirect(a, 0), R(i))))$
- (4) $Assign(Ind(*, reg_i), reg_j) \xrightarrow{[2]} q_f(Cons(reg_i, Cons(reg_j, Mov(Indirect(R(i), 0), R(j))))))$
- (5) $Ind(i, Plus(*, Const(*, c), reg_j)) \xrightarrow{[2]} reg_i(Cons(reg_j, Mov(R(i), Indirect(R(j), c))))$
- (6) $Plus(i, reg_i, Ind(*, Plus(*, Const(*, c), reg_j))) \xrightarrow{[2]} reg_i(Cons(reg_i, Cons(reg_j, Add(R(i), Indirect(R(j), c))))))$
- (7) $Plus(i, reg_i, reg_j) \xrightarrow{[1]} reg_i(Cons(reg_i, Cons(reg_j, Add(R(i), R(j))))))$
- (8) $Plus(i, reg_i, Const(1)) \xrightarrow{[1]} reg_i(Cons(reg_i, Incr(R(i))))$

Example from Twig paper (adapted)

Friday, March 13, 2020 12:13 AM



- (0) $Register(i) \xrightarrow{[0]} reg_i(NoOp())$
- (1) $Const(i, c) \xrightarrow{[2]} reg_i(Mov(R(i), Imm(c)))$
- (2) $Mem(i, a) \xrightarrow{[2]} reg_i(Mov(R(i), Indirect(a)))$
- (3) $Assign(Mem(*, a), reg_i) \xrightarrow{[2]} q_f(Cons(reg_i, Mov(Indirect(a, 0), R(i))))$
- (4) $Assign(Ind(*, reg_i), reg_j) \xrightarrow{[2]} q_f(Cons(reg_i, Cons(reg_j, Mov(Indirect(R(i), 0), R(j))))))$
- (5) $Ind(i, Plus(*, Const(*, c), reg_j)) \xrightarrow{[2]} reg_i(Cons(reg_j, Mov(R(i), Indirect(R(j), c))))$
- (6) $Plus(i, reg_i, Ind(*, Plus(*, Const(*, c), reg_j))) \xrightarrow{[2]} reg_i(Cons(reg_i, Cons(reg_j, Add(R(i), Indirect(R(j), c))))))$
- (7) $Plus(i, reg_i, reg_j) \xrightarrow{[1]} reg_i(Cons(reg_i, Cons(reg_j, Add(R(i), R(j)))))$
- (8) $Plus(i, reg_i, Const(1)) \xrightarrow{[1]} reg_i(Cons(reg_i, Incr(R(i))))$

$(0) (0) (1) (2) (6)$
 $0 + 0 + 1 + 2 + 2 = 5$

$0 + 2 + 0 + 1 + 2 + 2 = 7$
 $(0) (2) (0) (1) (5) (7)$

Normalization

Friday, March 13, 2020 11:45 AM

Multi-level patterns

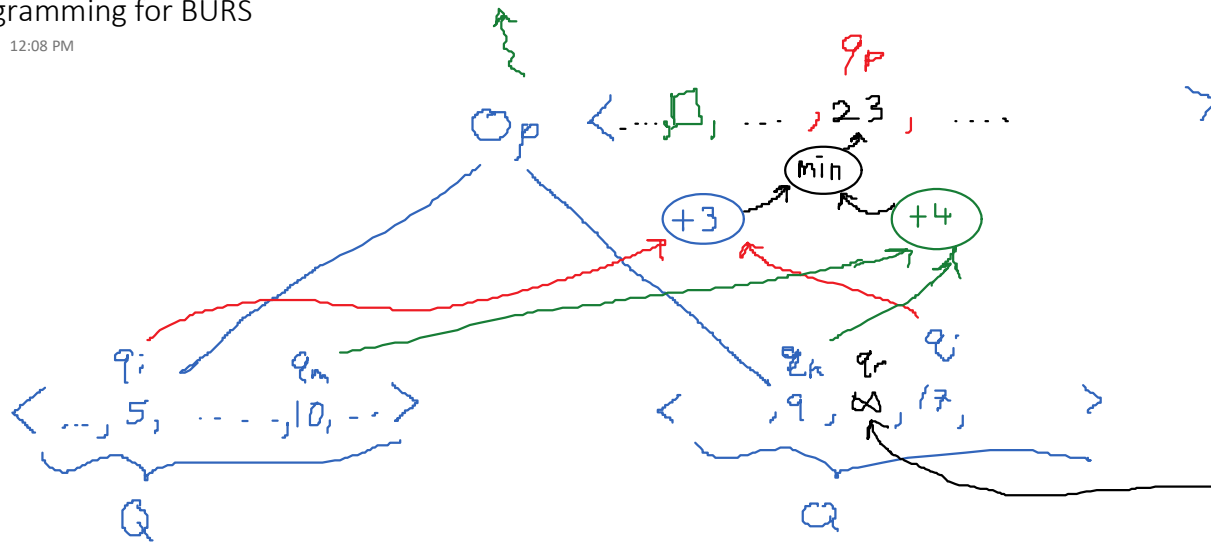
$$\text{Baz}(\text{Goo}(q_2, q_3), q_4) \xrightarrow{[w_1]} q_1(\dots)$$
$$\text{Baz}(q_5, \text{Gaz}(q_6)) \xrightarrow{[w_2]} q_9(\dots)$$

Single-level patterns

$$\text{Goo}(q_2, q_3) \xrightarrow{[0]} q_a(\dots) \quad // \text{ } q_a \text{ is a new "intermediate state"; the weight of such a rule is 0}$$
$$\text{Baz}(q_a, q_4) \xrightarrow{[w_1]} q_1(\dots) \quad // \text{ weight incurred only when topmost operator of original pattern reached}$$
$$\text{Gaz}(q_6) \xrightarrow{[0]} q_b(\dots)$$
$$\text{Baz}(q_5, q_b) \xrightarrow{[w_2]} q_9(\dots)$$

Dynamic programming for BURS

Friday, March 13, 2020 12:08 PM



∞ means no tiling of the subtree exists that leads to a tree in state q_r



$$\begin{array}{l} \xrightarrow{3} q_p(\text{Foo}(q_i, q_j)) \\ \hline 3 + 5 + 17 = 25 \end{array}$$

$$\begin{array}{l} \xrightarrow{4} q_p(\text{Bar}(q_m, q_k)) \\ 4 + 10 + 19 = 23 \\ 23 \end{array}$$

Last Thoughts

Friday, March 13, 2020 12:35 PM

String-automata/languages → Tree-automata/languages

The right conceptual model for BURS:

"A BURS is a weighted tree transducer, where weights are totally ordered (e.g., from $\mathbb{N} \cup \{\infty\}$)

Can test desirable properties, e.g., does the BURS handle all IR trees?

language containment: does $L(\text{IR}) \subseteq L(\text{Pat})$ hold?