

Problem 1

PART 1

In this section, given the dataword, we are expected to find the codeword. From the table, we have

$$C[0] = d[6] \wedge d[4] \wedge d[3] \wedge d[1] \wedge d[0]$$

$$C[1] = d[6] \wedge d[5] \wedge d[3] \wedge d[2] \wedge d[0]$$

$$C[2] = d[7] \wedge d[3] \wedge d[2] \wedge d[1]$$

$$C[3] = d[7] \wedge d[6] \wedge d[5] \wedge d[4]$$

- a. Dataword: **0000 0010**

From equations above

$$C[0] = 0 \wedge 0 \wedge 0 \wedge 1 \wedge 0 = 1$$

$$C[1] = 0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 = 0$$

$$C[2] = 0 \wedge 0 \wedge 0 \wedge 1 = 1$$

$$C[3] = 0 \wedge 0 \wedge 0 \wedge 0 = 0$$

Codeword: **0101 00000010**

- b. Dataword: **1010 1001**

From equations above

$$C[0] = 0 \wedge 0 \wedge 1 \wedge 0 \wedge 1 = 0$$

$$C[1] = 0 \wedge 1 \wedge 1 \wedge 0 \wedge 1 = 1$$

$$C[2] = 1 \wedge 1 \wedge 0 \wedge 0 = 0$$

$$C[3] = 1 \wedge 0 \wedge 1 \wedge 0 = 0$$

Codeword: **0010 10101001**

- c. Dataword: **0110 1110**

From equations above

$$C[0] = 1 \wedge 0 \wedge 1 \wedge 1 \wedge 0 = 1$$

$$C[1] = 1 \wedge 1 \wedge 1 \wedge 1 \wedge 0 = 0$$

$$C[2] = 0 \wedge 1 \wedge 1 \wedge 1 = 1$$

$$C[3] = 0 \wedge 1 \wedge 1 \wedge 0 = 0$$

Codeword: **0101 01101110**

- d. Dataword: **1101 1011**

From equations above

$$C[0] = 1 \wedge 1 \wedge 1 \wedge 1 \wedge 1 = 1$$

$$C[1] = 1 \wedge 0 \wedge 1 \wedge 0 \wedge 1 = 1$$

$$C[2] = 1 \wedge 1 \wedge 0 \wedge 1 = 1$$

$$C[3] = 1 \wedge 1 \wedge 0 \wedge 1 = 1$$

Codeword: **1111 11011011**

PART 2

In this section, given the codeword, we find the dataword assuming that at most 1 bit can be an error

- a. Codeword: **0000 0000111**

→ Given check bits = 0000

→ Given dataword = 0000 0111

Using table given, the original checkbits for this dataword are

$$C[0] = 0 \wedge 0 \wedge 0 \wedge 1 \wedge 1 = 0$$

$$C[1] = 0 \wedge 0 \wedge 0 \wedge 1 \wedge 1 = 0$$

$$C[2] = 0 \wedge 0 \wedge 1 \wedge 1 = 0$$

$$C[3] = 0 \wedge 0 \wedge 0 \wedge 0 = 0$$

$$\text{Syndrome} = \text{original check bits} \wedge \text{given checkbits} = 0000 \wedge 0000 = 0000 \rightarrow \text{No error}$$

- b. Codeword: **1011 1000111**

→ Given check bits = 1011

→ Given dataword = 1000 0111

Using table given, the original checkbits for this dataword are

$$C[0] = 0 \wedge 0 \wedge 0 \wedge 1 \wedge 1 = 0$$

$$C[1] = 0 \wedge 0 \wedge 0 \wedge 1 \wedge 1 = 0$$

$$C[2] = 1 \wedge 0 \wedge 1 \wedge 1 = 1$$

$$C[3] = 1 \wedge 0 \wedge 0 \wedge 0 = 1$$

$$\text{Syndrome} = \text{original check bits} \wedge \text{given checkbits} = 1100 \wedge 1011 = 0111 \rightarrow \text{Error in name3}$$

- c. Codeword: **0010 10101001**

→ Given check bits = 0010

→ Given dataword = 1010 1001

Using table given, the original checkbits for this dataword are

$$C[0] = 0 \wedge 0 \wedge 1 \wedge 0 \wedge 1 = 0$$

$$C[1] = 0 \wedge 1 \wedge 1 \wedge 0 \wedge 1 = 1$$

$$C[2] = 1 \wedge 1 \wedge 0 \wedge 0 = 0$$

$$C[3] = 1 \wedge 0 \wedge 1 \wedge 0 = 0$$

$$\text{Syndrome} = \text{original check bits} \wedge \text{given checkbits} = 0010 \wedge 0010 = 0000 \rightarrow \text{No error}$$

