Tree-Structured Indexes

Chapter 9
Introduction

- As for any index, 3 alternatives for data entries \( k^* \):
  1. Data record with key value \( k \)
  2. \( \langle k, \text{rid of data record with search key value } k \rangle \)
  3. \( \langle k, \text{list of rids of data records with search key } k \rangle \)
- Choice is orthogonal to the indexing technique used to locate data entries \( k^* \).
- Tree-structured indexing techniques support both range searches and equality searches.
- \textbf{ISAM}: static structure; \textbf{B+ tree}: dynamic, adjusts gracefully under inserts and deletes.
Range Searches

- "Find all students with gpa > 3.0"
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search can be quite high.
- Simple idea: Create an `index' file.

Can do binary search on (smaller) index file!
Index file may still be quite large. But we can apply the idea repeatedly!

Leaf pages contain data entries.

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Comments on ISAM

- **File creation**: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then space for overflow pages.
- **Index entries**: `<search key value, page id>`; they `direct` search for *data entries*, which are in leaf pages.
- **Search**: Start at root; use key comparisons to go to leaf. 
  \[ \text{Cost} \propto \log_F N \ ; \ F = \# \ \text{entries/index pg}, \ N = \# \ \text{leaf pgs} \]
- **Insert**: Find leaf data entry belongs to, and put it there.
- **Delete**: Find and remove from leaf; if empty overflow page, de-allocate.

**Static tree structure**: *inserts/deletes affect only leaf pages.*

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Example ISAM Tree

- Each node can hold 2 entries; no need for ‘next-leaf-page’ pointers. (Why?)
After Inserting 23*, 48*, 41*, 42* ...
... Then Deleting 42*, 51*, 97*

Note that 51* appears in index levels, but not in leaf!
B+ Tree: The Most Widely Used Index

- Insert/delete at $\log_F N$ cost; keep tree height-balanced. ($F =$ fanout, $N =$ # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains $d \leq m \leq 2d$ entries. The parameter $d$ is called the order of the tree.
- Supports equality and range-searches efficiently.
Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf (as in ISAM).
- Search for 5*, 15*, all data entries >= 24* ... 

Based on the search for 15*, we know it is not in the tree!

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**B+ Trees in Practice**

- Typical order: 100. Typical fill-factor: 67%.
  - Average fanout = 133

- Typical capacities:
  - Height 4: $133^4 = 312,900,700$ records
  - Height 3: $133^3 = 2,352,637$ records

- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes
Inserting a Data Entry into a B+ Tree

- Find correct leaf \( L \).
- Put data entry onto \( L \).
  - If \( L \) has enough space, done!
  - Else, must **split** \( L \) (into \( L \) and a new node \( L_2 \))
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to \( L_2 \) into parent of \( L \).
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits “grow” tree; root split increases height.
  - Tree growth: gets **wider** or **one level taller at top**.
**Inserting 8∗ into Example B+ Tree**

- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.

- Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.

(Images and diagrams showing the insertion process are not transcribed here due to the limitations of text-based representation.)
Example B+ Tree After Inserting 8*

- Notice that root was split, leading to increase in height.
- In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.
Deleting a Data Entry from a B+ Tree

- Start at root, find leaf \( L \) where entry belongs.
- Remove the entry.
  - If \( L \) is at least half-full, *done!*
  - If \( L \) has only \( d-1 \) entries,
    - Try to re-distribute, borrowing from *sibling* (adjacent node with same parent as \( L \)).
    - If re-distribution fails, *merge* \( L \) and sibling.
- If merge occurred, must delete entry (pointing to \( L \) or sibling) from parent of \( L \).
- Merge could propagate to root, decreasing height.
Example Tree After (Inserting 8*, Then) Deleting 19* and 20* ...

- Deleting 19* is easy.
- Deleting 20* is done with re-distribution.
  Notice how middle key is copied up.
... And Then Deleting 24*

- Must merge.
- Observe ‘toss’ of index entry (on right), and ‘pull down’ of index entry (below).
Example of Non-leaf Re-distribution

- Tree is shown below during deletion of $24^*$. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.
After Re-distribution

- Intuitively, entries are re-distributed by ‘pushing through’ the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we’ve re-distributed 17 as well for illustration.
Prefix Key Compression

- Important to increase fan-out. (Why?)
- Key values in index entries only `direct traffic`; can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt, David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.

- Insert/delete must be suitably modified.
Bulk Loading of a B+ Tree

- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- **Bulk Loading** can be done much more efficiently.
- **Initialization**: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.
Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!
Summary of Bulk Loading

- Option 1: multiple inserts.
  - Slow.
  - Does not give sequential storage of leaves.

- Option 2: **Bulk Loading**
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control “fill factor” on pages.
A Note on `Order’

- **Order (d)** concept replaced by physical space criterion in practice (`at least half-full’).
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (**duplicates**) can lead to variable-sized data entries (if we use Alternative (3)).
Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.
- ISAM is a static structure.
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; \( \log_F N \) cost.
  - High fanout (F) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.
Summary (Contd.)

- Typically, 67% occupancy on average.
- Usually preferable to ISAM, modulo locking considerations; adjusts to growth gracefully.
- If data entries are data records, splits can change rids!

- Key compression increases fanout, reduces height.
- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.