- d. Codeword: **0111 11001010**

→ Given check bits = 0111

→ Given dataword = 1100 1010

Using table given, the original checkbits for this dataword are

$$C[0] = 1 \wedge 0 \wedge 1 \wedge 1 \wedge 0 = 1$$

$$C[1] = 1 \wedge 0 \wedge 1 \wedge 0 \wedge 0 = 0$$

$$C[2] = 1 \wedge 1 \wedge 0 \wedge 1 = 1$$

$$C[3] = 1 \wedge 1 \wedge 0 \wedge 0 = 0$$

$$\text{Syndrome} = \text{original check bits} \wedge \text{given checkbits} = 0101 \wedge 0111 = 0010 \rightarrow \text{error in checkbit}$$

Problem 2

Multiplicand = 1100 0101 = -59

Multiplier = 0101 0101 = 85

Expected result = $-59 * 85 = -5015$

→ 2-bit Booth encoding of multiplier is **1 1 1 1** (Procedure illustrated in discussion session slides)

→ Actual Multiplication (sign extend multiplicand)

Multiplicand		2-bit encoding		
1111 1111 1100 0101	*	1	=	1111 1111 1100 0101 [PP1]
<i>Shift left by 2</i>				
1111 1111 0001 0100	*	1	=	1111 1111 0001 0100 [PP2]
<i>Shift left by 2</i>				
1111 1100 0101 0000	*	1	=	1111 1100 0101 0000 [PP3]
<i>Shift left by 2</i>				
1111 0001 0100 0000	*	1	=	1111 0001 0100 0000 [PP4]

→ Add all Partial products (PP1 to PP4)

Result is **11 1110 1100 0110 1001** = -5015 [Expected]

Problem 3

PART 1

17.384

(a) 17 → **1 0001**

(b) $0.384 = [0 * 2^{-1}] + [1 * 2^{-2}] + [1 * 2^{-3}] + [0 * 2^{-4}] + [0 * 2^{-5}] + [0 * 2^{-6}] + [1 * 2^{-7}] + \text{etc ...}$

(c) Hence, 0.384 in binary is **0.0110001001001101111**

(d) 17.384 would be **10001.0110001001001101111**

(e) Normalize to get **1.00010110001001001101111 * 2⁴**

(f) Excess 127 to exponent ⇒ $E = 4 + 127 = 131$. $E =$ **1000 0011**

(g) Since it is a positive number, $sign = 0$

(h) So **Sign = 0**, **exponent = 1000 0011** and **mantissa = 00010110001001001101111**

Hence representation is **0 10000011 00010110001001001101111**

PART 2

Given number is **1 10000001 100010010010000000000000**

- (a) We can observe sign bit is **1**
- (b) Exponent is **1000 0001 = 129**
Due to excess 127, we have $e = 129 - 127 = 2$ (i.e., $2+127$ will give us $E = 129$)
- (c) Mantissa is **100010010010000000000000**

Hence, number is

$$(-1)^{\text{sign}} * 1.(\text{mantissa}) * 2^{\text{exponent}}$$

$$(-1)^1 * 1.(100010010010000000000000) * 2^2$$

$$\text{Mantissa} = 100010010010000000000000 = 2^{-1} + 2^{-5} + 2^{-8} + 2^{-11} = 0.53564453125$$

Final result is

$$-(1 + 0.53564453125) * 2^2 = -6.142578125$$

The representation above is for the number **-6.142578125**

Problem 4

(a) Initial State

P0		P1		P2		Memory
State	Data	State	Data	State	Data	Data
I	0	I	0	I	0	7

- (b) **Operation:** Processor P0 executes **lw \$1, 100 (\$0)**
Action: P0 misses in the cache and reads the block from the memory
Memory State

P0		P1		P2		Memory
State	Data	State	Data	State	Data	Data
S	7	I	0	I	0	7

- (c) **Operation:** Processor P1 executes **addi \$1, \$0, 13** and **sw \$1, 100 (\$0)**
Action: P1 misses, reads the block from memory and invalidates P0
Memory State

P0		P1		P2		Memory
State	Data	State	Data	State	Data	Data
I	7	M	13	I	0	7

- (d) **Operation:** Processor P2 executes **lw \$1, 100 (\$0)**
Action: P2 misses, reads the block, P1 intervenes and supplies data, updating the memory
Memory State

P0		P1		P2		Memory
State	Data	State	Data	State	Data	Data
I	7	S	13	S	13	13

(e) **Operation:** Processor P1 executes `addi $1, $0, 19` and `sw $1, 100($0)`

Action: P1 hits, but does not have permission to write. P1 invalidates P2's copy

Memory State

P0		P1		P2		Memory
State	Data	State	Data	State	Data	Data
I	7	M	19	I	13	13